

# Three-echelon Inventory Model with Defective Product and Rework Considerations under Credit Period

M.F. Yang, M.C. Lo, Y.T. Chou, and W.H. Chen

**Abstract**—Multi-echelon transaction is a common situation in the supply chain nowadays. This research is to formulate a three-echelon integrated inventory model under defective products, reworking and credit period consideration. An algorithm and numerical analysis are used to observe the effect of defective rate and credit period time to the inventory policy and total profit.

**Index Terms**—Defective products, Reworking, Credit period

## I. INTRODUCTION

Recently, in a competitive market, how to satisfy customers' demand is one of critical issues for companies. In addition to constant and high quality, enough stock is an important fundamental factor to affect the level of customer satisfaction. Enterprises should frame appropriate inventory policies to perform inventory management well. Inventory policy describes how to stock inventory and when to replenish. It determines: (1) How much product is stored at a site, (2) when replenishment orders are generated, and (3) what quantity is replenished [1]. Started from Harris's [2] economic order quantity (EOQ) model, the researchers as well as practitioners are interested in optimal inventory policy. Harris [2] focused on inventory decisions of an individual firm, yet from supply chain management's (SCM) point of view, collaborating closely with the members of supply chain is certainly necessary. In the network (supply chain), each node's (the member in the supply chain) position is corresponding to its relative position in reality. These nodes serve external demand which generates orders to the down-stream echelon. Meanwhile, they are served by external supply which responds to the orders of the up-stream echelon [3]. Ben-Daya et al. [4] pointed out that the reason to collaborate with the other members of supply chain is to remain competitive. Better collaboration with customers and

suppliers will not only provide a better service to satisfy customer's demand but reduce the total cost of the whole supply.

In 1950s, Arrow et al. [5] have been focused on multi-echelon inventory problem. Burns and Sivazlian [6] investigated the dynamic response of a multi-echelon supply chain to various demands placed upon the system by a final consumer. Van der Heijden [7] determined a simple inventory control rule for multi-echelon distribution systems under periodic review without lot sizing. Pal et al. [8] developed a three-layer integrated production-inventory model considering out-of-control quality occurs in supplier and manufacturer stage. The defective products are reworked at a cost after the regular production time. Chung et al. [9] combined deteriorating items with two levels of trade credit under three-layer condition in supply chain system. A new economic production quantity (EPQ) inventory is proposed to minimize the total cost.

Yield rate is an important factor in manufacturing industry. In practice, imperfect production could be the result of insufficient process control, wrongly planned maintenance, inadequate work instructions, or damages that occur during handling [10]. High defective rate will not only waste production costs but also pay more inspecting costs and repair costs, even cause the shortage. In early researches, defective product was rarely considered in economic ordering quantity (EOQ) model; however, defective production is a common condition in real life. Schwaller [11] added fixed defective rate and inspecting costs to the traditional EOQ model. Salameh and Jaber [12] pointed that all products should be divided into good products and defective products. They also found EOQ will increase if defective products increase. Lin [13] assumed a random number of defective goods in buyer's arriving order lot with partial lost sales for the mixtures of distributions of the controllable lead time demand to accommodate more practical features of the real inventory systems.

Credit period is a common business strategy between vendors and buyers. It will bring additional interest or opportunity cost to each other, hence delayed period is a critical issue that researchers should consider when developing inventory models. In traditional EOQ assumptions, the buyer has to pay immediately when the vendor delivers products to the buyer; however, in real business transactions, the vendor usually gives a fixed delayed period to reduce the stress of capital. During the period, the buyer can keep selling products without paying the vendor; they can also earn extra interest from sales. Goyal [14] developed an EOQ model with delay in payments. Two situations were discussed in the research;

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time interval between successive orders was longer than or equal to permissible delay in settling accounts, or time interval between successive orders was shorter than permissible delay in settling accounts. Sarkar et al. [15] derived an EOQ model for various types of time-dependent demand when delay in payment and price discount are permitted by suppliers to retailers.

Our purpose in this article is to maximize the expected joint total profit. We will develop three-echelon inventory model with defective rate and rework considerations under credit period situation. We first defined the parameters and assumptions in Section II, and then we started to develop the integrated inventory model in Section III. In Section IV, we solved the model to get the optimal solution and showed numerical examples in Section V. In the end, we summarized the conclusions in Section VI.

## II. NOTATIONS AND ASSUMPTIONS

To develop a three-echelon inventory model with defective rate and permissible delay in payments, we divided the expected joint total annual profit of the model into three parts which are the annual profit of the supplier, the manufacturer, and the retailer, The following notations and assumptions below are used to develop the model:

### A. Notations

To establish the mathematical model, the following notations and assumptions are used. The notations are shown in Table I.

Table I. the parameters and the decision variable

$Q$  = Economic delivery quantity, a decision variable

$n$  = The number of lots delivered in a production cycle from the manufacturer to the retailer, a positive integer, a decision variable

### Supplier side

$P_s$  = Supplier's purchasing cost per unit

$A_s$  = Supplier's ordering cost per order

$h_s$  = Supplier's annual holding cost per unit

$I_{sp}$  = Supplier's opportunity cost per dollar per year

### Manufacturer side

$P$  = Manufacturer's production rate

$X$  = Manufacturer's permissible delay period

$P_m$  = Manufacturer's purchasing cost per unit

$A_m$  = Manufacturer's ordering cost per order

$Z$  = The probability of defective products from manufacturer

$W$  = Manufacturer's inspecting cost per unit

$G$  = Manufacturer's repair cost per unit

$t_m$  = The time for reworking defective products at manufacturer per unit

$F_m$  = Manufacturer's transportation cost per shipment

$h_m$  = Manufacturer's annual holding cost per unit

$I_{mp}$  = Manufacturer's opportunity cost per dollar per year

$I_{me}$  = Manufacturer's interest earned per dollar per year

### Retailer side

$D$  = Average annual demand per unit time

$Y$  = Retailer's permissible delay period

$P_c$  = Retailer's selling price per unit

$P_r$  = Retailer's purchasing cost per unit

$A_r$  = Retailer's ordering cost per order

$F_r$  = Retailer's transportation cost per shipment

$h_r$  = Retailer's annual holding cost per unit

$I_{rp}$  = Retailer's opportunity cost per dollar per year

$I_{re}$  = Retailer's interest earned per dollar per year

$TP_s$  = Supplier's total annual profit

$TP_m$  = Manufacturer's total annual profit

$TP_r$  = Retailer's total annual profit

$EJTP_i$  = The expected joint total annual profit,  $i = 1, 2, 3, 4^*$

\*"i" represents four different cases due to the relationship of replenishment time and permissible payment period of manufacturer and the relationship of replenishment time and permissible payment period of retailer. We will have more detailed discussion below in Section 3.

### B. Assumptions

- (i) This supply chain system consists of a single supplier, a single manufacturer, and a single retailer for a single product.
- (ii) Economic delivery quantity multiplies by the number of delivery per production run is economic order quantity (EOQ).
- (iii) Shortages are not allowed.
- (iv) The sale price must not be less than the purchasing cost at any echelon,  $P_c > P_r > P_m > P_s$ .
- (v) Defective products only occur in the production process.
- (vi) The inspecting time is ignored and defective products can be inspected immediately.
- (vii) Defective products are repaired after the production process is end.
- (viii) The time horizon is infinite.

## III. MODEL FORMULATION

In this section, we discuss the model of supplier, manufacture, and retailer and combined them into an integrated inventory model.

### A. The supplier's total annual profit

In each production run, the supplier's profit includes sales revenue, purchasing cost, ordering cost, holding cost, and opportunity cost. The supplier's total annual profit consists of the following elements.

- (i) Sales revenue =  $P_m D$
- (ii) Purchasing cost =  $P_s D$
- (iii) Ordering cost =  $\frac{A_s D}{nQ}$
- (iv) Holding cost =  $\frac{h_s D n Q}{2P}$

Under the condition of permissible delay in payments, the supplier offers the manufacturer a payment period. In the other words, the supplier doesn't receive the payment immediately. Opportunity cost will result from no interest during the period.

- (v) Opportunity cost =  $P_m I_{sp} D X$

The supplier's total annual profit is:

$$TP_s = P_m D - P_s D - \frac{A_s D}{nQ} - \frac{h_s D n Q}{2P} - P_m I_{sp} D X \quad (1)$$

### B. The manufacturer's total annual profit

In each production run, the manufacturer's profit includes sales revenue, purchasing cost, ordering cost, holding cost, transportation cost, inspecting cost, repair cost, interest income, and opportunity cost. The manufacturer's total annual profit consists of the following elements.

- (i) Sale revenue =  $P_r D$
- (ii) Purchasing cost =  $P_m D$

(iii) Ordering cost =  $\frac{A_m D}{nQ}$

When the production process is over, the defective products will be picked up to be reworked. The manufacturer's inventory level is shown in Fig. I. The average inventory  $\bar{I}_v$  in each production run is

$$\bar{I}_v = nQ \left[ \frac{Q}{P} + \frac{(n-1)Q}{D} - \frac{nQ}{2P} \right] - t_m Z n Q^2 - \frac{(n-1)nQ^2}{2D} = nQ^2 \left[ \frac{(2-n)}{2P} + \frac{(n-1)}{2D} \right] - t_m (ZnQ)^2$$

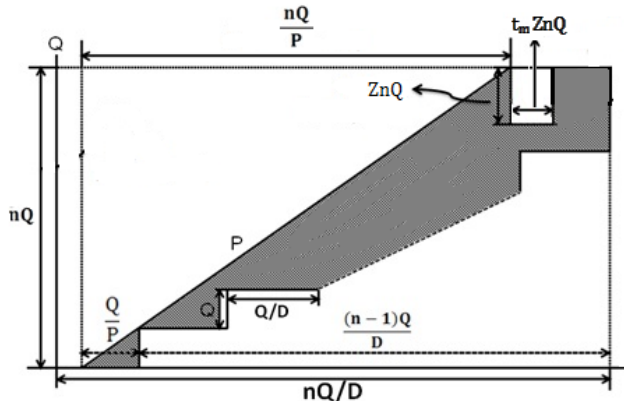


Fig. I. The manufacturer's inventory level

Thus, the annual holding cost is

(iv) Holding cost =  $\frac{h_m Q}{2} \left[ \frac{(2-n)}{P} + (n-1) - 2t_m Z^2 n D \right]$

(v) Inspecting cost =  $WD$

(vi) Repair cost =  $GZD$

(vii) Transportation cost =  $\frac{F_m D}{nQ}$

The situation at the manufacturer and the retailer's side is the same as the supplier and the manufacturer's side

(viii) Opportunity cost =  $P_r I_{mp} DY$

The length of payment period will affect the amount of interest income and opportunity cost. As the items are sold out before the deadline of the payment period, the manufacturer earns interest by sales revenue (see Fig. II). Contrarily, as the items are sold out after the deadline of the payment period, the manufacturer still earns interest by sales revenue during the replenishment time, yet the items in stock result in opportunity cost (see Fig. III).

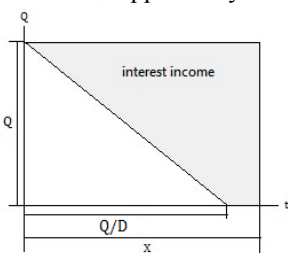


Fig. II.  $Q/D < X$

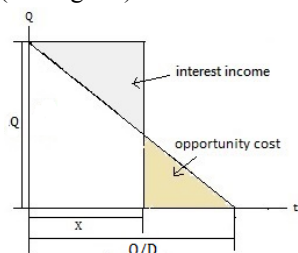


Fig. III.  $Q/D \geq X$

**Case 1. If  $Q/D < X$ , then**

(ix) Interest income =  $P_r I_{me} \left( DX - \frac{Q}{2} \right)$

The manufacturer's total annual profit is:

$$TP_{m1} = P_r D - P_m D - \frac{A_m D}{nQ} - \frac{h_m Q}{2} \left[ \frac{(2-n)}{P} + (n-1) - 2t_m Z^2 n D \right] - WD - GZD - \frac{F_m D}{nQ} + P_r I_{me} \left( DX - \frac{Q}{2} \right) \quad (2)$$

**Case 2. If  $Q/D \geq X$ , then**

(x) Interest income =  $\frac{P_r I_{me} (DX)^2}{2Q}$

(xi) Opportunity cost =  $\frac{P_m I_{mp} (nQ - DX)^2}{2Q}$

The manufacturer's total annual profit is:

$$TP_{m2} = P_r D - P_m D - \frac{A_m D}{nQ} - \frac{h_m Q}{2} \left[ \frac{(2-n)}{P} + (n-1) - 2t_m Z^2 n D \right] - WD - GZD - \frac{F_m D}{nQ} + \frac{P_r I_{me} (DX)^2}{2Q} - \frac{P_m I_{mp} (nQ - DX)^2}{2Q} \quad (3)$$

**C. The retailer's total annual profit**

In each production run, the retailer's profit includes sales revenue, purchasing cost, ordering cost, holding cost, transportation cost, interest income, and opportunity cost. The retailer's total annual profit consists of the following elements.

(i) Sales revenue =  $P_c D$

(ii) Purchasing cost =  $P_r D$

(iii) Ordering cost =  $\frac{A_r D}{nQ}$

(iv) Holding cost =  $\frac{h_r Q}{2}$

(v) Transportation cost =  $\frac{F_r D}{Q}$

At the supplier's side, the length of payment period will also affect the amount of interest income and opportunity cost.

**Case 1. If  $Q/D < Y$ , then**

(vi) Interest income =  $P_c I_{re} \left( DY - \frac{Q}{2} \right)$

The retailer's total annual profit is:

$$TP_{r1} = P_c D - P_r D - \frac{A_r D}{nQ} - \frac{h_r Q}{2} - \frac{F_r Q}{2} + P_c I_{re} \left( DY - \frac{Q}{2} \right) \quad (4)$$

**Case 2. If  $Q/D \geq Y$ , then**

(vii) Interest cost =  $\frac{P_c I_{re} (DY)^2}{2Q}$

(viii) Opportunity cost =  $\frac{P_r I_{rp} (Q - DY)^2}{2Q}$

The retailer's total annual profit is:

$$TP_{r2} = P_c D - P_r D - \frac{A_r D}{nQ} - \frac{h_r Q}{2} - \frac{F_r Q}{2} + \frac{P_c I_{re} (DY)^2}{2Q} - \frac{P_r I_{rp} (Q - DY)^2}{2Q} \quad (5)$$

**D. The expected joint total annual profit**

According to the four different conditions, the expected joint total annual profit function,  $EJTP_i(Q, n)$ , can be expressed as

$$EJTP_i = \begin{cases} EJTP_1 = TP_s + TP_{m1} + TP_{r1} & \text{if } Q/D < X, Q/D < Y \\ EJTP_2 = TP_s + TP_{m2} + TP_{r1} & \text{if } Q/D \geq X, Q/D < Y \\ EJTP_3 = TP_s + TP_{m1} + TP_{r2} & \text{if } Q/D < X, Q/D \geq Y \\ EJTP_4 = TP_s + TP_{m2} + TP_{r2} & \text{if } Q/D \geq X, Q/D \geq Y \end{cases}$$

where

$$EJTP_1(n, Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ} (A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2} \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n-1 - 2t_m Z^2 n D) + h_r \right] - P_m I_{sp} DX - P_r I_{mp} DY + P_r I_{me} \left( DX - \frac{Q}{2} \right) + P_c I_{re} \left( DY - \frac{Q}{2} \right) \quad (6)$$

$$EJTP_2(n, Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ} (A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2} \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n-1 - 2t_m Z^2 n D) + h_r \right] - P_m I_{sp} DX - P_r I_{mp} DY + \frac{P_r I_{me} (DX)^2}{2Q} - \frac{P_m I_{mp} (Q - DX)^2}{2Q} + P_c I_{re} \left( DY - \frac{Q}{2} \right) \quad (7)$$

$$EJTP_3(n, Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ} (A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2} \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n-1 - 2t_m Z^2 n D) + h_r \right] - P_m I_{sp} DX - P_r I_{mp} DY +$$

$$P_r I_{me} \left( DX - \frac{Q}{2} \right) + \frac{P_c I_{re} (DY)^2}{2Q} - \frac{P_r I_{rp} (Q - DY)^2}{2Q} \quad (8)$$

$$EJTP_4(n, Q) = D(P - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ} (A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2} \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n - 1 - 2t_m Z^2 n D) + h_r \right] - P_m I_{sp} DX - P_r I_{mp} DY + \frac{P_r I_{me} (DX)^2}{2Q} - \frac{P_m I_{mp} (Q - DX)^2}{2Q} + \frac{P_c I_{re} (DY)^2}{2Q} - \frac{P_r I_{rp} (Q - DY)^2}{2Q} \quad (9)$$

IV. SOLUTION PROCEDURE

In order to maximize  $EJTP_i(Q, n)$ , we set  $[\partial EJTP_i(Q, n) / \partial Q] = 0$  and obtain the economic value of  $Q = Q_1^*, Q_2^*, Q_3^*$ , and  $Q_4^*$ . To prevent the equations are too long to read, we set  $[2D(A_s + A_m + F_m + A_r + F_r n)] = U$ .

$$Q_1^* = \left( \frac{U}{n \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n - 1 - 2t_m Z^2 n D) + h_r \right] + P_r I_{me} + P_c I_{re}} \right)^{0.5} \quad (10)$$

$$Q_2^* = \left( \frac{U + n(P_m I_{mp} - P_r I_{me})(DX)^2}{n \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n - 1 - 2t_m Z^2 n D) + h_r \right] + P_m I_{mp} + P_c I_{re}} \right)^{0.5} \quad (11)$$

$$Q_3^* = \left( \frac{U + n(P_r I_{rp} - P_c I_{re})(DY)^2}{n \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n - 1 - 2t_m Z^2 n D) + h_r \right] + P_r I_{me} + P_r I_{rp}} \right)^{0.5} \quad (12)$$

$$Q_4^* = \left( \frac{U + n[(P_m I_{mp} - P_r I_{me})(DX)^2 + (P_r I_{rp} - P_c I_{re})(DY)^2]}{n \left[ \frac{h_s D n + h_m (2-n)}{P} + h_m (n - 1 - 2t_m Z^2 n D) + h_r \right] + P_m I_{mp} + P_r I_{rp}} \right)^{0.5} \quad (13)$$

Algorithm

In order to obtain the optimal values of  $EJTP_i(Q, n)$ , follow these steps:

- Step 1. Set  $n = n_i = 1$  and substitute into (10), (11), (12), and (13) to obtain  $Q_1, Q_2, Q_3$ , and  $Q_4, \forall_i = 1, 2, 3$ , and 4.
- Step 2. Find  $EJTP_i$  by substituting  $n_i$  and  $Q_i$  into (6), (7), (8), (9),  $\forall_i = 1, 2, 3$ , and 4.
- Step 3. Let  $n_i = n_i + 1$  and repeat step 1 to step 2 until  $EJTP_i(n_i) > EJTP_i(n_i + 1)$ . The optimal  $n_i^* = n_i; Q_i^* = Q(n_i^*), \forall_i = 1, 2, 3$ , and 4.
- Step 4. Compute the replenishment time and compare with payment period. Examine the relationship whether is conform to the situation and select the most expected joint total profit.

V. NUMERICAL EXAMPLE

A numerical example is used to demonstrate the proposed models in this section.

Given  $D = 1000$  units/year,  $P_s = 20$  \$/per unit,  $A_s = 50$  \$/per order,  $h_s = 2$  \$/per unit,  $I_{sp} = 0.02$  \$/year,  $P = 2000$  units/year,  $X = 0.205479$  year (i.e 75 days),  $P_m = 35$  \$/per unit,  $A_m = 70$  \$/per order,  $h_m = 3$  \$/per unit,  $F_m = 50$  \$/per shipment,  $Z = 0.1$ ,  $W = 0.5$  \$/per unit,  $G = 1$  \$/per unit,  $t_m = 0.000274$  year/per unit (i.e 0.1 day),  $I_{mp} = 0.035$  \$/year,  $I_{me} = 0.03$  \$/year,  $Y = 0.041096$  year (i.e 15 days),  $P_r = 50$  \$/per unit,  $P_c = 70$  \$/per unit,  $A_r = 100$  \$/per order,  $F_r = 65$  \$/per shipment,  $h_r = 5$  \$/per order,  $I_{rp} = 0.04$  \$/year,  $I_{re} = 0.035$  \$/year; following the equation and algorithm already given in this paper, the economic ordering policy is shown in Table II.

Table II. The optimal solution for given parameters

	Case 1	Case 2	Case 3	Case4
$n_i^*$	2	2	2	2
$Q_i^*$	162	169	173	172
$EJTP_i$	47109.71	47114.03	47126.05	47134.49
$Q_i^*/D$	0.162	0.169	0.173	0.172
Optimal solution			⊗	

We also did sensitivity analysis to observe the economic ordering policies under different values of  $X, Y$ , and  $Z$ , and showed the results in Table III and Table IV.

Table III. The inventory policy under different X and Z

X↓	Z→	0.1	0.2	0.3
65 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47104.13	47012.58	46925.99
75 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47126.05	47034.5	46947.91
85 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47147.96	47056.41	46969.83

Table IV. The inventory policy under different Y and Z

Y↓	Z→	0.1	0.2	0.3
10 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47121.4	47029.85	46943.27
15 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47126.05	47034.5	46947.91
20 days	$n_i^*$	2	2	2
	$Q_i^*$	173	173	174
	$EJTP_i$	47131.18	47039.03	46953.04

VI. CONCLUSION

In this paper, we formulated three-echelon inventory model with defective rate and rework considerations under credit period situation. Both defective rate and credit period are important factor to impact the inventory policy. From Table 3 and Table 4, we can know that

- (i) as the credit period ( $X$  and  $Y$ ) increases, there is a marginal increase in expected joint total profit. The economic delivery quantity remains the same when  $X$  increases while there is little decrease when  $Y$  increases.
- (ii) as defective rate ( $Z$ ) increases, of course, there is significant decrease in expected joint total profit, and the economic delivery quantity increases softly.
- (iii) changing  $X$  has a greater effect than changing  $Y$ .

From managerial point of view, the decision maker or the enterpriser should put more attention to the length of the supplier's credit period more than the manufacturer's. Although offering a credit period to the down-stream firm leads the cost for the up-stream firm, it can release the pressure of the down-stream firm's capital using. If the down-stream firm controls the sale revenue well, there will be additional interest that enhances the performance of the

whole supply chain. Also, reworking the defective products is important in production process. Defective products cause additional time and cost on purchasing and production. The rate of storage will be higher, too. The decision maker should reach the situation of production line at any time.

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