Optimization of the Size of Minimal Invariant Ellipsoid with Providing the Desired Modal Properties

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***Abstract*— In view of rapid development of computer technologies digital systems of automatic control are installed on the modern ships for performance of various manoeuvres at optimal trajectories taking into account the features of the ship and active disturbances. In the paper, the problem of suppression of exogenous disturbance, about which we have no information except its boundedness, is considered. In this situation, it requires to choose the parameters of the controller, which will give the best possible result under the worst bounded disturbance, and also ensure the implementation of additional requirements to dynamic processes.**

***Index Terms*— control law, stability, invariant ellipsoid, optimization, disturbances**

I. Introduction

The level of development of modern computer technologies, the continuous increasing of computing power, the appearance of new program tools – all this contributes to the widespread automation of the functioning of various marine objects using autonomous onboard systems.

In particular, this allows installing on modern marine moving objects highly efficient systems for automatic motion control, thereby facilitating and making it more safe trips and expeditions in the open sea. Such systems can reduce the appearance of accidents due to human factors, conserve energy resources, and precisely follow to the specified routes, avoiding various obstacles, to compensate for the influence of the external disturbances with regard to the peculiarities of the dynamics of the vessel.

This raises a number of substantive and formal problems associated with systems engineering and automatic control of the motion, namely, the problem of minimizing the time of manoeuvre and fuel consumption, the problem of construction of optimal trajectories of motion, the problem of suppression of external influences caused by gusts of wind and rough sea. Often these problems effectively solved separately in [1] and [2], but in practice, we often deal with multiple tasks simultaneously.

Special attention should be paid to the situation when in the framework of a formalized setting external disturbances are uncertainties, and control system must not simply to compensate such influences, in a sense, but also to ensure the implementation of additional requirements to dynamic processes. This fact significantly complicates the analysis and design of control systems, one of the main functions of which is to suppress the influence of impacts on the vessel. Some aspects of suppression of bounded external disturbances are considered in [3–8].

The theory of multipurpose control laws' synthesis with the set of additional modal requirements to the dynamic process is presented in papers [9–12, 14–21].

Particularly, in the paper much prominence is given to the questions, associated with computer synthesis and modelling of the control laws those suppress bounded exogenous disturbances. The example of modelling of the control system for the marine ship with displacement ton 6000 is performed.

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II. Problem Statement

Let consider linear model of marine vessel

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{\delta} + \mathbf{D}\mathbf{w}(t), \\ \dot{\boldsymbol{\delta}} &= \mathbf{u}, \\ \mathbf{y} &= \mathbf{C}\mathbf{x}, \end{aligned} \tag{1}$$

where $\mathbf{x} \in E^n$ is a state vector (it defines the deviation from the equilibrium position), $\mathbf{y} \in E^{k_y}$ is a vector of controllable coordinates, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{C}$ and $\mathbf{M}$ are constant matrices of corresponding dimensions.

Feedback for the system (1) we form as a state controller

$$\mathbf{u} = \mathbf{K}_x\mathbf{x} + \mathbf{K}_\delta\boldsymbol{\delta} = \mathbf{K}\begin{pmatrix}\mathbf{x} \\ \cdots \\ \boldsymbol{\delta}\end{pmatrix}, \ \mathbf{K} = \left(\mathbf{K}_x \ \vdots \ \mathbf{K}_\delta\right), \tag{2}$$

where matrices $\mathbf{K}_x$ and $\mathbf{K}_\delta$ are constant.

Further, the set of stabilizing controllers (2) will be denoted as Ω_k, identifying it with the set of such matrices $\mathbf{K}$, for which the characteristic polynomial

$$\Delta_3(s,\mathbf{K}) = \det\left(\mathbf{E}_{n+m}s - \mathbf{A}_0 - \mathbf{B}_0\mathbf{K}\right) \tag{3}$$

of the closed-loop system (1), (2) is hurwitzian. Here the following notation is used:

$$\mathbf{A}_0 = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{pmatrix},\ \mathbf{B}_0 = \begin{pmatrix} \mathbf{0} \\ \mathbf{E}_m \end{pmatrix}.$$

Let introduce the narrowing Ω_{sk} of the set of stabilizing controllers, determining it by desired modal requirements for a closed-loop system:

$$\Omega_{sk} = \left\{ \mathbf{K} \in \Omega_k :\ \delta_i(\mathbf{K}) \in C_\Delta,\ i = \overline{1, n+m} \right\} \quad (4)$$

where $\delta_i(\mathbf{K})$ are the roots of the characteristic polynomial (3) of this system, C_Δ is defined area on the complex plane. In particular, as such area it can be taken a half-plane $C_\Delta = \{ s = x \pm jy \in \mathbf{C}^1 :\ x \le -\alpha_d \}$, where $\alpha_d > 0$ is a given real number that determines the degree of stability of the closed-loop system. However, other variants are also possible.

Let assume that in the assignment of the external disturbance there is uncertainty, however, accept its limitations, supposing that

$$\|\mathbf{w}(t)\| \le 1 \ \text{ at } 0 \le t < \infty, \quad (5)$$

where the Euclidean norm of the space E^l is used.

Let introduce the functional $J_d = J_d(\mathbf{K})$ characterizing the minimal invariant ellipsoid, including the set $\Re_{\mathbf{e}a}$ of reactions on the external influences for a closed-loop system (1), (2). In accordance with [7] in this case we have

$$J_d = J_d(\mathbf{K}) = f(\mathbf{P}(\alpha_0)) = \mathrm{tr}(\mathbf{C}_3 \mathbf{P}(\alpha_0) \mathbf{C}_3') \quad (6)$$

where $\alpha_0 = \arg\min_{\alpha>0} f(\mathbf{P}(\alpha))$, $f(\mathbf{P}(\alpha)) = \mathrm{tr}(\mathbf{C}_3 \mathbf{P}(\alpha) \mathbf{C}_3')$, $\mathbf{P}(\alpha)$ is a positive definite solution of linear matrix equation

$$\mathbf{A}_3 \mathbf{P} + \mathbf{P} \mathbf{A}_3' + \alpha \mathbf{P} + \alpha^{-1} \mathbf{D}_3 \mathbf{D}_3' = 0, \quad (7)$$

$$\mathbf{A}_3 = \mathbf{A}_0 + \mathbf{B}_0 \mathbf{K},\ \mathbf{C}_3 = (\mathbf{C} \ \vdots\ \mathbf{0}),\ \mathbf{D}_3 = \begin{pmatrix} \mathbf{D} \\ \mathbf{0} \end{pmatrix}.$$

Let consider the problem of choosing the stabilizing controller (2), which minimizes the size J_d of the invariant ellipsoid with regard to the desired modal properties of a closed-loop system

$$J_d = J_d(\mathbf{K}) \to \min_{\mathbf{K} \in \Omega_{sk} \subseteq \Omega_k}, \quad (8)$$

where the feasible set Ω_{sk} is defined by formula (4).

Note that if the modal properties are not taken into account, and it is required only the stability of the closed-loop system, we are led to the formulation of the well-known problem

$$J_d = J_d(\mathbf{K}) \to \min_{\mathbf{K} \in \Omega_k}, \quad (9)$$

which is discussed in details in [13]. He with co-authors provide a relatively simple and elegant solution, which is represented in the form

$$J_d^* = J_d(\mathbf{K}^*) = \min_{\mathbf{K} \in \Omega_k} J_d(\mathbf{K}) = F(\mathbf{P}(\alpha^*, \beta^*)) = \mathrm{tr}(\mathbf{C}_3 \mathbf{P}(\alpha^*, \beta^*) \mathbf{C}_3'), \quad (10)$$

where

$$(\alpha^*, \beta^*) = \arg\min_{\alpha>0,\ \beta>0} F(\mathbf{P}(\alpha, \beta)),$$

$$F(\mathbf{P}(\alpha, \beta)) = \mathrm{tr}(\mathbf{C}_3 \mathbf{P}(\alpha, \beta) \mathbf{C}_3'),$$

$\mathbf{P}(\alpha, \beta)$ is positive definite solution of linear matrix equations

$$\mathbf{A}_0 \mathbf{P} + \mathbf{P} \mathbf{A}_0' - \beta \mathbf{B}_0 \mathbf{B}_0' + \alpha \mathbf{P} + \alpha^{-1} \mathbf{D}_3 \mathbf{D}_3' = 0, \quad (11)$$

$$\mathbf{K}^* = \arg\min_{\mathbf{K} \in \Omega_k} J_d(\mathbf{K}) = -\beta \mathbf{B}_0' \mathbf{P}^{-1}(\alpha^*, \beta^*). \quad (12)$$

Because of the correctness of inclusion $\Omega_{sk} \subset \Omega_k$, we have

$$J_d^* = J_d(\mathbf{K}^*) = \min_{\mathbf{K} \in \Omega_k} J_d(\mathbf{K}) \le \min_{\mathbf{K} \in \Omega_{sk}} J_d(\mathbf{K}) = J_d(\mathbf{K}_0) = J_d^0, \quad (13)$$

i.e. the solution of (9) is a lower estimation for the solution to (8), that greatly simplifies the usage of approximate methods to search for its solution.

As it noticed in [7], for any pair $\alpha > 0$, $\beta > 0$, for which linear matrix equation (11) has the solution $\mathbf{P}(\alpha, \beta) > 0$, the controller (2) with the coefficient matrix

$$\mathbf{K} = \mathbf{K}(\alpha, \beta) = -\beta \mathbf{B}_0' \mathbf{P}^{-1}(\alpha, \beta) \quad (14)$$

is stabilizing.

Let introduce a special notation for the hurwitzian characteristic polynomial of the closed-loop system (1), (2) taking into account the condition (14):

$$\Delta_3(s, \alpha, \beta) = \det(\mathbf{E}_{n+m} s - \mathbf{A}_0 - \mathbf{B}_0 \mathbf{K}(\alpha, \beta)). \quad (15)$$

A set $\Omega_{\alpha\beta} \subset \Omega_k$ of stabilizing feedbacks with matrices $\mathbf{K}(\alpha, \beta)$, defined by the formula (14), further we will call *the set of parameterized controllers* for the problem (9), which solution is reduced to the search for the minimum of the function $F(\mathbf{P}(\alpha, \beta))$.

Let note that the process of solution of the problem (8) is convenient to connect with parameterization using real vectors $\boldsymbol{\gamma} \in E^{n+m}$ of characteristic polynomials of the system closed by controllers from the admissible set Ω_{sk}, entered by (4).

The parameterization of characteristic polynomials mentioned above define the corresponding parameterization of the set Ω_{sk} of controllers (2) for the object (1). Really, let define the arbitrary vector $\boldsymbol{\gamma} \in E^{n+m}$ and construct the polynomial $\Delta^*(s, \boldsymbol{\gamma})$. This polynomial will be characteristic for the closed-loop system, if the matrix $\mathbf{K}$ of the controller (2) will satisfy the identity

$$\Delta_3(s, \mathbf{K}) = \det(\mathbf{E}_{n+m} s - \mathbf{A}_0 - \mathbf{B}_0 \mathbf{K}) \equiv \Delta^*(s, \boldsymbol{\gamma}). \quad (16)$$

Collecting all components of the matrix $\mathbf{K}$ into vector $\mathbf{k} \in E^{m \times n + m \times m}$, we obtain that to the identity (16) will correspond an equivalent linear system

$$\boldsymbol{\Gamma} \mathbf{k} = \mathbf{m}(\boldsymbol{\gamma}), \quad (17)$$

where matrix $\boldsymbol{\Gamma}$ is defined by matrices $\mathbf{A}_0$ and $\mathbf{B}_0$, and vector $\mathbf{m}(\boldsymbol{\gamma})$ is defined by the coefficients of the polynomial $\Delta^*(s, \boldsymbol{\gamma})$ and characteristic polynomial of the matrix $\mathbf{A}_0$.

Note that the system (17) is always compatible due to the condition of full controllability. It contains $n+m$ equations and $m \times n + m \times m$ unknown variables, i.e. $n_c = m \times n + m \times m - n - m$ components of the vector $\mathbf{k}$, collected to the vector $\mathbf{h}_c \in E^{n_c}$, can be choose arbitrarily. Let consider vector $\boldsymbol{\varepsilon} = \{\boldsymbol{\gamma}, \mathbf{h}_c\} \in E^\lambda$, $\lambda = n + m + n_c$, and in addition, define the arbitrary vector $\mathbf{h}_c$. Then we can find the corresponding solution $\mathbf{k}(\boldsymbol{\varepsilon}) = \mathbf{k}(\boldsymbol{\gamma}, \mathbf{h}_c)$ of the system (17). Thus it is found the matrix $\mathbf{K} = \mathbf{K}(\boldsymbol{\varepsilon}) = \mathbf{K}(\boldsymbol{\gamma}, \mathbf{h}_c)$ of the controller coefficients (2) from the set Ω_{sk}, parameterized by vectors $\boldsymbol{\varepsilon} = \{\boldsymbol{\gamma}, \mathbf{h}_c\}$.

Let take an arbitrary controller from the set $\Omega_{\alpha\beta}$ and form the characteristic polynomial $\Delta_3(s,\alpha,\beta)$ of the closed-loop system by the formula (15), presenting it in the form

$$\Delta_3(s,\alpha,\beta) = \left(s^{n+m} \quad s^{n+m-1} \quad \dots \quad s \quad 1\right)(1 \quad \mathbf{v}(\alpha,\beta))',$$

$$\mathbf{v}(\alpha,\beta) = \left(\nu_{n+m-1}(\alpha,\beta) \quad \dots \quad \nu_1(\alpha,\beta) \quad \nu_0(\alpha,\beta)\right) \in E^{n+m}.$$

$\mathbf{v}(\alpha,\beta)$ is the vector of coefficients of the polynomial $\Delta_3(s,\alpha,\beta)$.

Similarly rewrite characteristic polynomial

$$\Delta_3(s,\mathbf{K}) = \left(s^{n+m} \quad s^{n+m-1} \quad \dots \quad s \quad 1\right)(1 \quad \mathbf{v}(\mathbf{K}))'$$

with the coefficients vector $\mathbf{v}(\mathbf{K}) \in E^{n+m}$, obtained using (3) for an arbitrary controller (2) taking into account the condition $\mathbf{K} \in \Omega_k$.

Let's call the non-negative real number

$$\rho_\Delta = \rho_\Delta(\alpha,\beta,\mathbf{K}) = \|\mathbf{v}(\alpha,\beta) - \mathbf{v}(\mathbf{K})\|. \qquad (18)$$

relative distance between regulators (2) with the matrices of coefficients $\mathbf{K}(\alpha,\beta) \in \Omega_{\alpha\beta}$ and $\mathbf{K} \in \Omega_k$ or between the corresponding characteristic polynomials $\Delta_3(s,\alpha,\beta)$ and $\Delta_3(s,\mathbf{K})$.

Let consider a finite dimensional problem on unconditional extremum

$$\rho_\Delta^* = \rho_\Delta^*(\boldsymbol{\varepsilon}) = \rho_\Delta(\alpha,\beta,\mathbf{K}(\boldsymbol{\varepsilon})) \to \min_{\boldsymbol{\varepsilon} \in E^\lambda}, \qquad (19)$$

where the dependence $\mathbf{K} = \mathbf{K}(\boldsymbol{\varepsilon}) = \mathbf{K}(\boldsymbol{\gamma},\mathbf{h}_c)$ from the vector $\boldsymbol{\varepsilon} = \{\boldsymbol{\gamma},\mathbf{h}_c\}$ is mentioned above.

Thus we can formulate the search algorithm of the approximate solution for the problem (8) using the ideology of the solution of the problem (9).

Algorithm 1.

1. Take any pair (α,β), $\alpha > 0$, $\beta > 0$.

2. Find the solution $\mathbf{P}(\alpha,\beta)$ of linear matrix equation (11).

3. If $\mathbf{P}(\alpha,\beta) \le 0$ (non positive defined) we need to change the selected pair (α,β) using some rule.

4. For given pair (α,β) construct the matrix $\mathbf{K}(\alpha,\beta)$ using (14) and calculate the value of the function $F(\mathbf{P}(\alpha,\beta)) = \mathrm{tr}\left(\mathbf{C}_3\mathbf{P}(\alpha,\beta)\mathbf{C}_3'\right)$.

5. For given pair (α,β) find the solution $\tilde{\boldsymbol{\varepsilon}}(\alpha,\beta)$ of the problem (19), obtain the matrix $\tilde{\mathbf{K}}(\alpha,\beta) = \mathbf{K}(\tilde{\boldsymbol{\varepsilon}}(\alpha,\beta)) \in \Omega_{sk}$ for the closest controller and calculate the value of the corresponding distance $\rho_\Delta = \rho_\Delta(\alpha,\beta)$.

6. Calculate the value of the auxiliary function

$$F_a(\alpha,\beta) = F(\mathbf{P}(\alpha,\beta)) + \mu\rho_\Delta(\alpha,\beta), \qquad (20)$$

where μ is a given weight factor.

7. Using any admissible numeric method of minimization the function $F_a(\alpha,\beta)$ on the set of parameters $\alpha > 0$, $\beta > 0$ define a new pair (α,β) and, repeating items 2,6, find the extremum (α^0,β^0) of the function $F_a(\alpha,\beta)$.

8. If $\overline{\mathbf{K}} = -\beta\mathbf{B}_0'\mathbf{P}^{-1}(\alpha^0,\beta^0) \in \Omega_{sk}$ and it is fulfilled the condition $\left(J_d(\overline{\mathbf{K}}) - J_d^*\right)/J_d^* < \varepsilon_d$, where ε_d is admissible deterioration of the size of minimal invariant ellipsoid by providing modal properties, then the problem is solved. If the last inequality is broken, it is necessary to decrease the value of the weight factor μ in (20) and repeat the search process.

III. Practical Example

Let us consider the mathematical model of the sea-going ship:

$$\begin{aligned} \dot\beta &= a_{11}\beta + a_{12}\omega + b_1\delta + h_1 d(t), \\ \dot\omega &= a_{21}\beta + a_{22}\omega + b_2\delta + h_2 d(t), \\ \dot\varphi &= \omega, \\ \dot\delta &= u \end{aligned} \qquad (21)$$

Here ω is an angular velocity relative to the vertical axis, φ is a course (the turn to port side is considered positive), δ is a deviation angle of the vertical rudders, β is a drift angle (angle between the velocity vector and longitudinal axis of the ship), u is a control, $d(t)$ is a bounded exogenous disturbance:

$$d'(t)d(t) \le 1, \quad 0 \le t < \infty \qquad (22)$$

Main parameters of the model are represented in the Fig. 1.

Coefficients in (21) for a fixed velocity are the following:

$a_{11} = -0.03408$, $a_{12} = 0.56$, $a_{21} = 0.015$, $a_{22} = -0.306$,
$b_1 = -0.0099$, $b_2 = -0.00417$, $h_1 = -0.0648$, $h_2 = -0.0046$,
$\mathbf{Q} = \mathrm{diag}(0.2, 0.2, 0.2, 0.2)$.

The graph of the test disturbance is represented in the Fig. 2.

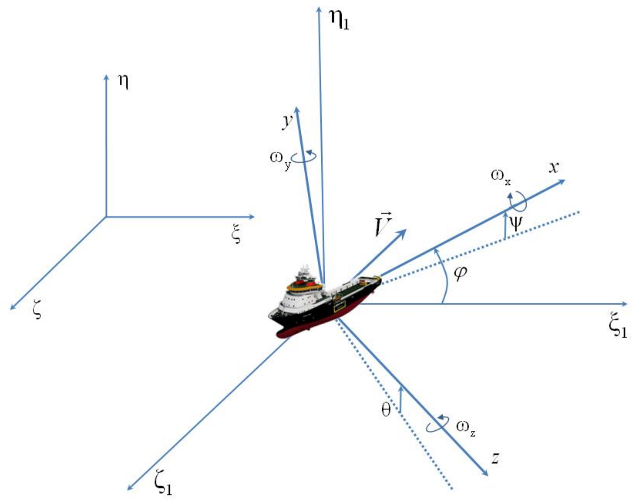

Fig. 1 Main parameters of the vessel

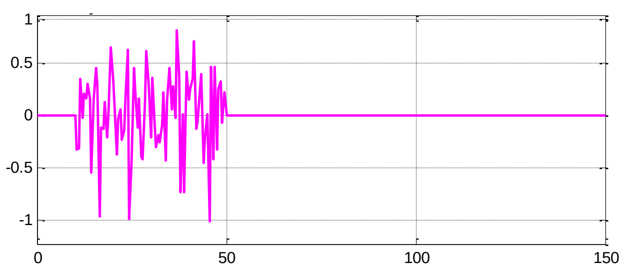

Fig. 2 External disturbance

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It is offered to search for the state controller with mathematical model

$$u = k_1\beta + k_2\omega + k_3\varphi + k_4\delta \tag{23}$$

where k_1, k_2, k_3, k_4 are parameters need to be found those provide the desired dynamics of the closed-loop system.

Deviation of the rudders and the its velocity turn (that is control) are constrained:

$$|\delta| \le 30^o, \quad |u| \le 3^o/\text{sec}.$$

Let consider the modal requirement, that consists in the providing of the desirable degree of stability, i.e. let C_Δ in (4) has the form $C_\Delta = \{s = x \pm jy \in \mathbf{C}^1 : \ x \le -0.05\}$ ($\alpha_d = 0.05$).

Using the Algorithm 1 stated above for the given ship we obtained the controller with the coefficients

$$k_1 = 2.44, k_2 = 66.6, k_3 = \text{-}0.0384, k_4 = 0.00133,$$

which provides the desired dynamics, is obtained. At the same time all requirements are taking into account.

Really, the eigenvalues of the matrix of the closed-loop system are

$$\lambda_1 = \text{-}0.3138, \lambda_2 = \text{-}0.0599 + 0.1886i,$$
$$\lambda_3 = \text{-}0.0599 \text{-} 0.1886i, \lambda_4 = \text{-}0.0574,$$

so the desired degree of stability is reahed.

For the performance testing of the found controller let use it for automatic control of a sea-going ship under bounded exogenous disturbance.

Fig. 3 and Fig. 4 show the graphs of yaw and deviation angle of the vertical rudders (solid line) in comparison with Polyak method (dashed line).

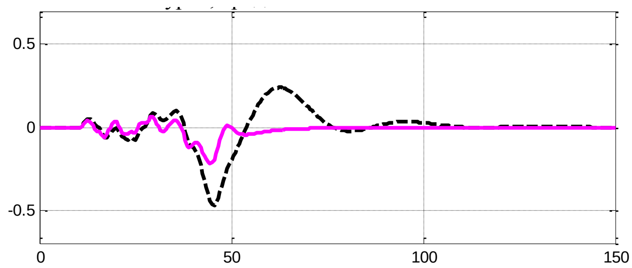

Fig. 3 Yaw variation

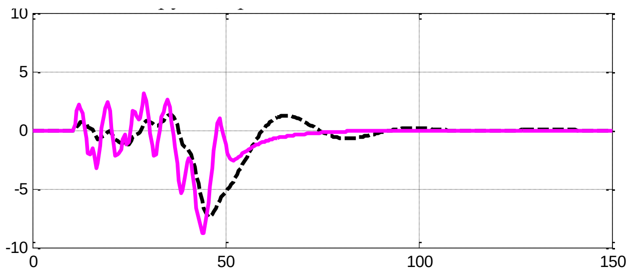

Fig. 4 Deviation angle of the rudders

As we can see, the developed algorithm gives better results than Polyak method.

IV. Conclusions

In the paper the method of suppression of bounded disturbances based on using of invariant ellipsoids and taking into consideration additional requirements to dynamic processes is stated. Its quality is illustrated on the example of the marine vessel.

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