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A Fuzzy-Neural Adaptive Iterative Learning Control for Freeway Traffic Flow Systems

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***Abstract*—In this paper, a fuzzy-neural adaptive iterative learning control (AILC) is proposed for traffic flow systems of a single lane freeway with random bounded off-ramp traffic volumes. It is assumed that the system dynamic functions and input gains are unknown for controller design. An adaptive fuzzy neural network (FNN) controller and an adaptive robust controller are applied to compensate for the unknown system nonlinearity and input gain respectively. On the other hand, to deal with the disturbance from random bounded off-ramp traffic volumes, a dead zone like auxiliary error with the time-varying boundary layer is introduced as a bounding parameter. This proposed auxiliary error is also utilized for the construction of adaptive laws without using the bound of the input gain for all the adaptation parameters. The traffic density tracking error is shown to converge along the axis of learning iteration to a residual set whose level of magnitude depends on the width of boundary layer.**

***Index Terms*—fuzzy neural network, adaptive iterative learning control, traffic flow systems, random bounded off-ramp traffic volumes.**

I. Introduction

IT is well-known that the traffic congestions on freeways are one of the main traffic problems in Taiwan. The freeway ramp metering [1] is one of the most typical control approaches to adjust the traffic flow of freeway. Besides, the PID-type control in [2], neural network control in [3], and optimal control in [4] are also some popular control methodologies in the research field of freeway ramp metering. The authors in [1], which is a good review of recent freeway ramp metering, have commented that the freeway ramp metering can be further divided into three classes of control strategies: 1. fixed-time ramp metering control, 2. local ramp metering control, 3. system ramp metering control. In the local ramp metering control strategies, ALINEA local ramp metering has been widely applied in the freeway traffic flow systems for a long period of time. In fact, ALINEA local ramp metering is a traditional PI-type controller which is not suitable for dealing with highly nonlinear systems with uncertainties. In addition, since very few strict mathematical analysis can be applied to design the controller gains of ALINEA local ramp metering, the system stability can not be guaranteed by ALINEA local ramp metering.

On the other hand, high repeatabilities often exist in the freeway traffic flow systems. For example, traffic congestions on the same freeway always repetitively appear in the same peak time interval from 7 to 9 AM every Monday. Unfortunately, the aforementioned freeway ramp metering control strategies are typical time-domain control approaches which do not consider the repetitive characteristics of freeway traffic flow systems for the design of ramp metering controller. This implies that these existing freeway ramp metering control approaches are not suitable to perform a repeated traffic control task for traffic flow systems. Recently, traditional discrete iterative learning control (ILC) schemes have been successfully applied for freeway traffic flow systems [5], [6], [7] with a repetitive task over a finite time interval. However, it is assumed that the system nonlinearities satisfy global Lipschitz continuous condition.

In this paper, the repetitive tracking control problem of traffic flow systems of a single lane freeway with random bounded off-ramp traffic volumes is studied. We consider a more general case in the sense that the system nonlinearities and system parameters are allowed to be unknown. An adaptive fuzzy neural network (FNN) controller and an adaptive robust controller are applied to compensate for the unknown system nonlinearity and input gain respectively. On the other hand, to deal with the disturbance from random bounded off-ramp traffic volumes, a dead zone like auxiliary error with the time-varying boundary layer is introduced as a bounding parameter. This proposed auxiliary error is also utilized for the construction of adaptive laws without using the bound of the input gain for all the adaptation parameters. The traffic density tracking error is shown to converge along the axis of learning iteration to a residual set whose level of magnitude depends on the width of boundary layer.

This paper is organized as follows. In section II, a problem formulation is given. The discrete AILC is then presented in section III. Based on the proposed AILC and a derived traffic density tracking error model, the analysis of closed-loop stability and learning performance will be studied extensively in Section IV. A simulation example is given in Section V to demonstrate the effectiveness of the proposed learning controller. Finally a conclusion is made in Section VI.

II. Problem Formulation

In this paper, we consider an uncertain traffic flow system [8] for a single lane freeway with n sections which can perform a given task repeatedly over a finite time sequence $t \in \{0, 1, 2, \cdots, N\}$. The traffic flow system for a single lane freeway with one on-ramp and one off-ramp in the ith section, $i = 1, \cdots, n$ is represented as follows:

$$\begin{aligned} \rho_i^j(t+1) &= f_i(q_{i-1}^j(t), \rho_i^j(t), q_i^j(t)) + \frac{T}{L_i}\left(r_i^j(t) - s_i^j(t)\right) \\ q_i^j(t) &= \rho_i^j(t)\nu_i^j(t) \\ \nu_i^j(t+1) &= g_i(\nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t), \rho_{i+1}^j(t+1)) \end{aligned} \quad (1)$$

where j and t denote the index of iteration and time, i is the ith section of a single lane freeway, n is the total number of sections, $\rho_i^j(t) \in \mathcal{R}$ is the traffic density in the

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ith section (in vehicles per lane per kilometer), $\nu_i^j(t) \in \mathcal{R}$ is the space mean speed in the ith section (in kilometers per hour), $q_i^j(t) \in \mathcal{R}$ is the traffic flow leaving the ith section and entering the $i+1$th section (in vehicles per hour), $r_i^j(t) \in \mathcal{R}$ is the on-ramp traffic volume in the ith section (in vehicles per hour), $s_i^j(t) \in \mathcal{R}$ is the off-ramp traffic volume of the ith section (in vehicles per hour) considered to be an unknown random bounded disturbance, $f_i(q_{i-1}^j(t), \rho_i^j(t), q_i^j(t))$ and $g_i(\nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t), \rho_{i+1}^j(t+1))$ are unknown real continuous nonlinear functions of $\nu_{i-1}^j(t)$, $q_{i-1}^j(t)$, $\rho_i^j(t)$, $\nu_i^j(t)$, $q_i^j(t)$, $\rho_{i+1}^j(t+1)$. Based on (1), the ith freeway traffic flow subsystem can be rewritten as follows:

$$\begin{aligned} &\rho_i^j(t+1) \\ &= f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) + \frac{T}{L_i}\left(r_i^j(t) - s_i^j(t)\right) \end{aligned} \tag{2}$$

Now, given a specified iteration-varying desired traffic density trajectory of the ith freeway traffic flow subsystem $\rho_{di}^j(t) \in \mathcal{R}$, $t \in \{0, 1, 2, \cdots, N+1\}$, the control objective is to design an AILC to adjust the on-ramp traffic volume $r_i^j(t)$ such that the traffic density $\rho_i^j(t)$ can follow $\rho_{di}^j(t)$ as close as possible $\forall t \in \{1, 2, \cdots, N+1\}$ when iteration j approaches infinity. In order to achieve this control objective, some assumptions on the freeway traffic flow system and desired traffic density trajectories are given as follows:

(A1) The freeway traffic flow system is a relaxed system whose on-ramp traffic volumes $r_i^j(t)$, traffic densities $\rho_i^j(t)$, space mean speeds $\nu_i^j(t)$ and traffic flows $q_i^j(t)$ are related by $\rho_i^j(t) = 0$, $\nu_i^j(t) = 0$ and $q_i^j(t) = 0$, $t < 0$.

(A2) The traffic flow rate entering the first section is $q_0^j(t)$ and the mean speed of the traffic entering the first section is assumed to be the mean speed in the first section, i.e., $\nu_0^j(t) = \nu_1^j(t)$. We also assume that the mean speed and traffic density of the traffic exiting the $n+1$th section are assumed to be those in nth section, i.e., $\nu_{n+1}^j(t) = \nu_n^j(t)$, $\rho_{n+1}^j(t) = \rho_n^j(t)$. The boundary conditions can be defined as $\rho_0^j(t) \equiv \frac{q_0^j(t)}{\nu_1^j(t)}$, $\nu_0^j(t) \equiv \nu_1^j(t)$, $\rho_{n+1}^j(t) \equiv \rho_n^j(t)$ and $\nu_{n+1}^j(t) \equiv \nu_n^j(t)$, respectively.

(A3) There exists a positive unknown constant s_U such that $|s_i^j(t)| \leq s_U$ for all $t \in \{0, 1, \cdots, N\}$, $j \geq 1$.

(A4) There exists a positive known constant ρ_d^U such that $|\rho_{di}^j(t)| \leq \rho_d^U$ for all $t \in \{0, 1, \cdots, N+1\}$, $j \geq 1$.

(A5) Let traffic density tracking errors be defined as $e_i^j(t) = \rho_i^j(t) - \rho_{di}^j(t)$. The initial traffic density tracking errors at each iteration $e_i^j(0)$ are bounded.

III. The Fuzzy-Neural AILC

In order to find the approach for controller design later, we first derive the traffic density tracking error equation as:

$$\begin{aligned} e_i^j(t+1) &= f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) + \frac{T}{L_i} r_i^j(t) \\ &\quad - \frac{T}{L_i} s_i^j(t) - \rho_{di}^j(t+1) \end{aligned} \tag{3}$$

In order to overcome the design problem due to unknown nonlinear function $f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ of the ith traffic flow subsystem, we apply the universal approximation technique to construct the basic structure of our AILC. An MIMO FNN [9] described by $\Theta^j(t)^\top \Phi(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ is utilized as the approximators of $f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$, $i = 1, 2, \cdots, n$. Here $\Phi(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) \in \mathcal{R}^{M \times 1}$ is the radial basis function vector in the rule layer of the MIMO FNN with M being the number of rules, $\Theta^j(t) \in \mathcal{R}^{M \times n}$ is the output weight matrix of the output layer with $\Theta_i^j(t) \in \mathcal{R}^{M \times 1}$ being the ith output weight vector. In other words, $\Theta_i^j(t)^\top \Phi(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ denotes the ith output of the MIMO FNN. In this work, we use the ith output of the MIMO FNN to uniformly approximate the nonlinear function $f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ of the ith traffic flow subsystem on a compact set $\mathcal{A}_c \subset \mathcal{R}^{4 \times 1}$. An important aspect of the above approximation property is that there exist an optimal parameter vector Θ_i^* for the ith output of the MIMO FNN such that the function approximation error $\epsilon_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ between the ith output of the optimal FNN $\Theta_i^{*\top} \Phi_i(t)$ and nonlinear function $f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ can be bounded by prescribed constants ϵ_i^* on the compact set $\mathcal{A}_c$. More precisely, if we define $\Phi^j(t) \equiv \Phi_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ and the $\epsilon_i^j(t) \equiv \epsilon_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t))$ for simplicity, then we have $f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) = \Theta_i^{*\top} \Phi_i(t) + \epsilon_i^j(t)$ and $|\epsilon_i^j(t)| \leq \epsilon_i^*$, $\forall (\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) \in \mathcal{A}_c$.

Based on the traffic density tracking error equation in (3) and the ith output of the MIMO FNN, we propose the fuzzy-neural AILC for the ith freeway traffic flow subsystem (2) as:

$$r_i^j(t) = \frac{\psi_i^j(t)}{\delta + \psi_i^j(t)^2}\left[-\Theta_i^j(t)^\top \Phi^j(t) + \rho_{di}^j(t+1)\right] \tag{4}$$

where $\delta > 0$. To further see the insight of the proposed AILC (4), we substitute (4) into (3) and find that

$$\begin{aligned} &e_i^j(t+1) \\ &= f_i(\rho_{i-1}^j(t), \nu_{i-1}^j(t), \rho_i^j(t), \nu_i^j(t)) - \Theta_i^j(t)^\top \Phi^j(t) \\ &\quad + \left(\frac{T}{L_i} - \psi_i^j(t)\right) r_i^j(t) + \Theta_i^j(t)^\top \Phi^j(t) - \rho_{di}^j(t+1) \\ &\quad + \frac{\psi_i^j(t)^2}{\delta + \psi_i^j(t)^2}\left[-\Theta_i^j(t)^\top \Phi^j(t) + \rho_{di}^j(t+1)\right] - \frac{T}{L_i} s_i^j(t) \\ &= \left(\Theta_i^* - \Theta_i^j(t)\right)^\top \Phi^j(t) + \left(\frac{T}{L_i} - \psi_i^j(t)\right) r_i^j(t) + \delta_{Li}^j(t) \end{aligned} \tag{5}$$

where

$$\begin{aligned} \delta_{Li}^j(t) &= \epsilon_i^j(t) - \frac{T}{L_i} s_i^j(t) \\ &\quad + \frac{\delta}{\delta + \psi_i^j(t)^2}\left[\Theta_i^j(t)^\top \Phi^j(t) - \rho_{di}^j(t+1)\right] \end{aligned} \tag{6}$$

It is clear that $\delta_{Li}^j(t)$ can be shown to be bounded by $\Theta_i^j(t)$ as follows:

$$\begin{aligned} |\delta_{Li}^j(t)| &\leq \left|\frac{\delta}{\delta + \psi_i^j(t)^2}\left[\Theta_i^j(t)^\top \Phi^j(t) - \rho_{di}^j(t+1)\right]\right| \\ &\quad + \left|\epsilon_i^j(t)\right| + \left|\frac{T}{L_i} s_i^j(t)\right| \end{aligned}$$

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$$\leq \quad \theta_i^*\big(\big|\Theta_i^j(t)\big|+1\big) \tag{7}$$

where θ_i^*, $i = 1,2,\cdots,n$ are some unknown positive constants. In order to overcome the uncertainty $\delta_{Li}^j(t)$ in (7), we now define an auxiliary error $e_{\phi i}^j(t+1)$ of the ith traffic flow subsystem as

$$e_{\phi i}^j(t+1) = e_i^j(t+1) - \phi_i^j(t+1)\mathbf{sat}\left(\frac{e_i^j(t+1)}{\phi_i^j(t+1)}\right) \tag{8}$$

for $t \in \{0,1,2,\cdots,N\}$. We don't define $e_{\phi i}^j(0)$ of the ith traffic flow subsystem since it will not be utilized in our design of controller and adaptive laws. In (8), **sat** is the saturation function defined as in [10] and $\phi_i^j(t+1)$ is the width of the time-varying boundary layer for the ith traffic flow subsystem which is to be designed later. It is noted that $e_{\phi i}^j(t+1)$ of the ith traffic flow subsystem which can be defined as in [10] and it can be easily shown that $e_{\phi i}^j(t+1)\mathbf{sat}\left(\frac{e_i^j(t+1)}{\phi_i^j(t+1)}\right) = |e_{\phi i}^j(t+1)|$, $\forall j \geq 1$.

Next, the time-varying boundary layer for the ith traffic flow subsystem will be designed as follows:

$$\phi_i^j(t+1) = \theta_i^j(t)\big(\big|\Theta_i^j(t)\big|+1\big) \tag{9}$$

where $\theta_i^j(t)$ is a parameter of the ith boundary to be updated later. In this AILC, $\Theta_i^j(t)$, $\psi_i^j(t)$ in (4) and $\theta_i^j(t)$ in (9) are designed to compensate the unknown optimal consequent parameter vectors Θ_i^*, input gains $\frac{T}{L_i}$ and θ_i^*, respectively. The adaptive laws for $\Theta_i^j(t)$, $\psi_i^j(t)$ and $\theta_i^j(t)$ at (next) $j+1$th iteration are given as follows :

$$\begin{aligned}&\Theta_i^{j+1}(t)\\ &= \quad \Theta_i^j(t) + \frac{\beta_1 e_{\phi i}^j(t+1)\,\Phi^j(t)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{10}$$

$$\begin{aligned}&\psi_i^{j+1}(t)\\ &= \quad \psi_i^j(t) + \frac{\beta_2 e_{\phi i}^j(t+1) r_i^j(t)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{11}$$

$$\begin{aligned}&\theta_i^{j+1}(t)\\ &= \quad \theta_i^j(t) + \frac{\beta_3 |e_{\phi i}^j(t+1)|\big(\big|\Theta_i^j(t)\big|+1\big)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{12}$$

for $t \in \{0,1,2,\cdots,N\}$, where $\beta_1,\beta_2,\beta_3 > 0$ are the adaptation gains. For the first iteration, we set $\Theta_i^1(t) = \Theta_i^1$ and $\psi_i^1(t) = \psi_i^1$ to be any constant vector and constant, respectively. $\theta_i^1(t) = \theta_i^1 > 0$ $\forall t \in \{0,1,2,\cdots,N\}$ to be a small fixed value $\forall t \in \{0,1,2,\cdots,N\}$. It is noted that $\theta_i^j(t) > 0, \forall t \in \{0,1,2,\cdots,N\}$ and $\forall j \geq 1$. Furthermore, we will choose $\psi^1(t) = \psi^1$ as a nonzero constant in order to prevent the controller (4) from being a zero input in the beginning of the learning process.

IV. Analysis of Stability and Convergence

In this section, we will analyze the closed loop stability and learning convergence. At first, define the parameter errors as $\widetilde{\Theta}_i^j(t) = \Theta_i^j(t) - \Theta_i^*$, $\widetilde{\theta}_g^j(t) = \psi_i^j(t) - \frac{T}{L_i}$, $\widetilde{\theta}_i^j(t) = \theta_i^j(t) - \theta_i^*$. Then it is easy to show, by subtracting the optimal control gains on both sides of (10)-(12), that

$$\begin{aligned}&\widetilde{\Theta}_i^{j+1}(t)\\ &= \quad \widetilde{\Theta}_i^j(t) + \frac{\beta_1 e_{\phi i}^j(t+1)\,\Phi^j(t)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{13}$$

$$\begin{aligned}&\widetilde{\psi}_i^{j+1}(t)\\ &= \quad \widetilde{\psi}_i^j(t) + \frac{\beta_2 e_{\phi i}^j(t+1) r_i^j(t)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{14}$$

$$\begin{aligned}&\widetilde{\theta}_i^{j+1}(t)\\ &= \quad \widetilde{\theta}_i^j(t) + \frac{\beta_3 |e_{\phi i}^j(t+1)|\big(\big|\Theta_i^j(t)\big|+1\big)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\end{aligned} \tag{15}$$

Now we are ready to state the main results in the following theorem.

Main Theorem. Consider the traffic flow systems in (1) satisfying the assumptions (A1)-(A5). If the fuzzy-neural AILC is designed as in (4), (8), (10), (11) and (12) with adaptive laws (10), (11) and (12) for the ith freeway traffic flow subsystem and the following condition can be satisfied:

$$2-\beta_1-\beta_2-\beta_3 > 0, \tag{16}$$

then the tracking performance and system stability will be guaranteed as follows:

(t1) The adjustable parameters $\Theta_i^j(t)$, $\psi_i^j(t)$, $\theta_i^j(t)$ and control inputs $r_i^j(t)$ are bounded $\forall t \in \{0,1,\cdots,N\}, j \geq 1$.

(t2) The auxiliary traffic density tracking errors $e_{\phi i}^j(t+1)$ are bounded $\forall t \in \{0,1,\cdots,N\}, j \geq 1$ and $\lim_{j\to\infty} e_{\phi i}^j(t+1) = 0$, $\forall t \in \{0,1,\cdots,N\}$

(t3) The traffic density tracking error $e_i^j(t+1)$ are bounded $\forall t \in \{0,1,\cdots,N\}$, $j \geq 1$ and $\lim_{j\to\infty} |e_i^j(t+1)| \leq \theta_i^\infty(t)\,(|\Theta_i^\infty(t)|+1)$, $\forall t \in \{0,1,\cdots,N\}$

Proof :

(t1) Define the cost functions of performance as follows

$$V_i^j(t) \quad = \quad \frac{1}{\beta_1}\widetilde{\Theta}_i^j(t)^\top \widetilde{\Theta}_i^j(t) + \frac{1}{\beta_2}\widetilde{\psi}_i^j(t)^2 + \frac{1}{\beta_3}\widetilde{\theta}_i^j(t)^2$$

Then, the difference between $V^{j+1}(t)$ and $V^j(t)$ can be derived as follows :

$$\begin{aligned}&V_i^{j+1}(t) - V_i^j(t)\\ &= \frac{1}{\beta_1}\left(\widetilde{\Theta}_i^{j+1}(t)^\top \widetilde{\Theta}_i^{j+1}(t) - \widetilde{\Theta}_i^j(t)^\top \widetilde{\Theta}_i^j(t)\right)\\ &\quad +\frac{1}{\beta_2}\left(\widetilde{\psi}_i^{j+1}(t)^2 - \widetilde{\psi}_i^j(t)^2\right)\\ &\quad +\frac{1}{\beta_3}\left(\widetilde{\theta}_i^{j+1}(t)^2 - \widetilde{\theta}_i^j(t)^2\right)\\ &= \frac{2e_{\phi i}^j(t+1)\,\widetilde{\Theta}_i^j(t)^\top \Phi^j(t)}{1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2}\\ &\quad + \frac{\beta_1 e_{\phi i}^j(t+1)^2\big|\Phi^j(t)\big|^2}{\left(1+\big|\Phi^j(t)\big|^2+\big|r_i^j(t)\big|^2+\big(\big|\Theta_i^j(t)\big|+1\big)^2\right)^2}\end{aligned}$$

$$
\begin{aligned}
&+\frac{2e_{\phi i}^j(t+1)\widetilde{\psi}_i^j(t)r_i^j(t)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{\beta_2 e_{\phi i}^j(t+1)^2\left|r_i^j(t)\right|^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{2|e_{\phi i}^j(t+1)|\widetilde{\theta}_i^j(t)\left(\left|\varTheta_i^j(t)\right|+1\right)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{\beta_3 e_{\phi i}^j(t+1)^2\left(\left|\varTheta_i^j(t)\right|+1\right)^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}
\end{aligned}
\tag{17}
$$

Since (5) can be rewritten as

$$
\widetilde{\varTheta}_i^j(t)^\top\varPhi^j(t)+\widetilde{\psi}_i^j(t)r_i^j(t)=-e_i^j(t+1)+\delta_{Li}^j(t) \tag{18}
$$

This implies that

$$
\begin{aligned}
&e_{\phi i}^j(t+1)\widetilde{\varTheta}_i^j(t)^\top\varPhi^j(t)+e_{\phi i}^j(t+1)\widetilde{\psi}_i^j(t)r_i^j(t)\\
&= -e_i^j(t+1)e_{\phi i}^j(t+1)+e_{\phi i}^j(t+1)\delta_{Li}^j(t)
\end{aligned}
\tag{19}
$$

Substituting (19) into (17), we have

$$
\begin{aligned}
&V_i^{j+1}(t)-V_i^j(t)\\
\le\ &\frac{-2e_i^j(t+1)e_{\phi i}^j(t+1)+2e_{\phi i}^j(t+1)\delta_{Li}^j(t)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{\beta_1 e_{\phi i}^j(t+1)^2\left|\varPhi^j(t)\right|^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_2 e_{\phi i}^j(t+1)^2\left|r_i^j(t)\right|^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{2|e_{\phi i}^j(t+1)|\widetilde{\theta}_i^j(t)\left(\left|\varTheta_i^j(t)\right|+1\right)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{\beta_3 e_{\phi i}^j(t+1)^2\left(\left|\varTheta_i^j(t)\right|+1\right)^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}
\end{aligned}
\tag{20}
$$

If we substitue (8) into (20) and using the fact that $|\delta_{Li}^j(t)|\le\theta_i^*\left(\left|\varTheta_i^j(t)\right|+1\right)$ in (7), we can derive that

$$
\begin{aligned}
&V_i^{j+1}(t)-V_i^j(t)\\
\le\ &\frac{-2e_{\phi i}^j(t+1)^2}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&-\frac{2|e_{\phi i}^j(t+1)|\theta_i^j(t)\left(\left|\varTheta_i^j(t)\right|+1\right)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{2|e_{\phi i}^j(t+1)|\theta_i^*\left(\left|\varTheta_i^j(t)\right|+1\right)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{2|e_{\phi i}^j(t+1)|\widetilde{\theta}_i^j(t)\left(\left|\varTheta_i^j(t)\right|+1\right)}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\\
&+\frac{\beta_1 e_{\phi i}^j(t+1)^2\left|\varPhi^j(t)\right|^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_2 e_{\phi i}^j(t+1)^2\left|r_i^j(t)\right|^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_3 e_{\phi i}^j(t+1)^2}{\left(1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2\right)^2}\\
\le\ &\frac{-(2-\beta_1-\beta_2-\beta_3)e_{\phi i}^j(t+1)^2}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}
\end{aligned}
\tag{21}
$$

If we choose β_1, β_2 and β_3 such that $k\equiv 2-\beta_1-\beta_2-\beta_3>0$, then we have

$$
\begin{aligned}
&V_i^{j+1}(t)-V_i^j(t)\\
\le\ &\frac{-ke_{\phi i}^j(t+1)^2}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}\le 0
\end{aligned}
\tag{22}
$$

for $j\ge 1$. Since $V^1(t)$ is bounded $\forall t\in\{0,1,2,\cdots,N\}$ due to $\widetilde{\varTheta}^1(t)=\varTheta^1(t)-\varTheta_i^*=\varTheta_i^1-\varTheta_i^*$, $\widetilde{\psi}^1(t)=\psi^1(t)-\frac{T}{L_i}=\psi^1-\frac{T}{L_i}$ and $\widetilde{\theta}_i^1(t)=\theta_i^1(t)-\theta_i^*=\theta_i^1-\theta_i^*$ are bounded $\forall t\in\{0,1,2,\cdots,N\}$, we conclude that from (22) that $V^j(t)$, and hence $\widetilde{\varTheta}_i^j(t)$, $\widetilde{\psi}_i^j(t)$ and $\widetilde{\theta}_i^j(t)$, are bounded $\forall j\ge 1$. The boundedness of $r_i^j(t)$ is then guaranteed by using (4). This proves **(t1)** of the main theorem.

(t2) By summing (22) from 1 to j leads to

$$
V_i^j(t)\le V_i^1(t)-\sum_{i=1}^{j}\frac{-ke_{\phi i}^j(t+1)^2}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}
$$

Since $V_i^1(t)$ is bounded and $V^j(t)$ must be nonnegative, we have

$$
\lim_{j\to\infty}\frac{e_{\phi i}^j(t+1)^2}{1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2}=0
$$

$\forall t\in\{0,1,2,\cdots,N\}$. Since $1+\left|\varPhi^j(t)\right|^2+\left|r_i^j(t)\right|^2+\left(\left|\varTheta_i^j(t)\right|+1\right)^2$ are bounded for all $j\ge 1$ and $t\in\{0,1,2,\cdots,N\}$, this readily implies that $\lim_{j\to\infty}e_{\phi i}^j(t+1)^2=0$

(t3) The boundedness of $e_i^j(t+1)$ at each iteration over $\{0,1,2,\cdots,N\}$ can be concluded from (8) because $\phi_i^j(t+1)$ is bounded. This implies that the bound of $e^\infty(t+1)$ will satisfy $\lim_{j\to\infty}|e_i^j(t+1)|=|e_i^\infty(t+1)|\le\phi_i^\infty(t+1)=\theta_i^\infty(t)\left(|\varTheta_i^\infty(t)|+1\right)$, $\forall\ t\in\{0,1,2,\cdots,N\}$. This proves **(t3)** of the main theorem. Q.E.D.

Remark 1 : According to **(t3)** of the main theorem, it is necessary to prevent the boundary layers to be large values in the learning process. Hence we usually set the initial values of θ_i^1 and the adaptation gain β_3 in (12) as small constants. This implies that $\theta_i^j(t)(|\varTheta_i^j(t)|+1), t\in\{0,1,\cdots,N\}$ will remain in a reasonable small value for all $j\ge 1$.

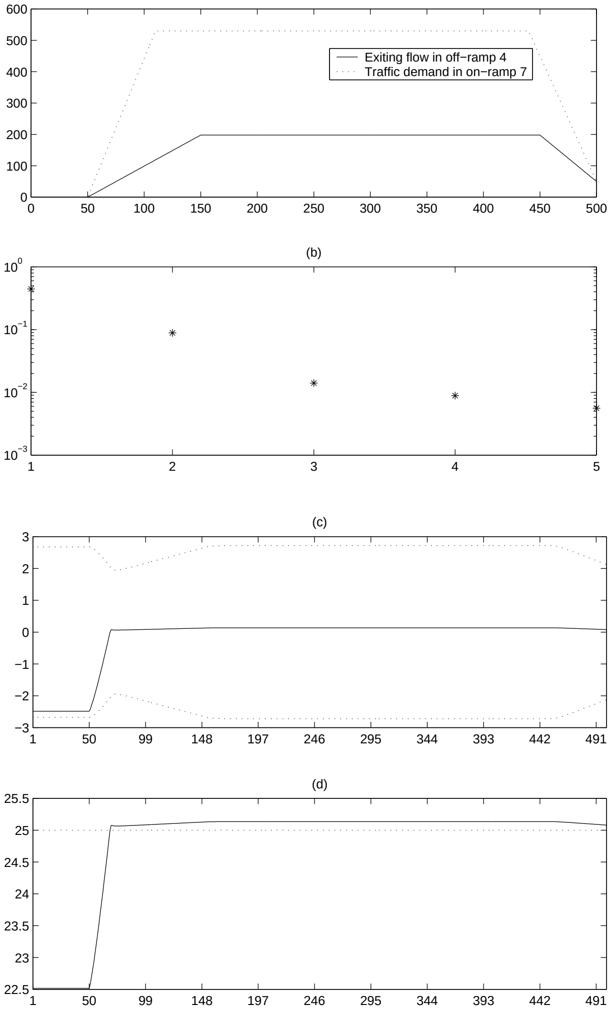
Remark 2 : In our early work [10], the design of adaptation gain is dependent of the upper bound of the input gain function. However, in this proposed controller, the upper bounds of input gains $\frac{T}{L_i}$ are not necessary for our fuzzy neural AILC design. In other words, the convergent condition in 16 is less restricted than that given in our previous work [10].

V. Simulation Example

In this section, we apply the proposed AILC for an unknown long segment of a single lane freeway in [5], [6], [7] which is subdivided into 12 sections. The difference equation of the ith traffic flow subsystem of a single lane freeway with one on-ramp and one off-ramp is given as follows,

$$\begin{aligned} \rho_i^j(t+1) &= \rho_i^j(t) + \frac{T}{L_i}\left[q_{i-1}^j(t) - q_i^j(t) + r_i^j(t) - s_i^j(t)\right] \\ q_i^j(t) &= \rho_i^j(t)\nu_i^j(t) \\ \nu_i^j(t+1) &= \nu_i^j(t) + \frac{T}{\tau}\left[V(\rho_i^j(t)) - \nu_i^j(t)\right] \\ &\quad + \frac{T}{L_i}\nu_i^j(t)\left[\nu_{i-1}^j(t) - \nu_i^j(t)\right] \\ &\quad - \frac{\nu T}{\tau L_i}\frac{\left[\rho_{i+1}^j(t) - \rho_i^j(t)\right]}{\left[\rho_i^j(t) + \kappa\right]} \\ V\left(\rho_i^j(t)\right) &= \nu_{\text{free}}\left(1 - \left[\frac{\rho_i^j(t)}{\rho_{\text{jam}}}\right]^l\right)^m \end{aligned}$$

where $\rho_i^j(t), \nu_i^j(t), q_i^j(t), r_i^j(t), s_i^j(t)$ are respectively the traffic density, space mean speed, traffic flow, on-ramp traffic volume, off-ramp traffic volume, $i = 1, \cdots, 12$. Here, the iterative-varying desired traffic density trajectory of the ith traffic flow subsystem is chosen as $\rho_{di}^j(t) = 25 + 0.1\sin(2\pi j/5)$veh/km. In this simulation, we select the length of the ith section, the sampling period, the free speed and maximum possible density per lane to be $L_i = 0.5$km, $T = 15/3600$h, $\nu_{\text{free}} = 80$km/h and $\rho_{\text{jam}} = 80$veh/km respectively. The freeway traffic flow system parameters $\tau = 0.01$h, $\nu = 35\text{km}^2/\text{h}$, $\kappa = 13$veh/km, $l = 1.8$, $m = 1.7 \in \mathcal{R}$ are respectively the street geometry, vehicle characteristics, drivers' behaviors, etc.. Besides, we assume that the traffic flow entering the first section is $q_0^j(t) = 1500$veh/h. Furthermore, the initial traffic density and space mean speed of the ith traffic flow subsystem at the beginning of each iteration are chosen as $\rho_i^j(0) = 22.516 + 0.1\sin(2\pi j/5)$veh/km, $\nu_i^j(0) = 66.619 + 0.3\sin(2\pi j/5)$km/h, respectively. The off-ramp traffic volume of the ith section is $s_i^j(t) = 0$ for $i = 1, \cdots, 3, 5, \cdots, 12$ and the off-ramp traffic volume of the 4th section $s_4^j(t)$ is shown in Figure 1(a). The control objective is to make the traffic density $\rho_i^j(t)$ of the ith traffic flow subsystem to track as close as possible the desired iterative-varying traffic density trajectory $\rho_{di}^j(t)$ for all $t \in \{1, \cdots, 500\}$. In order to achieve the control objective, the fuzzy-neural discrete AILC in (4), (8), (10), (11), and (12) is applied with the design parameters $\beta_1 = 0.9499$, $\beta_2 = 0.9499$, $\beta_3 = 0.0001$ so that $k \equiv 2 - \beta_1 - \beta_2 - \beta_3 = 0.1$. Furthermore, we set $\delta = 0.00001$ in (4) and the initial control parameters at the first iteration are chosen as $\Theta_i^1(t) = \Theta_i^1 = [0.5, 0.5, 0.5, 0.5, 0.5]^\top$, $\psi_i^1(t) = \psi_i^1 = 0.1$ and $\theta_i^1(t) = \theta_i^1 = 1.5$, $i = 1, \cdots, 12$, respectively. In the following, we only investigate the learning performance of the 7th traffic flow subsystem due to the limitations on length of the paper. In order to verify the robustness against iteration-varying initial resetting traffic density errors $e_7^j(0)$ and the bounded off-ramp traffic volumes $s_7^j(t)$ of the 7th traffic flow subsystem, we show $\max_{t\in\{1,\cdots,500\}} |e_{\phi 7}^j(t)|$ with respective to iteration j in Figure 1 (b). It implies that the



asymptotical convergence proves the technical result given in **(t2)** of the main theorem. Because the learning process is almost completed at the 5th iteration, the traffic density errors of the 7th section $e_7^5(t)$ is shown in Figure 1(c) to prove the result in **(t3)** of the main theorem. It is clear that the trajectory of $e_7^5(t)$ satisfies $-\theta_7^5(t)\left(\left|\Theta_7^5(t)\right| + 1\right) \le e_7^5(t) \le \theta_7^5(t)\left(\left|\Theta_7^5(t)\right| + 1\right)$, $t \in \{1, \cdots, 500\}$ in Figure 1 (c). In order to verify the nice traffic density tracking performance at the 5th iteration, we show the relation between traffic density $\rho_7^5(t)$ and desired traffic density trajectory $\rho_{d7}^5(t)$ in Figure 1 (d) for $t \in \{0, 1, 2, \cdots, 500\}$. To see the control behavior that $\rho_7^5(t)$ is close to $\rho_{d7}^5(t)$ for $t \in \{0, 1, 2, \cdots, 500\}$ except the initial fifty discrete-time, the trajectories between $\rho_7^5(t)$ and $\rho_{d7}^5(t)$ are shown again in Figure 1 (e) only for the time sequence $t \in \{0, 1, 2, \cdots, 100\}$. It is clear that $\rho_7^5(t)$ converges to $\rho_{d7}^5(t)$ after $t \ge 50$. Finally, Figure 1(f) shows the bounded learned control input $r_7^5(t)$ for the 7th traffic flow subsystem.

VI. Conclusion

A discrete fuzzy neural AILC is proposed in this paper for repeatable traffic flow systems with initial resetting traffic density errors, iteration-varying desired trajectories and random off-ramp traffic volumes. We first derive a tracking error model to establish the main control structure. The MIMO FNN is applied in the main structure to compensate for the lumped uncertainties from unknown system nonlinearities.

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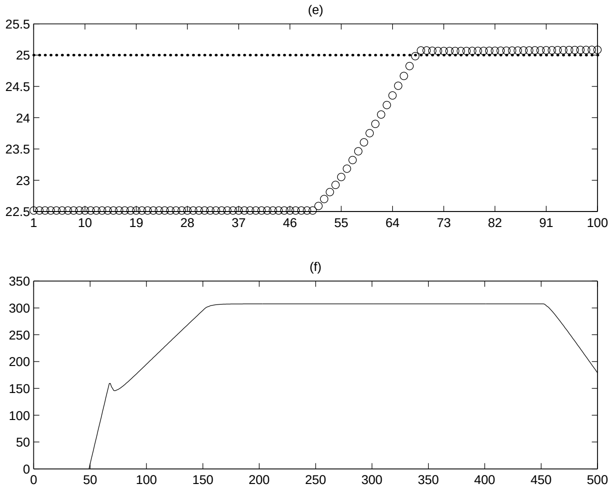

Fig. 1. (a) $s_4^j(t)$ versus time t; (b)$\max_{t\in\{1,\cdots,500\}} |e^j_{\phi 7}(t)|$ versus control iteration j; (c)$e_7^5(t)$ (solid line) and $\theta_7^5(t)\left(\left|\Theta_7^5(t)\right| + 1\right), -\theta_7^5(t)\left(\left|\Theta_7^5(t)\right| + 1\right)$ (dotted lines) versus time $t \in \{1, 2, \cdots, 500\}$; (d)$\rho_7^5(t)$ (solid line) and $\rho^j_{d7}(t)$ (dotted line) versus time $t \in \{0, 1, \cdots, 500\}$ at the 5th control iteration; (e)$\rho_7^5(t)$ ($\circ \circ \circ$) and $\rho^5_{d7}(t)$ ($\cdots$) versus time $t \in \{0, 1, \cdots, 100\}$ at the 5th control iteration; (f)$r_7^5(t)$ versus time t.

For further compensation of the lumped uncertainties induced by function approximation errors and random off-ramp traffic volumes of the freeway, a dead-zone like auxiliary traffic density error functions with time-varying boundaries are then constructed. By the auxiliary traffic density error functions, the adaptive laws for the control parameters and time-varying boundary layer are designed to guarantee the closed-loop stability and learning error convergence. Based on a Lyapunov like analysis, we show that all adjustable parameters and the internal signals remain bounded and the traffic density tracking errors asymptotically converge to a residual set whose size depends on the width of boundary layer as iteration goes to infinity.

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