Gravitational Stabilization and Reorientation of a Dumbbell Shaped Artificial Satellite on the Principle of Swing in a Circular Orbit

Sergey P. Bezglasnyi, Alena A. Mukhametzyanova

***Abstract*— The problem of control of plane motions of a dumbbell shaped artificial satellite is considered in this note. The satellite is modeled by weight rod with two point masses. They are fixed on the rod. A third mass point can move along the rod. The control is realized by varying continuously the distance from the centre of mass of the satellite to the movable mass. New limited control laws processes of excitation and damping, diametrically reorientation and gravitational stabilization to the local vertical of the dumbbell shaped artificial satellite pendulum are constructed. The problem is solved by the method of Lyapunov's functions of the classical theory of stability. The theoretical results are illustrated by graphical representation of the numerical results.**

***Index Terms*—Orbit, dumbbell shaped satellite, gravitational torque, asymptotic stability**

I. Introduction

THE problem of the stability of the relative equilibria and different motions of a satellite about the centre of mass in a Keplerian orbit under the action of gravitational, aerodynamic, and other torques has been the subject of publications by numerous investigators [1-8]. In this paper we study the problem of gravitational stabilization of the relative equilibrium of the dumbbell shaped satellite in a circular orbit. The problem of its reorientation by a movable mass swings principle solved. Swing can be simulated single-mass [9] or the two-mass [10, 11] pendulum of variable length. Many mechanical systems include flat pendulum motion, so swing models can be applied in the study of the dynamics and methods of control of such systems. Problems of swing and damping dual-mass pendulum resolved in [11], using the original continuous control law moving mass. The problem of the diametrical reorientation and stabilization of the gravitational plane motion of the satellite in a circular orbit was solved by the authors [12], using the same law [11]. In [13], the solution of the problem of orbital maneuvering the satellite using space tether system with a movable mass. The problem of gravitational stabilization of two opposite radial position of relative equilibrium of a dumbbell shaped satellite in a circular orbit solved a similar control in [14]. There is flaw in the [11-14]. The movable mass can move without limitations. Limited control on the principle of double pendulum swing was constructed in [15].

In this paper, the model of the dumbbell shaped satellite as well as [14]. We build the limited laws of the movable mass motion for the control plane motion of the satellite in a circular orbit. The satellite consists of two point masses connected by a weighty tether, along which a fourth point mass can be moved. Satellite is modeled rigid rod. The center of mass of the satellite moves in an orbit under the action of forces of central Newtonian gravity. Control is the distance from the common center of mass of the two ends of the rod to the cargo and moving cargo. Control imposed limitation. On the movement of the movable mass limitation imposed on both sides.

The new law for the model of the satellite [14] solves the problem of gravitational stabilization of the radial equilibrium position of the satellite relative to flat perturbations.

Control is built, which solves the problem of "swing" of the dumbbell shaped satellite and its reorientation in a diametrically opposite position with respect to an asymptotically stable equilibrium (a revolution of the satellite at an angle π) in orbit.

II. The Equation of Plane Motion of Dumbbell Shaped Satellite with a Movable Mass

Consider dumbbell shaped satellite motion in a central Newtonian gravitational field with center O. Dumbbell shaped satellite is modeled rigid rod with a mass m_3, as well as in the article [14]. Point mass m_1 and m_2 fixed to the ends of the rod. The movable mass m_4 is moved along the rod (Fig. 1). The common center of mass of payload and the rod is at the point O_1. We denote the distance from the point O_1 to the load m_4 as l and distance from the point O_1 to the center of mass O_2 as d. For them, the following relation holds true:

$$\left(m_1+m_2+m_3\right)d=m_4(l-d) \tag{1}$$

The orbital coordinate system O_2XYZ was selected. The axis O_2X is tangential to the orbit. The axis O_2Y is perpendicular to the plane of the orbit. The axis O_2Z completes the system of coordinates to the right hand third

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axis. O_1xyz is coordinate system associated with the dumbbell shaped satellite. The axes of the system are directed at the main central axes of inertia of a satellite.

The movement of the coordinate system O_1xyz relative to the orbital coordinates will be described by Euler angles ψ, θ, φ. we assume that the principal central moments of inertia of the system without movable mass: $B_1 = 0$,

$$A_1 = C_1 = \frac{L^2 m_3}{12} + L^2 \frac{4m_1m_2 + m_1m_3 + m_2m_3}{4(m_1 + m_2 + m_3)},$$

here L – length of the rod.

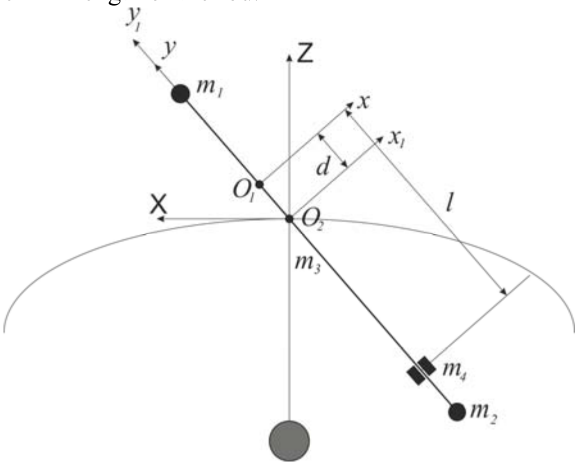

Fig. 1. Satellite.

We obtain from relation (1):

$$d = \frac{m_4 l}{m_1 + m_2 + m_3 + m_4} \tag{2}$$

here

$$m = \frac{(m_1 + m_2 + m_3)m_4}{m_1 + m_2 + m_3 + m_4}. \tag{3}$$

Equation of plane motion about the center of mass of the dumbbell shaped satellite in a circular orbit by the gravitational moment of the article [14]:

$$\varphi'' = -2\frac{mll'}{A_1 + ml^2}(\varphi' + 1) - 3\sin\varphi\cos\varphi \tag{4}$$

here the prime denotes the derivative with respect to new variable ν - true anomaly.

The distance from O_1 to the moving mass m_4, is considered the control:

$$l = l(\varphi, \varphi') \tag{5}$$

III. The Gravitational Stabilization of a Dumbbell Shaped Satellite

We will solve the problem of stabilization of the planar oscillations of a dumbbell shaped satellite relative radial position of equilibrium using a movable mass by swings principle. Control (5) are constructed according to the equations:

$$l = \begin{cases} l_0 + a\varphi'\sin\varphi, \\ \text{when } -b \le a\varphi'\sin\varphi \le b,\ \text{a} \ge \text{b}>0; \\ l_0 + b\,sign(\sin\varphi)\cdot sign(\varphi), \\ \text{when } a\varphi'\sin\varphi \le -b \cup a\varphi'\sin\varphi \ge b. \end{cases} \tag{6}$$

here $l_0 = const > 0,\ a = const > 0$. Taking into account the equality:

$$l' = \begin{cases} a\varphi''\sin\varphi + a\varphi'^2\cos\varphi, \\ \text{when } -b \le a\varphi'\sin\varphi \le b,\ \text{a} \ge \text{b>0}; \\ 0,\ \text{when } a\varphi'\sin\varphi \le -b \cup a\varphi'\sin\varphi \ge b. \end{cases}$$

rewrite (4):

$$\begin{cases} \varphi''(A_1 + ml(l_0 + 3a\varphi'\sin\varphi + 2a\sin\varphi)) = \\ = -2mla\cos\varphi(\varphi' + 1)\varphi'^2 - \\ -3(A_1 + ml^2)\sin\varphi\cos\varphi, \\ \text{when } -b \le a\varphi'\sin\varphi \le b,\ \text{a} \ge \text{b>0}; \end{cases} \tag{7}$$

$$\begin{cases} \varphi'' = -3\sin\varphi\cos\varphi, \\ \text{when } a\varphi'\sin\varphi \le -b \cup a\varphi'\sin\varphi \ge b. \end{cases} \tag{8}$$

Equation (7) has a zero solution $\varphi = \varphi' = 0$ corresponding to the investigated relative satellite equilibrium. It is the equation of perturbed motion in the neighborhood of this equilibrium state. We will find a solution to the problem by using the second method of the classical theory of stability. We chose the Lyapunov function:

$$\begin{aligned} V &= \frac{A_1 + ml_0(l_0 + 3a\varphi'\sin\varphi + 4a\sin\varphi)}{2}\varphi'^2 + \\ &+ \frac{3}{4}\left(A_1 + ml_0(l_0 + \frac{4}{3}a\varphi + \frac{a}{2}\varphi'\sin\varphi)\right)\times \\ &\times(1 - \cos 2\varphi) \approx \frac{A_1 + ml_0^2}{2}\left(\varphi'^2 + 3\varphi^2\right) \end{aligned} \tag{9}$$

The function $V(\varphi, \varphi')$ can be represented by a power series in the neighborhood of the relative equilibrium state $\varphi = \varphi' = 0$. The power series begins a positive definite quadratic form, so the function is positive definite according to the basis of definite function [16].

The derivative function (9) looks up to terms of the fourth degree in the variables φ, φ', by virtue of (7):

$$\frac{1}{Fn}\dot{V} = -\frac{9}{4}\varphi^4 + \frac{12F}{G}\varphi^3\varphi' - \frac{21}{4}\varphi^2\varphi'^2 - \frac{4F}{G}\varphi\varphi'^3 - \frac{1}{2}\varphi'^4$$

here $F = mal_0 > 0$, $G = A_1 + ml_0^2 > 0$, n – the mean motion [2]. Derivative of Lyapunov function will be determined negatively function of its variables, if the equality $G > \sqrt{6,4}F$ holds, according to the Sylvester's criterion. The relative equilibrium state $\varphi = \varphi' = 0$ dumbbell shaped satellite in a circular orbit is asymptotically stable based on Lyapunov's theorem on asymptotic stability [16]. Trivial solution $\varphi = \varphi' = 0$ is not asymptotic stability in general, but numerical calculations showed, that the for any initial deviations and velocity motion in the vicinity of the lower equilibrium position dumbbell shaped satellite is damped.

A numerical integration equations of motion performed in the interval $\nu \in [0; 300]$ *rad* for the following numerical values of system parameters $m_1 = 400$ *kg*, $m_2 = 300$ *kg*, $m_3 = 100$ *kg*, $m_4 = 200$ *kg*, $L = 32$ *m*, $l_0 = 9$ *m*, $a = 5$ *m•s*, $b = 1,5$ and initial data: $\varphi(t_0) = 1,5$ *rad*, $\dot{\varphi}(t_0) = 0.1$ *rad/s*. These values were taken as an illustrative example. The phase portrait of system (7) with control (6) is shown in

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figure 2, which illustrating asymptotic damping of the amplitude and velocity vibrations of the of the dumbbell shaped satellite around a zero equilibrium state. Amplitude and speed begins with sufficiently large initial deviations.

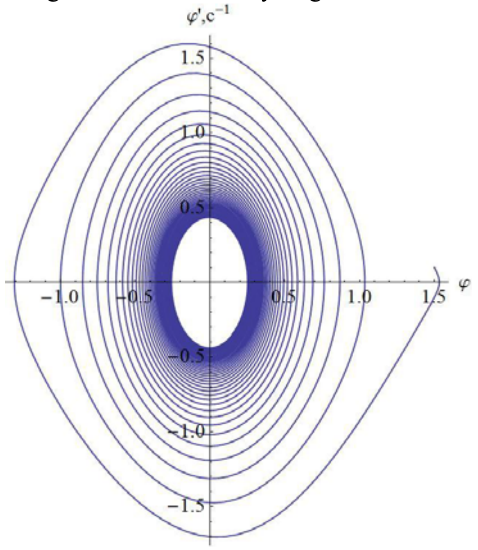

Fig. 2. Phase portrait.

IV. Swinging and Reorientation of the Dumbbell Shaped Satellite

It is known [2] that the satellite has two radial equilibrium state. The first, is the relative equilibrium state in the orbit at which rod is directed along the radius of the local vertical. The second, the diametrically opposite equilibrium state. We solve the problem of the swinging of a satellite from an arbitrary neighborhood of the relative equilibrium position and its diametrical reorientation. We will assume that in control law (2.1) the parameter.

$$a = const < 0 \tag{10}$$

The equation of controlled motion of the satellite maintains the form (4). The function (9) is positive-definite in the vicinity of equilibrium $\varphi = \varphi' = 0$. We calculate the derivative of this function with respect to time by virtue of (1.4) up to terms of the fourth order we have the relation:

$$\frac{G}{n|F|}\dot{V}_{(2.3)} = G\varphi'^2\left(\frac{1}{2}\varphi'^2 + \frac{15}{4}\varphi^2 + 4\frac{F}{G}\varphi\varphi'\right) + \\ +3H\varphi^2\left(\frac{5}{2}\varphi'^2 + \frac{3}{4}\varphi^2 + 4\frac{F}{G}\varphi\varphi'\right) \tag{11}$$

here $F = mal_0 < 0$.

Derivative (11) will be negative-definite, when the inequality $G > 4\sqrt{\frac{2}{15}}|F|$ satisfying, by Sylvester's criterion [16]. According to the Chetayev theorem of instability [16] the relative equilibrium state $\varphi = \varphi' = 0$ of the satellite in a circular orbit is unstable. Thus, the process of swinging of the satellite with respect to radial position is implemented. Numerical calculations show that as a result of this swing comes diametric reversal of the satellite relative to its center of mass.

Let us show that after the satellite diametric reversal control (6) under the conditions (10) stabilizes the satellite in the neighborhood of the opposite equilibrium state $\varphi = \pi$, $\varphi' = 0$. We write the equation of perturbation motion by introducing the deviation $\varphi = \pi + x$:

$$x''(A_1 + ml\ (l_0 - 3ax'\sin x - 2a\sin x)) = \\ = 2mla\cos x(x'+1)x'^2 - \frac{3}{2}(A_1 + ml^2)\sin 2x \tag{12}$$

Because now $a = const < 0$, then equation (12) with control (6) coincides with the first equation of the system (7) where $a = const > 0$. The solution $x = x' = 0$ will be asymptotically stable according to the results obtained in Section 2.

Thus, the control (6) under condition (10) implements asymptotically stable reorientation diametrically dumbbell shaped satellite.

This process in Fig. 3-4 is illustrated by graphs of numerical calculations. The integration was performed in the range $\nu \in [0;200]$ *rad*, parameter $a = -5$ *m•s* and the initial values: $\varphi(\nu_0) = 0.4$ *rad*, $\varphi'(\nu_0) = 0.1$ *rad/s*. Other parameters of the system were the same as they were before in Section 2. In the phase portrait (Fig.3) shows the behavior of the angle φ. It shows swinging around the zero equilibrium state $\varphi = \varphi' = 0$ followed by an asymptotic approach to a new equilibrium state $\varphi = \pi$, $\varphi' = 0$.

Fig. 4 shows the behavior of the distance l as a function of the angle φ. Initially, as the satellite swings, the deviations of the distance l from the value l_0 in the vicinity of equilibrium $\varphi = 0$ increase periodically, and after the turning of the satellite and its transit into the vicinity of position $\varphi = \pi$. The distance l converges asymptotically to l_0. The value l is in the vicinity of the value l_0, which is

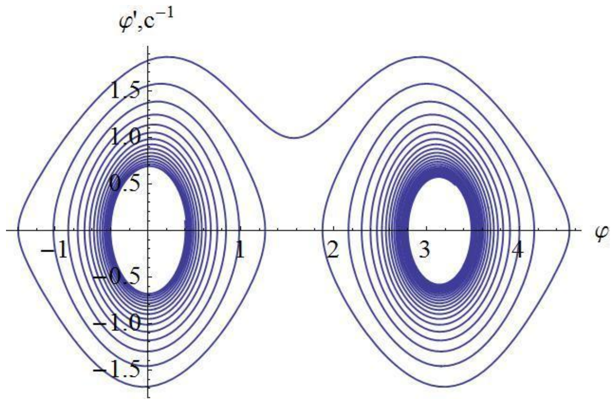

Fig. 3. Phase portrait.

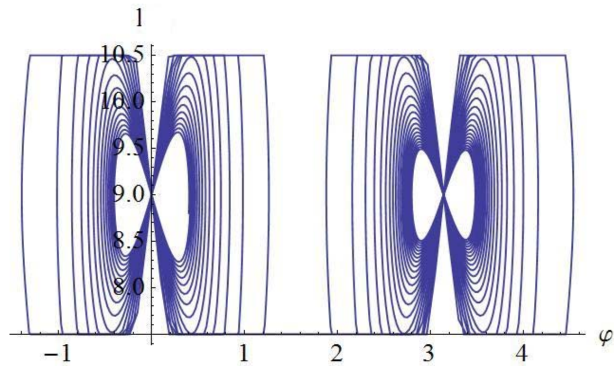

Fig. 4. Value $l(\varphi)$.

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defined by a constant $b = 1{,}5$. The satellite reorientation is carried out counterclockwise.

Another result of the numerical research is a numerical integration of controlled motions for different values of the parameter a. It showed the choice of the values of this parameter can be used for control the direction of rotation of the satellite at the same initial conditions.

V. Conclusion

The equation of controlled planar motions relative to the center of mass of the dumbbell shaped satellite with a moving mass in a circular orbit under the action of the gravitational torque was obtained in this paper. New control laws of a moving mass under the conditions limitations this motion was constructed. These laws are solve problems of gravitational stabilization with respect to planar perturbations of relative equilibrium of the dumbbell shaped satellite in a circular orbit and its diametrical reorientation by controlling motion of a movable mass. The Lyapunov functions necessary for a rigorous proof of the asymptotic stability and instability of studied movements were constructed for the proposed control. A numerical integration confirmed the findings.

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