Processing Medical Images by New Several Mathematics Shearlet Transform

Luis Cadena, Nikolai Espinosa, Franklin Cadena, Svetlana Kirillova, Daria Barkova, Alexander Zotin

***Abstract* — A new methodology is shown to perform medical image processing by the shearlet transform.**
The contours of the processed images are obtained and compared with those obtained with classic processing filters. Thus it is shown that the shearlet transform performs processing images with greater precision.
The results obtained with the filter shearlet, compared with Prewitt filter and Sobel for dental, urological and lung images.

***Index Terms* – image processing, shearlet transform, contour detect, filter Sobel, filter Prewitt.**

I. INTRODUCTION

Natural images are governed by anisotropic structure. The image basically consist of smooth regions separated by edges, it is suggestive to use a model consisting of piecewise regular functions [1-2, 9].

A simple image with one discontinuity along a smooth curve is represented by the two types of basis functions: isotropic and anisotropic. Isotropic basis functions generate a large number of significant coefficients around the discontinuity. Anisotropic basis functions trace the discontinuity line and produce just a few significant coefficients [3].

Shearlets were introduced by Guo, Kutyniok, Labate, Lim and Weiss in [1-3, 5-14] to address this problem.

II. SHEARLET TRANSFORM

Shearlets are obtained by translating, dilating and shearing a single mother function. Thus, the elements of a shearlet system are distributed not only at various scales and locations - as in classical wavelet theory - but also at various orientations. Thanks to this directional sensitivity property, shearlets are able to capture anisotropic features, like edges, that frequently dominate multidimensional phenomena, and to obtain optimally sparse approximations. Moreover, the simple mathematical structure of shearlets allows for the generalization to higher dimensions and to treat uniformly the continuum and the discrete realms, as well as fast algorithmic implementation [11-16, 18].

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The shearlets $\psi_{a,s,t}$ emerge by dilation, shearing and translation of a function $\psi \in L^2(\mathbf{R}^2)$ as follows

$$\left\{ \psi_{a,s,t} := a^{-\frac{3}{4}}\psi\left(A_a^{-1}S_s^{-1}(\cdot - t)\right) = a^{-\frac{3}{4}}\psi\left(\begin{pmatrix} \frac{1}{a} & -\frac{s}{a} \\ 0 & \frac{1}{\sqrt{a}} \end{pmatrix}(\cdot - t)\right) : a \in \mathbf{R}^+, s \in \mathbf{R}, t \in \mathbf{R}^2 \right\}$$

The description of the equation is detailed in [18]

In Figure 1 show the splitting of frequency plane for cone-adapted continuous shearlet system

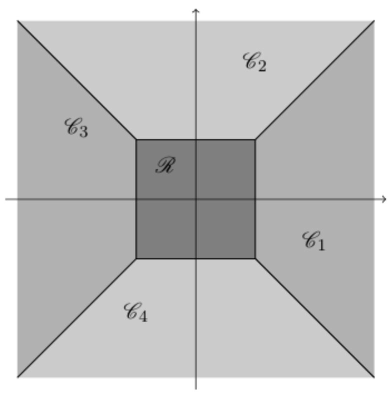


Figure 1. Splitting of frequency plane for cone-adapted continuous shearlet system

Definition 1. For $\phi, \psi, \tilde{\psi} \in L^2(\mathbb{R}^2)$, the cone-adapted continuous shearlet system $\mathcal{SH}(\phi, \psi, \tilde{\psi})$ is defined by [9]

$$\mathcal{SH}(\phi, \psi, \tilde{\psi}) = \Phi(\phi) \cup \Psi(\psi) \cup \tilde{\Psi}(\tilde{\psi}),$$

where

$\Phi(\phi) = \{\phi_t = \phi(\cdot - t) : t \in \mathbb{R}\},$

$\Psi(\psi) = \left\{\psi_{a,s,t} = a^{-\frac{3}{4}}\psi(A_a^{-1}S_s^{-1}(\cdot - t)) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\right\}$,

$\tilde{\Psi}(\tilde{\psi}) = \left\{\tilde{\psi}_{a,s,t} = a^{-\frac{3}{4}}\psi(\tilde{A}_a^{-1}S_s^{-1}(\cdot - t)) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\right\}$,

and $\tilde{A}_a = diag(a^{\frac{1}{2}}, a)$.

In this case shear parameter s has only finite set of possible values, so we can define a subset of possible shears.

In the following, the function ϕ will be chosen to have compact frequency support near the origin, which ensures that the system Φ(ϕ) is associated with the low frequency region[9].

Similar to the situation of continuous shearlet systems, an associated transform can be defined for cone-adapted continuous shearlet systems[9].

Definition 2. Then, for $, \psi, \psi^{\tilde{}} \in L^2(\mathbb{R}^2)$, the Cone-Adapted Continuous Shearlet Transform of $f \in L^2(\mathbb{R}^2)$ is the mapping [9]

$$f \to \mathcal{sh}_{\phi,\psi,\tilde{\psi}} f\left(t', (a,s,t), (\tilde{a},\tilde{s},\tilde{t})\right) = \left(\langle f, \phi_t\rangle, \langle f, \psi_{a,s,t}\rangle, \langle f, \tilde{\psi}_{\tilde{a},\tilde{s},\tilde{t}}\rangle\right),$$

where $\left(t', (a,s,t), (\tilde{a},\tilde{s},\tilde{t})\right) \in \mathbb{R}^2 \times \mathbb{S}^2_{cone}$,

$$\mathbb{S}_{cone} = \left\{(a,s,t) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\right\}.$$

It is shown that shearlet transform can be obtained by the following formula [6]:

$$\mathcal{sh}_\psi \widehat{f(a,s},t)(x) = a^{\frac{3}{4}}\hat{f}(x)\overline{\hat{\psi}(A_a S_s^T x)}.$$

Discrete Shearlet Transform is detailed in [18]

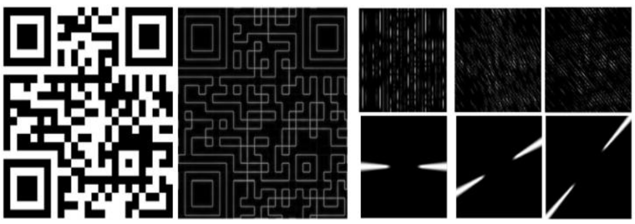
III.- CONTOUR DETECT OF OBJECTS IN THE IMAGE

Consider the problem - contour detection of objects in the image. Investigation of the algorithm FFST [15-16] found that the contours of objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the scale and the last of all possible values of the shift parameter. In this regard, it is proposed to use this feature in solving our problems:

$$f_{cont} = \sum_{k=0}^{k_{max}} \sum_{m=0}^{m_{max}} \mathcal{sh}_\psi(f(j*, k, m)),$$

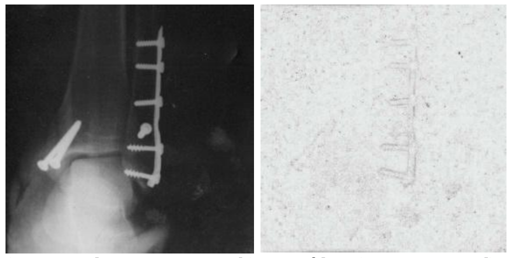
where sh_ψ assigns the coefficients of the function f sh_ψ f (j *, k, m), obtained for the last scale j*, orientation k and displacement m, where k_{max} - the maximum number of turns, m_{max} - the maximum number of displacements:

The results of this task using a modified algorithm FFST shown in various data (Fig. 2-4). The modified algorithm is proposed to be used for contour detection (Fig. 2).

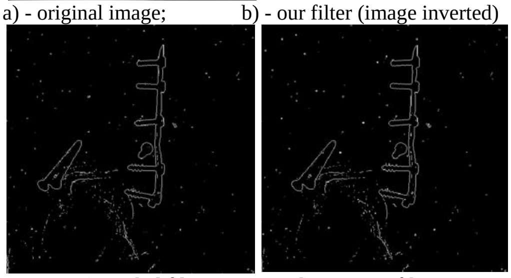


(a): Shearlet transform for model image b) for detect contour
Figure 2 – detect contour for model image.

The results of this task using a modified algorithm for data processing FFST tomography are shown in Figure 3 (a specialized system solves the problem of human ecology [17]). Table 1 shows the results of the corresponding calculations for some images and a comparison with Sobel filters and Prewitt. The modified algorithm is comparable in accuracy to the classical algorithms Sobel and Prewitt (Fig. 4).



a) - original image; b) - our filter (image inverted)



c) - Sobel filter d) - Prewitt filter
Figure 3 - edge detect in the foot X-ray

TABLE I
VALUE METRICS PSNR (dB)

image (512x512)	Our algorithm (modify FFST)	Algorithm Sobel	Algorithm Prewitt
dental	24.7998286	24.7508017	24.7508017
foot	24.4680013	24.3784720	24.3784744
legs	24.1742415	24.1690470	24.1690490
skull	25.0442384	24.9960827	24.9960827
lung1	24.5130034	24.4768184	24.4768148
lung2	24.1099857	24.1079805	24.1079786
urology1	27.0594257	27.0041530	27.0041667
urology2	28.1983404	28.1227822	28.1227840

Value metrics PSNR (dB) for solving the problem edge detect for different X-rays medical images

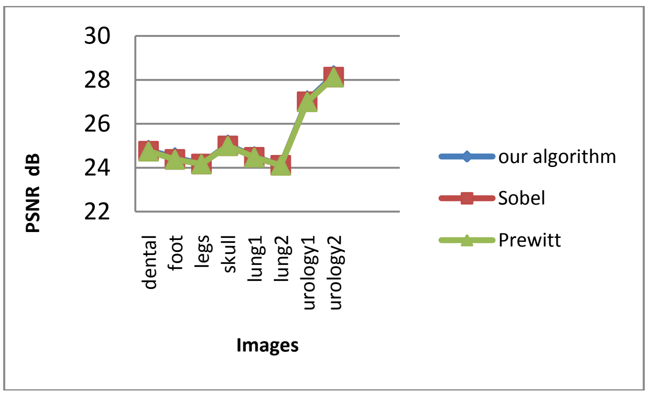


Figure 4 – Comparison our algorithm, Sobel and Prewitt in the problem contour detect for different images, in the figure line "our algorithm" overlaps with others.

IV. CONCLUSIONS

We take advantages of the Shearlet transform to find solution to the problem of contour in the image, to undertake using the modified algorithm FFST, where the contours of

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objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the last scale and the of all possible values of the shift parameter. The modified algorithm is comparable in accuracy to the classical algorithms Sobel and Prewitt. From table 1 our filter is 0.17% best than filter Sobel and Prewitt and because in Figure 4 line “our algorithm” overlaps with others, difference Sobel and Prewitt is 0.000007 %.

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