# Inverse Sine Function Circuit with Temperature Compensation

Perm Apisitticharoonlert, Wandee Petchmaneelumka, and Vanchai Riewruja

*Abstract*—A technique to realize inverse sine function circuit with temperature compensation is presented in this paper. The hyperbolic tangent characteristic of bipolar-transistor differential pair existed in operational transconductance amplifier (OTA) is utilized for the proposed realization method. The proposed scheme provides a simple configuration and low cost. Simulation and experimental results confirming the performance of the proposed scheme are agreed with the theoretical values.

*Index Terms*—sine-to-triangular waveform converter, operational transconductance amplifier, temperature compensation, hyperbolic tangent

## I. INTRODUCTION

Inverse sine function circuit or sine-to-triangular waveform converter is important circuit building block in electronic signal processing, instrumentation and measurement system. Many applications of sine-to-triangular waveform converter can be found in the literatures [1-4]. The techniques for implementation of sine-to-triangular waveform converter based on the use of hyperbolic tangent characteristic of bipolar-transistor differential pair existed in OTA have been reported [4-5]. Unfortunately, the performance of these approaches is disturbed by ambient temperature which exists in the behavior of OTA. Therefore, the aim of this paper is to propose a sine-to-triangular waveform converter based on the use of OTA's characteristic. The temperature effect of OTA used in the proposed scheme is compensated. Moreover, the circuit configuration is simple and low cost. PSPICE simulation and experimental results verifying the performance of the proposed circuit agreed with theoretical values are given.

## II. CIRCUIT DESCRIPTION

#### A. Principle of OTA

Basic scheme of BJT-based OTA and its symbol is shown

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in Fig. 1, where  $V_{in}$ ,  $I_B$  and  $I_o$  define the input voltage, bias current and output current of OTA, respectively.





Fig. 1. BJT-based of OTA.

Transistors  $Q_1 - Q_2$  form the differential pair. Transistors  $Q_3 - Q_4$ ,  $Q_5 - Q_6$  and  $Q_7 - Q_8$  function as the current mirrors CM<sub>1</sub>, CM<sub>2</sub> and CM<sub>3</sub>, respectively, with unity gain. The relation between  $V_{in}$  and  $I_o$  can be given by

$$I_o = I_B \tanh(V_{in} / 2V_T) \tag{1a}$$

or

$$V_{in} = 2V_T \tanh^{-1}(I_o / I_B) \tag{1b}$$

where  $V_T$  is the thermal voltage. From (1b), the hyperbolic tangent term in (1a) can be expressed as

$$I_{o} = I_{B} \left( \frac{V_{in}}{2V_{T}} - \frac{1}{3} \left( \frac{V_{in}}{2V_{T}} \right)^{3} + \frac{2}{15} \left( \frac{V_{in}}{2V_{T}} \right)^{5} - \dots \right)$$
(2)

It can be seen that the series covers to a sine function as

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$
(3)

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If the input voltage of OTA  $V_{in}$  is weighted with the appropriated value of the factor *m* for 0 < m < 1, then the output current  $I_o$  can be approximated as

$$I_o \cong I_B \sin\left(\frac{V_{in}}{2V_T}\right) \tag{4}$$

## B. Proposed circuit

The proposed principle is based on the use of the inverse function technique of an inverting amplifier using operational amplifier (opamp) as shown in Fig. 2.



Fig. 2 Simple inverse sine function.

From routine circuit analysis, the relation between the currents  $i_1$  and  $i_2$  can be stated as

$$I_1 + I_2 = 0 (5)$$

For (1a), (5) can be rewritten as

$$\frac{V_{in}}{R_1} + I_B \tanh\left(\frac{V_o}{2V_T}\right) = 0$$
(6a)

or

$$V_o = -2V_T \tanh^{-1} \left( \frac{V_{in}}{I_B R} \right)$$
(6b)

The power series of (6b) can be stated as

$$V_{o} = -2V_{T} \left( \frac{V_{in}}{I_{B}R} + \frac{1}{3} \left( \frac{V_{in}}{I_{B}R} \right)^{3} + \frac{1}{5} \left( \frac{V_{in}}{I_{B}R} \right)^{5} + \dots \right)$$
(7)

From (7), the series is corresponded to an inverse sine function as [6]

$$K\sin^{-1}(x) = K_T \left( x + B_3 \frac{x^3}{3} + B_5 \frac{x^5}{5} + \dots \right)$$
 (8a)

and

$$B_{i} = \prod_{j=1}^{\left(\frac{i-1}{2}\right)} \left[\frac{(i-2j)}{i-2j+1}\right] \qquad for \quad i = 3, 5, 7, \dots$$
(8b)

If the current  $I_2$  is chosen with the appropriated value by the weighting factor *m* at the output voltage  $V_o$ , then the series in (7) can then be expressed as

$$V_o = -2V_T \sin^{-1} \left( \frac{mV_{in}}{I_B R} \right) \tag{9}$$

From (9), the term of thermal voltage  $V_T$  causes the output voltage  $V_o$  depended on the ambient temperature. The proposed inverse sine function with temperature compensation is shown in Fig. 3. It consists of opamp OA<sub>1</sub>, OTAs A<sub>1</sub> – A<sub>2</sub>, constant resistors  $R_1$  and  $R_L$  and variable resistor  $R_v$ . The operation of the proposed scheme can be explained as follow. The input voltage  $V_{in}$  is applied to the proposed circuit in Fig. 3.



Fig. 3. Proposed inverse sine function circuit.

The voltage  $V_2$  can be expressed as

$$V_2 = -2V_T \sin^{-1} \left(\frac{mV_{in}}{I_{B1}R}\right) \tag{10}$$

The voltage  $V_2$  is attenuated by the variable resistor  $R_v$  to obtain the appropriated value for OTA A<sub>2</sub>. The output voltage  $V_{out}$  can be expressed as

$$V_{out} = -\frac{I_{B2}}{2V_T} R_L k V_2 \tag{11}$$

where k is the optimal gain of the voltages  $V_2$  for OTA A<sub>2</sub>. Substitute V<sub>2</sub> from (10) into (11), the output voltage  $V_{out}$  of the proposed converter can be rewritten as

$$V_{out} = -\frac{I_{B2}}{2V_T} R_L k \left( -2V_T \sin^{-1} \left( \frac{mV_{in}}{I_{B1}R} \right) \right)$$
  
$$= I_{B2} R_L k \sin^{-1} \left( \frac{mV_{in}}{I_{B1}R} \right)$$
(12)

It should be noted that the output of the proposed sine-totriangular converter is in the form of inverse sine. Moreover, temperature existing in thermal voltage  $V_T$  is compensated.

#### **III. SIMULATION RESULTS**

To verify the performance of the proposed inverse sine function, the circuit in Fig. 3 was simulated using PSPICE simulation program. The commercial opamp and OTA

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models are selected for OA<sub>1</sub> and A<sub>1</sub> – A<sub>2</sub>, respectively. The circuit parameters  $R_1 = 1k\Omega$ ,  $R_L = 10k\Omega$ ,  $I_{B1} = 100\mu$ A and  $I_{B2} = 400\mu$ A were chosen. The supply voltage was set to ±10V. Fig. 4 demonstrates the simulation result of the proposed inverse sine function circuit, where  $V_{in}$  is 500Hz sinusoidal with 200mV<sub>pp</sub>.



Fig. 4. Simulation result of the proposed sine-to-triangular waveform converter.



Fig. 5. Simulated frequency spectrum of the proposed sine-to-triangular waveform converter.





Fig. 6. Simulation results of temperature change by 3 difference values. (a) obtained triangular wave (b) zoom in at the peak of the obtained output

Fig. 5 shows simulated frequency spectrum of the obtained triangular signal in Fig. 4. Fig. 6 illustrates the simulated results by changing temperature with 3 difference values  $(30^{\circ}C, 50^{\circ}C \text{ and } 70^{\circ}C)$ . From Fig. 6(b), the proposed circuit can minimize the effect from the ambient temperature, which the error caused by temperature changing is about 1.9%. The error of the It is evident that the performance of the proposed scheme is agreed with the expected values.

#### IV. EXPERIMENTAL RESULTS

The proposed inverse sine function circuit in Fig. 3 was also experimentally implemented using commercial available devices LM351 and CA3280 for opamp OA<sub>1</sub> and OTAs  $A_1 - A_2$ , respectively. The circuit parameters are summarized in Table I. Experimental result of the proposed inverse sine function circuit is shown in Fig. 7(a). Frequency spectrum of the obtained triangular wave provided from the proposed circuit is shown in Fig. 7(b). It is clearly seen that the proposed converter is close agreement with the expected value.

TABLE I           PARAMETERS SET IN PRACTICE REALIZATION IN FIG. 3	
parameter	value
Supply voltage	±10V
$I_{B1}$	300µA
$I_{B2}$	700µA
$R_1$	1kΩ
$R_L$	$10k\Omega$

#### V. CONCLUSION

Simple technique to realize inverse sine function or sineto-triangular waveform converter with temperature compensation has been introduced in this paper. The proposed circuit provides simple configuration and low cost. Simulation and experimental results confirming the circuit performance are agreed with the expected values. Proceedings of the International MultiConference of Engineers and Computer Scientists 2016 Vol II, IMECS 2016, March 16 - 18, 2016, Hong Kong



Fig. 7. Experimental results of the proposed sine-to-triangular waveform converter.

(a) behavior of the proposed circuit

(b) frequency spectrum

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