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A Discrete Robust Adaptive Iterative Learning Control for a Class of Nonlinear Systems with Unknown Control Direction

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***Abstract*—In this paper, a discrete robust adaptive iterative learning control is proposed for a class of uncertain nonlinear systems with unknown control direction and random bounded disturbances. Based on a new design methodology, the problem of unknown sign and upper bound of the time-varying input gain parameter can be solved. In order to deal with the uncertainties from random bounded disturbance and unknown input gain parameter, a dead zone like auxiliary error with a time-varying boundary layer is introduced. This proposed auxiliary error is utilized for the construction of adaptive laws and the time-varying boundary layer is applied as a bounding parameter. By using a Lyapunov like analysis, it is shown that the closed-loop is stable and the internal signals are bounded for all the iterations. Besides, the norm of output tracking error will asymptotically converge to a residual set which is bounded by the width of boundary layer.**

***Index Terms*—robust adaptive iterative learning control, nonlinear systems, random bounded disturbance, unknown input gain parameter, unknown control direction.**

I. Introduction

In order to perform the tasks of repeated tracking control or periodic disturbance rejection in a finite time interval, adaptive iterative learning control (AILC) is one of the most successful and attractive ILC approaches [1], [2], [3] in the past two decades. In the research field of AILC, most of the AILC algorithms [4], [5], [6], [7], [8] were studied for continuous-time linear or nonlinear systems. However, a subsequent real implementation of the AILC algorithm needs to store the data of desired trajectory, system output and control parameters in memory. Therefore, it is more practical to design the AILC in discrete-time domain. Recently, some discrete AILC schemes have been studied for SISO [9], [10], [11] or MIMO [12] discrete-time nonlinear systems. In these aforementioned discrete AILC, a main required condition on the plant is that the plant nonlinearities are linearly parameterizable and the unknown parameters must be linear with respective to some known nonlinear functions in order to design suitable adaptive laws. Furthermore, the disturbance must be assumed to be repeatable or small enough for the technical analysis. In our previous work [13], another discrete AILC was proposed for the similar uncertain discrete-time nonlinear systems which can deal with not only the problems of iteration-varying reference trajectories and random bounded initial resetting error but also the problem of random bounded disturbance.

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However, the system control direction or even the upper bound of the input gain parameter is required to be known for the design of discrete AILC in the above works [9], [10], [11], [12], [13]. In [14], [15], [16], the ILC algorithms were applied for nonlinear systems without prior knowledge on system control direction. Since the system control direction is unknown, Nussbaum-type gain function was used to design the AILC algorithms for SISO [14], [15] and MIMO [16] continuous-time nonlinear systems. Based on the motivation of continuous Nussbaum-type gain function, the discrete Nussbaum-type gain function and n-step ahead predictor approach were presented in [17], [18] for discrete AILC of nonlinear systems with unknown control direction. Recently, a modified projection based adaptive law without using Nussbaum-type gain function was introduced in a discrete AILC for nonlinear systems whose control direction is unknown. But unfortunately, the disturbance is also required to be repeatable as those in [9], [10], [11], [12].

In this paper, a discrete robust adaptive iterative learning control (RAILC) is proposed for a class of uncertain nonlinear systems with unknown control direction and random bounded disturbance. Based on a new design methodology, the problem of unknown sign and upper bound of the time-varying input gain parameter is successfully solved. In order to deal with the uncertainties from random bounded disturbance and unknown input gain parameter, a new dead zone like auxiliary error with a time-varying boundary layer is introduced in this paper. This proposed auxiliary error is utilized for the construction of adaptive laws and the time-varying boundary layer is applied as a bounding parameter for the uncertainty. By using a Lyapunov like analysis, it is shown that the closed-loop is stable and the internal signals are bounded for all the iterations. Besides, the norm of output tracking error will asymptotically converge to a residual set whose size is bounded by the width of boundary layer.

This paper is organized as follows. In section II, a problem formulation is given. The discrete RAILC is presented in section III. Based on the proposed discrete RAILC and a derived error model, the analysis of closed-loop stability and learning performance will be studied extensively in Section IV. A simulation example will be given in Section V to demonstrate the effectiveness of the proposed learning controller. Finally a conclusion is made in Section VI.

II. Problem Formulation

In this paper, we consider a class of nonlinear discrete-time systems which can perform a given task repeatedly over a finite time sequence $t \in \{0, 1, \cdots, N\}$ as follows:

$$y^j(t+1) = \theta(t)^\top f(y^j(t), t) + b(t)u^j(t) + d^j(t) \quad (1)$$

where $y^j(t) \in R^1$ is the system output, $u^j(t) \in R^1$ is the control input, $\theta(t) \in R^{n\times 1}$ is an unknown time-varying system parameter vector, $f(y^j(t), t) \in R^{n\times 1}$ is a known nonlinear function vector, $b(t) \in R$ is an unknown time-varying input gain parameter and $d^j(t) \in R$ is an unknown non-repeatable disturbance. Here, j denotes the index of iteration and $t \in \{0, 1, \cdots, N\}$. Given a specified desired trajectory $y_d^j(t)$, $t \in \{0, 1, \cdots, N+1\}$, the control objective is to force the output $y^j(t)$ to follow $y_d^j(t)$ such that $\lim_{j\to\infty} |y_d^j(t) - y^j(t)| \le \epsilon$ for some small positive error tolerance bound ϵ and for $t \in \{1, 2, \cdots, N+1\}$. In order to achieve this control objective, some assumptions on the nonlinear discrete-time system and desired trajectory are given as follows:

(A1) The nonlinear discrete-time system is a relaxed system whose input $u^j(t)$ and output $y^j(t)$ are related by $y^j(t) = 0$, $t < 0$.
(A2) The nonlinear function $f(y^j(t), t)$ is bounded if $y^j(t)$ is bounded.
(A3) Let output tracking error be defined as $e^j(t) = y^j(t) - y_d(t)$. The initial output error at each iteration $e^j(0)$ is bounded.
(A4) The unknown non-repeatable disturbance is bounded, i.e., $|d^j(t)| \le d_U$ for an unknown positive constant d_U and for all $j \ge 1$.

III. Robust Adaptive Iterative Learning Controller

The output tracking error satisfies

$$\begin{aligned} &e^j(t+1) \\ &= y^j(t+1) - y_d^j(t+1) \\ &= \theta(t)^\top f(y^j(t), t) + b(t)u^j(t) + d^j(t) - y_d^j(t+1) \\ &= \theta^*(t)^\top \xi(y^j(t), y_d^j(t+1), t) + b(t)u^j(t) + d^j(t) \quad (2) \end{aligned}$$

where $\theta^*(t) = [\theta(t)^\top, -1]^\top \in R^{(n+1)\times 1}$ and $\xi(y^j(t), y_d^j(t+1), t) = [f(y^j(t), t)^\top, y_d^j(t+1)]^\top \in R^{(n+1)\times 1}$. In the following discussions, we will define $\xi^j(t) \equiv \xi(y^j(t), y_d^j(t+1), t)$ for simplicity. Based on the error equation in (2), we propose the adaptive iterative learning controller for the class of repeatable discrete-time nonlinear systems (1) as follows :

$$u^j(t) = \frac{\widehat{b}^j(t)}{\delta + \widehat{b}^j(t)^2} \left[-\theta^j(t)^\top \xi^j(t)\right] \quad (3)$$

where $\delta > 0$. Substituting (3) into (2), we can find that

$$\begin{aligned} &e^j(t+1) \\ &= \theta^*(t)^\top \xi^j(t) - \theta^j(t)^\top \xi^j(t) + b(t)u^j(t) - \widehat{b}(t)u^j(t) \\ &\quad + \theta^j(t)^\top \xi^j(t) + \widehat{b}(t)u^j(t) + d^j(t) \\ &= \left(\theta^*(t) - \theta^j(t)\right)^\top \xi^j(t) + \left(b(t) - \widehat{b}^j(t)\right)u^j(t) \\ &\quad + \theta^j(t)^\top \xi^j(t) + \frac{\widehat{b}^j(t)^2}{\delta + \widehat{b}^j(t)^2}\left[-\theta^j(t)^\top \xi^j(t)\right] + d^j(t) \\ &= \left(\theta^*(t) - \theta^j(t)\right)^\top \xi^j(t) + \left(b(t) - \widehat{b}^j(t)\right)u^j(t) \\ &\quad + \frac{\delta}{\delta + \widehat{b}^j(t)^2}\left[\theta^j(t)^\top \xi^j(t)\right] + d^j(t) \\ &= \left(\theta^*(t) - \theta^j(t)\right)^\top \xi^j(t) + \left(b(t) - \widehat{b}^j(t)\right)u^j(t) + \delta_L^j(t) \quad (4) \end{aligned}$$

where

$$\delta_L^j(t) = \frac{\delta}{\delta + \widehat{b}^j(t)^2}\left[\theta^j(t)^\top \xi^j(t)\right] + d^j(t)$$

The bounding function of $\delta_L^j(t)$ can be shown to satisfy the following result,

$$\begin{aligned} |\delta_L^j(t)| &\le \left|\frac{\delta}{\delta + \widehat{b}^j(t)^2}\left[\theta^j(t)^\top \xi^j(t)\right]\right| + \left|d^j(t)\right| \\ &\le \left|\theta^j(t)\right|\left|\xi^j(t)\right| + d_U \\ &\le \psi^*\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right| + 1\right) \quad (5) \end{aligned}$$

where $\psi^* = \max\{1, d_U\}$ is a positive constant.

In order to overcome the uncertainty $\delta_L^j(t)$, we now define an auxiliary error $e_\phi^j(t+1)$ as follows:

$$e_\phi^j(t+1) = e^j(t+1) - \phi^j(t)\mathbf{sat}\left(\frac{e^j(t+1)}{\phi^j(t)}\right) \quad (6)$$

for $t \in \{0, 1, \cdots, N\}$. We don't define $e_\phi^j(0)$ since it will not be utilized in our design of controller and adaptive laws. In (6), **sat** is the saturation function defined as

$$\mathbf{sat}\left(\frac{e^j(t+1)}{\phi^j(t)}\right) = \begin{cases} 1 & \text{if } e^j(t+1) > \phi^j(t) \\ \frac{e^j(t+1)}{\phi^j(t)} & \text{if } |e^j(t+1)| \le \phi^j(t) \\ -1 & \text{if } e^j(t+1) < -\phi^j(t) \end{cases}$$

and $\phi^j(t)$ is the width of the time-varying boundary layer designed as

$$\phi^j(t) = \psi^j(t)\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right| + 1\right) \quad (7)$$

where $\psi^j(t)$ is a parameter to be updated later. It is noted that $e_\phi^j(t+1)$ can be rewritten as

$$e_\phi^j(t+1) = \begin{cases} e^j(t+1) - \phi^j(t) & \text{if } e^j(t+1) > \phi^j(t) \\ 0 & \text{if } |e^j(t+1)| \le \phi^j(t) \\ e^j(t+1) + \phi^j(t) & \text{if } e^j(t+1) < -\phi^j(t) \end{cases}$$

and it can be easily shown that $e_\phi^j(t+1)\mathbf{sat}\left(\frac{e^j(t+1)}{\phi^j(t)}\right) = |e_\phi^j(t+1)|$, $\forall j \ge 1$.

In this RAILC, $\theta^j(t)$, $\widehat{b}^j(t)$ in (3) and $\psi^j(t)$ in (7) are designed to compensate the unknown optimal control parameter vector $\theta^*(t)$, $b(t)$ and ψ^*, respectively. The adaptive laws for $\theta^j(t)$, $\widehat{b}^j(t)$ and $\psi^j(t)$ at (next) $j+1$th iteration are given as follows :

$$\theta^{j+1}(t) = \theta^j(t) + \frac{\beta_1 e_\phi^j(t+1)\xi^j(t)}{1 + \left|\xi^j(t)\right|^2 + \left|u^j(t)\right|^2 + \left(\left|\theta^j(t)\right|\left|\xi^j(t)\right| + 1\right)^2} \quad (8)$$

$$\widehat{b}^{j+1}(t) = \widehat{b}^j(t) + \frac{\beta_2 e_\phi^j(t+1)u^j(t)}{1 + \left|\xi^j(t)\right|^2 + \left|u^j(t)\right|^2 + \left(\left|\theta^j(t)\right|\left|\xi^j(t)\right| + 1\right)^2} \quad (9)$$

$\psi^{j+1}(t)$

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$$= \psi^j(t) + \frac{\beta_3|e_\phi^j(t+1)|\big(|\theta^j(t)||\xi^j(t)|+1\big)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \quad (10)$$

for $t \in \{0,1,\cdots,N\}$, where $\beta_1,\beta_2,\beta_3 > 0$ are the adaptation gains. For the first iteration, we set $\theta^1(t)=\theta^1$, $\widehat{b}^1(t)=\widehat{b}^1$ to be any constant vector and $\psi^1(t)=\psi^1>0$ to be a small fixed value $\forall t \in \{0,1,2,\cdots,N\}$. It is noted that $\psi^j(t)>0, \forall t \in \{0,1,\cdots,N\}$ and $\forall j \geq 1$. In general, we will choose $\widehat{b}^1(t)=\widehat{b}^1$ as a nonzero vector in order to prevent the controller (3) from being a zero input in the beginning of the learning process.

IV. Analysis of Stability and Convergence

Define the parameter errors as $\widetilde{\theta}^j(t)=\theta^j(t)-\theta^*(t)$, $\widetilde{\widehat{b}}^j(t)=\widehat{b}^j(t)-b(t)$, $\widetilde{\psi}^j(t)=\psi^j(t)-\psi^*$. Then it is easy to show, by subtracting the optimal control gains on both sides of (8)-(10), that

$$\widetilde{\theta}^{j+1}(t) = \widetilde{\theta}^j(t) + \frac{\beta_1 e_\phi^j(t+1)\xi^j(t)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \quad (11)$$

$$\widetilde{\widehat{b}}^{j+1}(t) = \widetilde{\widehat{b}}^j(t) + \frac{\beta_2 e_\phi^j(t+1)\widehat{b}^j(t)u^j(t)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \quad (12)$$

$$\widetilde{\psi}^{j+1}(t) = \widetilde{\psi}^j(t) + \frac{\beta_3|e_\phi^j(t+1)|\big(|\theta^j(t)||\xi^j(t)|+1\big)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \quad (13)$$

The following theorem states the main results of this paper.

Main Theorem. Consider the nonlinear system (1) satisfying the assumptions (A1)-(A3). If the robust adaptive iterative learning controller, designed as in (3), (6), (8), (9) and (10), is applied and the following condition is satisfied

$$2-\beta_1-\beta_2-\beta_3>0, \quad (14)$$

then we can get,

(t1) The adjustable parameters $\theta^j(t)$, $\widehat{b}^j(t)$, $\psi^j(t)$ are bounded $\forall t \in \{0,1,\cdots,N\}, j \geq 1$.

(t2) The auxiliary error $e_\phi^j(t+1)$ are bounded $\forall t \in \{0,1,\cdots,N\}, j \geq 1$ and

$$\lim_{j\to\infty} e_\phi^j(t+1)=0, \forall t \in \{0,1,\cdots,N\}$$

(t3) The output tracking error $e^j(t+1)$ and the input $u^j((t)$ are bounded $\forall t \in \{0,1,\cdots,N\}$, $j \geq 1$ and

$$\lim_{j\to\infty} |e^j(t+1)| \leq \psi^\infty(t)\left(|\theta^\infty(t)|\,|\xi^\infty(t)|+1\right), \forall\, t \in \{0,1,\cdots,N\}$$

Proof :

(t1) Define the cost functions of performance as follows

$$V^j(t) = \frac{1}{\beta_1}\widetilde{\theta}^j(t)^\top\widetilde{\theta}^j(t) + \frac{1}{\beta_2}\widetilde{\widehat{b}}^j(t)^2 + \frac{1}{\beta_3}\widetilde{\psi}^j(t)^2$$

The difference between $V^{j+1}(t)$ and $V^j(t)$ can be derived as follows :

$$\begin{aligned} V^{j+1}(t)-V^j(t) &= \frac{1}{\beta_1}\Big(\widetilde{\theta}^{j+1}(t)^\top\widetilde{\theta}^{j+1}(t)-\widetilde{\theta}^j(t)^\top\widetilde{\theta}^j(t)\Big) \\ &+\frac{1}{\beta_2}\Big(\widetilde{\widehat{b}}^{j+1}(t)^2-\widetilde{\widehat{b}}^j(t)^2\Big) \\ &+\frac{1}{\beta_3}\Big(\widetilde{\psi}^{j+1}(t)^2-\widetilde{\psi}^j(t)^2\Big) \\ &= \frac{2e_\phi^j(t+1)\widetilde{\theta}^j(t)^\top\xi^j(t)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \\ &+\frac{\beta_1 e_\phi^j(t+1)^2|\xi^j(t)|^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \\ &+\frac{2e_\phi^j(t+1)\widetilde{\widehat{b}}^j(t)u^j(t)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \\ &+\frac{\beta_2 e_\phi^j(t+1)^2\,|u^j(t)|^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \\ &+\frac{2|e_\phi^j(t+1)|\widetilde{\psi}^j(t)\big(|\theta^j(t)||\xi^j(t)|+1\big)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \\ &+\frac{\beta_3 e_\phi^j(t+1)^2\big(|\theta^j(t)||\xi^j(t)|+1\big)^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \end{aligned} \quad (15)$$

By using (4), it implies that

$$\begin{aligned} &e_\phi^j(t+1)\widetilde{\theta}^j(t)^\top\xi^j(t)+e_\phi^j(t+1)\widetilde{\widehat{b}}^j(t)u^j(t) \\ &= -e^j(t+1)e_\phi^j(t+1)+e_\phi^j(t+1)\delta_L^j(t) \end{aligned} \quad (16)$$

Substituting (16) into (15), it yields

$$\begin{aligned} V^{j+1}(t)-V^j(t) &\leq \frac{-2e^j(t+1)e_\phi^j(t+1)+2e_\phi^j(t+1)\delta_L^j(t)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \\ &+\frac{\beta_1 e_\phi^j(t+1)^2|\xi^j(t)|^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \\ &+\frac{\beta_2 e_\phi^j(t+1)^2|u^j(t)|^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \\ &+\frac{2|e_\phi^j(t+1)|\widetilde{\psi}^j(t)\big(|\theta^j(t)||\xi^j(t)|+1\big)}{1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2} \\ &+\frac{\beta_3 e_\phi^j(t+1)^2\big(|\theta^j(t)||\xi^j(t)|+1\big)^2}{\Big(1+|\xi^j(t)|^2+|u^j(t)|^2+\big(|\theta^j(t)||\xi^j(t)|+1\big)^2\Big)^2} \end{aligned} \quad (17)$$

Substituting (6) into (17) and using the fact that $|\delta_L^j(t)| \leq$

$\psi^*\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)$ in (5), we can derive that

$$\begin{aligned}
&V^{j+1}(t)-V^j(t)\\
\leq\ & \frac{-2e_\phi^j(t+1)^2}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
&-\frac{2|e_\phi^j(t+1)|\psi^j(t)\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
&+\frac{2|e_\phi^j(t+1)|\psi^*\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
&+\frac{2|e_\phi^j(t+1)|\widetilde{\psi}^j(t)\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
&+\frac{\beta_1 e_\phi^j(t+1)^2\left|\xi^j(t)\right|^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_2 e_\phi^j(t+1)^2\left|u^j(t)\right|^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_3 e_\phi^j(t+1)^2\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
=\ & \frac{-2e_\phi^j(t+1)^2}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
&+\frac{\beta_1 e_\phi^j(t+1)^2\left|\xi^j(t)\right|^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_2 e_\phi^j(t+1)^2\left|u^j(t)\right|^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
&+\frac{\beta_3 e_\phi^j(t+1)^2\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}{\left(1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2\right)^2}\\
\leq\ & \frac{-(2-\beta_1-\beta_2-\beta_3)e_\phi^j(t+1)^2}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}
\end{aligned} \tag{18}$$

If β_1, β_2 and β_3 are chosen such that $k\equiv 2-\beta_1-\beta_2-\beta_3>0$, then we have

$$\begin{aligned}
&V^{j+1}(t)-V^j(t)\\
\leq\ & \frac{-ke_\phi^j(t+1)^2}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}\\
\leq\ & 0
\end{aligned} \tag{19}$$

for $j\geq 1$. Since $V^1(t)$ is bounded $\forall t\in\{0,1,\cdots,N\}$ due to $\widetilde{\theta}^1(t)=\theta^1(t)-\theta^*(t)=\theta^1-\theta^*(t)$, $\widetilde{\widehat{b}}^1(t)=\widehat{b}^1(t)-b(t)=\widehat{b}^1-b(t)$ and $\widetilde{\psi}^1(t)=\psi^1(t)-\psi^*=\psi^1-\psi^*$ are bounded $\forall t\in\{0,1,\cdots,N\}$. We conclude from (19) that $V^j(t)$, and hence $\widetilde{\theta}^j(t)$, $\widetilde{\widehat{b}}^j(t)$ and $\widetilde{\psi}^j(t)$ are bounded $\forall j\geq 1$. This proves (t1) of the main theorem.

(t2) By summing (19) from 1 to j leads to

$$\begin{aligned}
&V^j(t)\\
\leq\ & V^1(t)\\
&-\sum_{i=1}^{j}\frac{ke_\phi^i(t+1)^2}{1+\left|\xi^i(t)\right|^2+\left|u^i(t)\right|^2+\left(\left|\theta^i(t)\right|\left|\xi^i(t)\right|+1\right)^2}
\end{aligned} \tag{20}$$

Since $V^1(t)$ is bounded and $V^j(t)$ must be nonnegative, we have

$$\lim_{j\to\infty}\frac{e_\phi^j(t+1)^2}{1+\left|\xi^j(t)\right|^2+\left|u^j(t)\right|^2+\left(\left|\theta^j(t)\right|\left|\xi^j(t)\right|+1\right)^2}=0 \tag{21}$$

$\forall t\in\{0,1,\cdots,N\}$. In order to prove that $u^j(t)$ and $e_\phi^j(t+1)$ are bounded and $e_\phi^j(t+1)$ will converge to zero $\forall t\in\{0,1,\cdots,N\}$, we take the following discussions.

(1) Since $y^j(0)$, $y_d^j(1)$, $\theta^j(0)$, $\widehat{b}^j(0)$, $\psi^j(0)$ are bounded $\forall j\geq 1$, we conclude that $\xi^j(0)=\xi(y^j(0),y_d^j(1),0)$ is bounded by using assumption (A2). The boundedness of $\xi^j(0)$ readily implies that $u^j(0)$ is bounded, and hence $1+\left|\xi^j(0)\right|^2+\left|u^j(0)\right|^2+\left(\left|\theta^j(0)\right|\left|\xi^j(0)\right|+1\right)^2$ as well as $e_\phi^j(1)$ are bounded $\forall j\geq 1$. If we let $t=0$ in (21), we have

$$\lim_{j\to\infty}e_\phi^j(1)^2=0 \tag{22}$$

(2) Since $e_\phi^j(1)$ is bounded $\forall j\geq 1$, it can be easily shown by using (6) that $e^j(1)$, $y^j(1)$ and hence $\xi^j(1)=\xi(y^j(1),y_d^j(2),1)$ are bounded $\forall j\geq 1$. Due to the fact that $\theta^j(1)$, $\widehat{b}^j(1)$ are bounded $\forall j\geq 1$ by (t1), we have $1+\left|\xi^j(1)\right|^2+\left|u^j(1)\right|^2+\left(\left|\theta^j(1)\right|\left|\xi^j(1)\right|+1\right)^2$ and $e_\phi^j(2)$ are bounded $\forall j\geq 1$. If we let $t=1$ in (21), we have

$$\lim_{j\to\infty}e_\phi^j(2)^2=0 \tag{23}$$

(3) Assume that $e_\phi^j(t')$ is bounded $\forall j\geq 1$ for some $t'\in\{2,3,\cdots,N\}$. Then $e^j(t'),y^j(t')$ and $\xi^j(t')=\xi(y^j(t'),y_d^j(t'+1),t')$ are bounded $\forall j\geq 1$. Due to the fact that $\theta^j(t')$, $\widehat{b}^j(t')$ are bounded $\forall j\geq 1$ by (t1), we have $1+\left|\xi^j(t')\right|^2+\left|u^j(t')\right|^2+\left(\left|\theta^j(t')\right|\left|\xi^j(t')\right|+1\right)^2$ and $e_\phi^j(t'+1)$ are bounded $\forall j\geq 1$. If we let $t=t'$ in (21), we have

$$\lim_{j\to\infty}e_\phi^j(t'+1)^2=0 \tag{24}$$

By mathematical induction, we now conclude that

$$\lim_{j\to\infty}e_\phi^j(t+1)^2=0,\forall t\in\{0,1,\cdots,N\} \tag{25}$$

and $e_\phi^j(t+1)$ is bounded $\forall t\in\{0,1,\cdots,N\}, j\geq 1$.

(t3) The boundedness of $e^j(t+1)$ at each iteration over $\{0,1,\cdots,N\}$ can be concluded from equation (6) because $\phi^j(t)$ is always bounded. Furthermore, the bound of $e^\infty(t+1)$ will satisfy

$$\begin{aligned}
&\lim_{j\to\infty}|e^j(t+1)|=|e^\infty(t+1)|\\
\leq\ & \psi^\infty(t)\left(\left|\theta^\infty(t)\right|\left|\xi^\infty(t)\right|+1\right),\forall\ t\in\{0,1,\cdots,N\}
\end{aligned}$$

This proves (t3) of the main theorem. Q.E.D.

Remark 1 : Since the output tracking error $e^j(t+1)$ can be shown to converge to a residual set which is bounded by the boundary layer $\psi^\infty(t)\left(\left|\theta^\infty(t)\right|\left|\xi^\infty(t)\right|+1\right)$, it is necessary to make $\psi^\infty(t)\left(\left|\theta^\infty(t)\right|\left|\xi^\infty(t)\right|+1\right)$ as small as possible for all $t\in\{0,1,\cdots,N\}$. This is why we set the initial value of ψ^1 as a small constant. The adaptation gain β_3 in (10)

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will be chosen as a small one such that $\psi^j(t)$ and hence, $\psi^j(t)\big(|\theta^j(t)||\xi^j(t)|+1\big), t \in \{0,1,\cdots,N\}$ will remain in a reasonable small value for all $j \geq 1$. Fortunately, the adaptation gain β_3 can be chosen as small as possible since it is required to satisfy the convergent condition (14).

Remark 2 : In this theorem, we derive the sufficient condition $2-\beta_1-\beta_2-\beta_3 > 0$ to guarantee the learning convergence. Compared with a similar convergent condition $2-b_U\beta_1-\beta_2 > 0$ given in our previous work [13] with b_U being the upper bound of the input gain function, it is clear that the proposed result is less restricted when choosing the learning gains β_1, β_2 and β_3. More importantly, we don't need the sign and upper bound of $b(t)$ for the controller design.

V. SIMULATION EXAMPLE

In this section, we use the proposed RAILC to iteratively control a nonlinear discrete-time plant [10]. The difference equation of the nonlinear dynamic plant is given as

$$y^j(t+1) = \theta(t)\sin^2(y^j(t)) + b(t)u^j(t) + d^j(t)$$

where $y^j(t)$ is the system output, $u^j(t)$ is the control input, $\theta(t) = 2 + 0.5\sin(t)$ is a time-varying parameter, $b(t) = 3 + 0.5\sin(t\pi/50)$ is a time-varying input gain and $d^j(t) = m^j \sin^3(t\pi/50)$ with $m^j = 0.1rand$ is a non-repeatable random disturbance. Here $rand$ is a uniform distribution on the interval $(0,1)$. Here the reference model is chosen as

$$y_d^j(t+1) = 0.3y_d^j(t) + r^j(t), \quad y_d(0) = 0.5rand$$

where $r^j(t) = 5\sin(2\pi t/5) + 0.3\sin(2\pi j/50)$ is an iteration-dependent bounded reference input.

The control objective is to make the system output $y^j(t)$ to track as close as possible the desired trajectory $y_d^j(t)$ for all $t \in \{1,\cdots,200\}$. To achieve the control objective, the discrete RAILC in (3), (6), (8), (9), and (10) is applied with the design parameters $\beta_1 = 0.9499$, $\beta_2 = 0.9499$, $\beta_3 = 0.0001$ so that $k \equiv 2-\beta_1-\beta_2-\beta_3 = 0.1$. Furthermore, we set $\delta = 0.1$ and choose the initial values in (3) as $\theta^1(t) = [0.1,-1]^\top$, $\xi(y^1(t), y_d^1(t+1), t) = [\sin^2(y^j(t)), y_d^1(t+1)]^\top$, $\hat{b}^j(t) = 0.1$ and $\psi^1(t) = 0.0015$ for $t \in \{0,1,2,\cdots,200\}$, respectively. In order to verify the robustness against varying initial resetting error $e^j(0)$ and the non-repeatable random disturbance $d^j(t)$, we show $\max_{t\in\{1,\cdots,200\}} |e_\phi^j(t)|$ with respective to iteration j in Fig. 1(a). The asymptotical convergence proves the technical result given in (t2) of main theorem. Since the learning process is almost completed at the 50th iteration, the learning error $e^{50}(t)$ is shown in Fig. 1(b). It clearly proves (t3) of the main theorem since the trajectory of $e^{50}(t)$ satisfies $-\psi^{50}(t)\big(|\theta^{50}(t)||\xi^{50}(t)|+1\big) \leq e^{50}(t) \leq \psi^{50}(t)\big(|\theta^{50}(t)||\xi^{50}(t)|+1\big)$, $t \in \{1,\cdots,200\}$ in Fig. 1(b). Since the nice output tracking performance at the 50th iteration are achieved, we show the relation between system output $y^{50}(t)$ and desired trajectory $y_d^{50}(t)$ in Fig. 1(c) for $t \in \{0,1,2,\cdots,200\}$. To see the control behavior that $y^{50}(t)$ is close to $y_d^{50}(t)$ for $t \in \{0,1,2,\cdots,200\}$ except the initial one $y^{50}(0)$, the trajectories between $y^{50}(t)$ and $y_d^{50}(t)$ are shown again in Fig. 1(d) but only for the time sequence $t \in \{0,1,2,\cdots,10\}$. It is clear that $y^{50}(t)$ converges to $y_d^{50}(t)$ after $t \geq 1$. Finally, Fig. 1(e) shows the bounded learned control force $u^{50}(t)$.

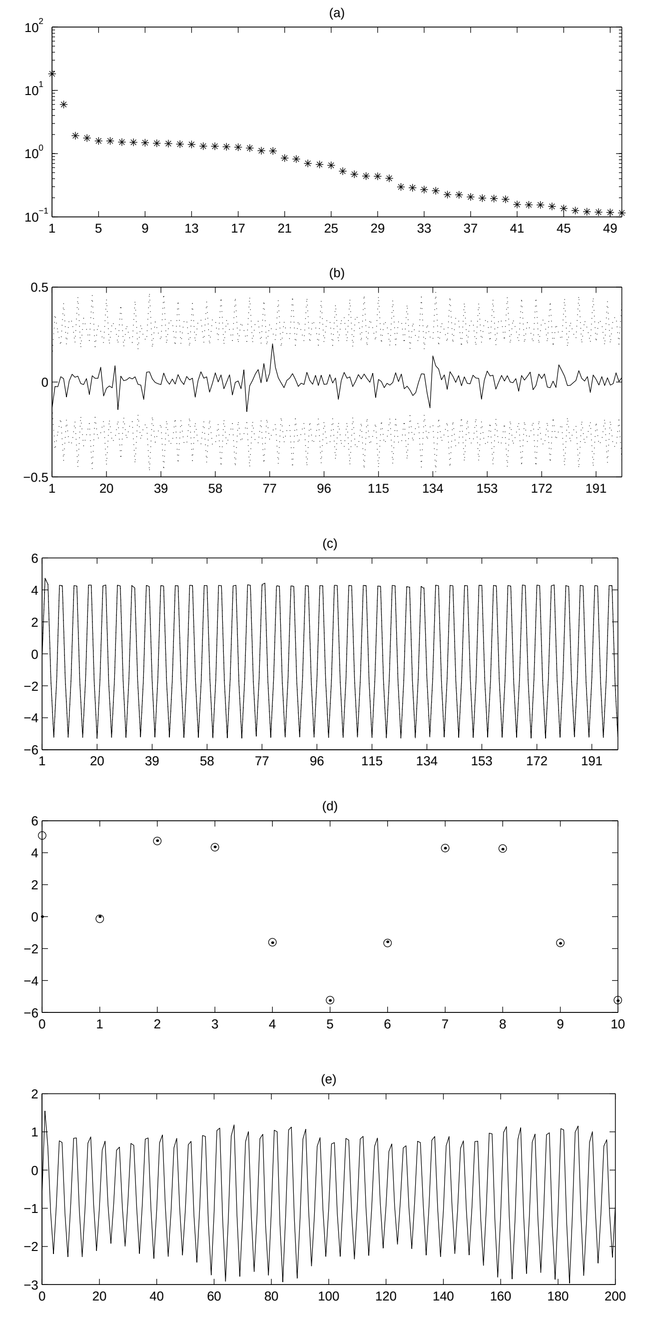

Fig. 1. (a)$\max_{t\in\{1,\cdots,200\}} |e_\phi^j(t)|$ versus control iteration j; (b)$e^{50}(t)$ (solid line) and $\psi^{50}(t)\big(|\theta^{50}(t)||\xi^{50}(t)|+1\big), -\psi^{50}(t)\big(|\theta^{50}(t)||\xi^{50}(t)|+1\big)$ (dotted lines) versus time $t \in \{1,2,\cdots,200\}$; (c)$y^{50}(t)$ (solid line) and $y_d^j(t)$ (dotted line) versus time $t \in \{0,1,\cdots,200\}$ at the 50th control iteration; (d)$y^{50}(t)$ ($\circ\circ\circ$) and $y_d^j(t)$ ($\cdots$) versus time $t \in \{0,1,\cdots,10\}$ at the 50th control iteration; (e)$u^{50}(t)$ versus time t.

VI. CONCLUSION

In this paper, we propose a discrete RAILC for a class of uncertain nonlinear systems with initial resetting output errors, iteration-varying reference trajectories, random bounded disturbances and unknown control direction. The RAILC is derived from an output tracking error model which successfully solve the possible singularity control problem due to unknown input gain parameter and its control direction. Compared with all the existing works dealing with similar discrete AILC problem, the class of nonlinear systems in this work can be more general in the sense

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that the control direction can be unknown and the input disturbance can be non-repeatable. Three control parameters in this RAILC are applied to compensate for the uncertainties from the unknown system parameters and input disturbance. By using a Lyapunov like analysis, it is shown that the control parameters and internal signals are bounded along the time axis for all iterations and the tracking error will asymptotically converge to a tunable residual set which is bounded by the width of boundary layer.

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