A Relaxed Explicit Extragradient-Like Method for Solving Generalized Mixed Equilibria, Variational Inequalities and Constrained Convex Minimization

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Abstract—**In this paper, we introduce a relaxed explicit extragradient-like scheme for finding a common element of the set of solutions of the minimization problem for a convex and continuously Fréchet differentiable functional, the set of solutions of a finite family of generalized mixed equilibrium problems and the set of solutions of a finite family of variational inequalities for inverse strongly monotone mappings in a real Hilbert space. Under suitable control conditions, we establish the strong convergence of the proposed scheme to the same common element of the above three sets, which is also the unique solution of a variational inequality defined over the intersection of the above three sets. Our results improve and extend the corresponding ones given by some authors recently in this area.**

Keywords: ***Multistep relaxed extragradient-like method; Averaged mapping approach to the gradient-projection algorithm; Convex minimization problem; Generalized mixed equilibrium problem; Variational inequality; Inverse-strongly monotone mapping.***

1 Introduction

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, I be the identity mapping on H, C be a nonempty closed convex subset of H and P_C be the metric (or nearest point) projection of H onto C, that is, the mapping $P_C : H \to C$ assigns to each point $x \in H$ the unique point $P_C x \in C$ satisfying the property

$$\|x - P_C x\| = \inf_{y\in C} \|x - y\|.$$

Let $T : C \to C$ be a self-mapping on C. We denote by $\mathrm{Fix}(T)$ the set of fixed points of T and by $\mathbf{R}$ the set of all real numbers. A mapping $A : H \to H$ is called $\bar{\gamma}$-strongly positive on H if there exists a constant $\bar{\gamma} > 0$ such that

$$\langle Ax, x\rangle \geq \bar{\gamma}\|x\|^2, \quad \forall x \in H.$$

A mapping $F : C \to H$ is called L-Lipschitz continuous if there exists a constant $L \geq 0$ such that

$$\|Fx - Fy\| \leq L\|x - y\|, \quad \forall x, y \in C.$$

In particular, if $L = 1$ then F is called a nonexpansive mapping; if $L \in [0, 1)$ then F is called a contraction. A mapping $T : C \to C$ is called k-strictly pseudocontractive (or a k-strict pseudocontraction) if there exists a constant $k \in [0, 1)$ such that

$$\|Tx-Ty\|^2 \leq \|x-y\|^2+k\|(I-T)x-(I-T)y\|^2, \quad \forall x, y \in C.$$

In particular, if $k = 0$, then T is a nonexpansive mapping. The mapping T is pseudocontractive if and only if

$$\langle Tx - Ty, x - y\rangle \leq \|x - y\|^2, \quad \forall x, y \in C.$$

T is strongly pseudocontractive if and only if there exists a constant $\lambda \in (0, 1)$ such that

$$\langle Tx - Ty, x - y\rangle \leq \lambda\|x - y\|^2, \quad \forall x, y \in C.$$

Note that the class of strictly pseudocontractive mappings includes the class of nonexpansive mappings as a subclass. That is, T is nonexpansive if and only if T is 0-strictly pseudocontractive. The mapping T is also said to be pseudocontractive if $k = 1$ and T is said to be strongly pseudocontractive if there exists a positive constant $\lambda \in (0, 1)$ such that $T + (1 - \lambda)I$ is pseudocontractive. Clearly, the class of strictly pseudocontractive mappings falls into the one between classes of nonexpansive mappings and pseudocontractive mappings. Also it

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is clear that the class of strongly pseudocontractive mappings is independent of the class of strictly pseudocontractive mappings (see [23]).The class of pseudocontractive mappings is one of the most important classes of mappings among nonlinear mappings. Recently, many authors have been devoting the study of the problem of finding fixed points of pseudocontractive mappings; see e.g., [12, 13, 14, 15, 16, 17, 22, 26] and the references therein.

Let $\mathcal{A} : C \to H$ be a nonlinear mapping on C. The variational inequality problem (in short, VIP) associated with the set C and the mapping $\mathcal{A}$ is stated as follows: find $x^* \in C$ such that

$$\langle \mathcal{A}x^*, x - x^* \rangle \geq 0, \quad \forall x \in C. \tag{1}$$

The solution set of VIP (1) is denoted by $\mathrm{VI}(C, \mathcal{A})$. The VIP (1) was first discussed by Lions [25]. There are many applications of VIP (1) in various fields. In 1976, Korpelevich [24] proposed an iterative algorithm, which is known as the extragradient method, for solving VIP (1) in Euclidean space $\mathbf{R}^n$. The literature on the VIP is vast and Korpelevich's extragradient method has received great attention given by many authors, who improved it in various ways; see e.g., [2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 18, 19, 20, 21, 27]and references therein, to name but a few.

On the other hand, let $\varphi : C \to \mathbf{R}$ be a real-valued function, $\mathcal{A} : C \to H$ be a nonlinear mapping and $\Theta : C \times C \to \mathbf{R}$ be a bifunction. In 2008, Peng and Yao [27] introduced the following generalized mixed equilibrium problem (in short, GMEP) of finding $x \in C$ such that

$$\Theta(x, y) + \varphi(y) - \varphi(x) + \langle \mathcal{A}x, y - x \rangle \geq 0, \quad \forall y \in C. \tag{2}$$

We denote the set of solutions of GMEP (2) by $\mathrm{GMEP}(\Theta, \varphi, \mathcal{A})$. The GMEP (2) is very general in the sense that it includes, as special cases, optimization problems, variational inequalities, minimax problems, Nash equilibrium problems in noncooperative games and others. The GMEP (2) is further considered and studied; see e.g., [3, 4, 5, 11, 19, 21].

Furthermore, let $f : C \to \mathbf{R}$ be a convex and continuously Fréchet differentiable functional. Consider the convex minimization problem (in short, CMP) of minimizing f over the constraint set C

$$\min_{x \in C} f(x) \tag{3}$$

(assuming the existence of minimizers). We denote by Ξ the set of minimizers of CMP (3). It is well known that the gradient-projection algorithm (GPA) generates a sequence $\{x_n\}$ determined by the gradient ∇f and the metric projection P_C:

$$x_{n+1} := P_C(x_n - \lambda \nabla f(x_n)), \quad \forall n \geq 0, \tag{4}$$

or more generally,

$$x_{n+1} := P_C(x_n - \lambda_n \nabla f(x_n)), \quad \forall n \geq 0, \tag{5}$$

where, in both (4) and (5), the initial guess x_0 is taken from C arbitrarily, the parameters λ or λ_n are positive real numbers. The convergence of algorithms (4) and (5) depends on the behavior of the gradient ∇f. As a matter of fact, it is known that, if ∇f is α-strongly monotone and L-Lipschitz continuous, then, for $0 < \lambda < \frac{2\alpha}{L^2}$, the sequence $\{x_n\}$ defined by the GPA (4) converges in norm to the unique solution of CMP (3). More generally, if the sequence $\{\lambda_n\}$ is chosen to satisfy the property

$$0 < \liminf_{n \to \infty} \lambda_n \leq \limsup_{n \to \infty} \lambda_n < \frac{2\alpha}{L^2},$$

then the sequence $\{x_n\}$ defined by the GPA ((5)) converges in norm to the unique minimizer of CMP (3). Since the Lipschitz continuity of the gradient ∇f implies that it is actually $\frac{1}{L}$-inverse strongly monotone (ism) [1], its complement can be an averaged mapping (that is, it can be expressed as a proper convex combination of the identity mapping and a nonexpansive mapping). Consequently, the GPA can be rewritten as the composite of a projection and an averaged mapping, which is again an averaged mapping. This shows that averaged mappings play an important role in the GPA. Recently, Xu [28] used averaged mappings to study the convergence analysis of the GPA, which is hence an operator-oriented approach.

Very recently, Ceng and Al-Homidan [4] proposed and analyzed an implicit iterative algorithm for finding a common element of the solution set $\cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j)$ of a finite family of GMEPs, the solution set $\cap_{i=1}^{N} \mathrm{VI}(C, A_i)$ of a finite family of variational inequalities and the solution set Ξ of CMP (3).

Motivated and inspired by the above facts, we introduce a relaxed explicit extragradient-like scheme for finding a common element of the set of solutions of the CMP (3) for a convex functional $f : C \to \mathbf{R}$ with L-Lipschitz continuous gradient ∇f, the set of solutions of a finite family of GMEPs and the set of solutions of a finite family of variational inequalities for inverse-strongly monotone mappings in a real Hilbert space. Under suitable control conditions, we establish the strong convergence of the proposed relaxed extragradient-like scheme to the same common element of the above three sets, which is also the unique solution of a variational inequality defined over the intersection of the above three sets. Our results improve and extend the corresponding ones given by some authors recently in this area.

2 Main Results

In this section, the general composite explicit scheme for a nonexpansive mapping $T : H \to H$ (see (3.5) in [9])

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and the general composite explicit one for a strict pseudocontraction (see (3.3) in [23]) are extended to develop the multistep relaxed explicit extragradient-like one for finding a common element of the set of solutions of the CMP (3) for a convex functional $f : C \to \mathbf{R}$ with L-Lipschitz continuous gradient ∇f, the set of solutions of a finite family of GMEPs and the set of solutions of a finite family of variational inequalities for inverse-strongly monotone mappings in a real Hilbert space.

To see this, we first recall the following definition:

Definition 1 *A mapping $F : C \to H$ is said to be*

(i) *monotone if*

$$\langle Fx - Fy, x - y\rangle \geq 0, \quad \forall x, y \in C;$$

(ii) *η-strongly monotone if there exists a constant $\eta > 0$ such that*

$$\langle Fx - Fy, x - y\rangle \geq \eta\|x - y\|^2, \quad \forall x, y \in C;$$

(iii) *α-inverse-strongly monotone if there exists a constant $\alpha > 0$ such that*

$$\langle Fx - Fy, x - y\rangle \geq \alpha\|Fx - Fy\|^2, \quad \forall x, y \in C.$$

Let M, N be two integers. Throughout the remainder of this section, we always assume the following:

- $F : C \to H$ is a κ-Lipschitzian and η-strongly monotone operator with positive constants $\kappa, \eta > 0$;
- and $f : C \to \mathbf{R}$ is a convex functional with L-Lipschitz continuous gradient ∇f;
- For each $i = 1, ..., N$, $A_i : C \to H$ is η_i-inverse strongly monotone, and for each $j = 1, ..., M$, $B_j : C \to H$ is μ_j-inverse strongly monotone;
- A is a $\bar{\gamma}$-strongly positive bounded linear operator on H with $\bar{\gamma} \in (1, 2)$ and $V : C \to H$ is an l-Lipschitzian mapping with $l \geq 0$;
- For each $j = 1, ..., M$, $\Theta_j : C \times C \to \mathbf{R}$ is a bifunction satisfying the following conditions:

  (A1) $\Theta_j(x, x) = 0$ for all $x \in C$;

  (A2) Θ_j is monotone, i.e., $\Theta_j(x, y) + \Theta_j(y, x) \leq 0$ for any $x, y \in C$;

  (A3) Θ_j is upper-hemicontinuous, i.e., for each $x, y, z \in C$,

  $$\limsup_{t \to 0^+} \Theta_j(tz + (1 - t)x, y) \leq \Theta_j(x, y);$$

  (A4) $\Theta_j(x, \cdot)$ is convex and lower semicontinuous for each $x \in C$,

  and $\varphi_j : C \to \mathbf{R} \cup \{+\infty\}$ is a proper lower semicontinuous and convex function with the following restrictions:

  (B1) for each $x \in H$ and $r > 0$, there exists a bounded subset $D_x \subset C$ and $y_x \in C$ such that for any $z \in C \setminus D_x$,

  $$\Theta_j(z, y_x) + \varphi_j(y_x) - \varphi_j(z) + \frac{1}{r}\langle y_x - z, z - x\rangle < 0;$$

  (B2) C is a bounded set;
- μ and γ are real numbers such that $0 < \mu < \frac{2\eta}{\kappa^2}$ and $0 \leq \gamma l < \tau$ with $\tau = 1 - \sqrt{1 - \mu(2\eta - \mu\kappa^2)}$;
- $P_C(I - \lambda_n \nabla f) = s_n I + (1 - s_n)T_n$ where T_n is nonexpansive, $s_n = \frac{2 - \lambda_n L}{4} \in (0, \frac{1}{2})$ and $\{\lambda_n\} \subset (0, \frac{2}{L})$ with $\lim_{n\to\infty} \lambda_n = \frac{2}{L}$;
- For $n = 0, 1, \cdots$, $\Lambda_n^N : C \to C$ is a mapping defined by $\Lambda_n^N(x) = P_C(I - \lambda_{N,n}A_N) \cdots P_C(I - \lambda_{1,n}A_1)(x)$ for all $x \in C$, where the sequence $\{\lambda_{i,n}\}_{n=0}^\infty$ is contained in $[a_i, b_i] \subset (0, 2\eta_i)$ and $\lim_{n\to\infty} \lambda_{i,n} = \lambda_i$, for each $i = 1, ..., N$;
- For $n = 0, 1, \cdots$, $\Delta_n^M : C \to C$ is a mapping defined by $\Delta_n^M(x) = T_{r_{M,n}}^{(\Theta_M, \varphi_M)}(I - r_{M,n}B_M) \cdots T_{r_{1,n}}^{(\Theta_1, \varphi_1)}(I - r_{1,n}B_1)(x)$ for all $x \in C$, where the sequence $\{r_{j,n}\}_{n=0}^\infty$ is contained in $[c_j, d_j] \subset (0, 2\mu_j)$ and $\lim_{n\to\infty} r_{j,n} = r_j$, for each $j = 1, ..., M$, and the mapping $T_r^{(\Theta,\varphi)} : H \to C$ is defined as follows:

  $$\begin{aligned} T_r^{(\Theta,\varphi)}x \quad := \quad & \{y \in C : \Theta(y, z) + \varphi(z) - \varphi(y) \\ & + \frac{1}{r}\langle y - x, z - y\rangle \geq 0, \forall z \in C\}. \end{aligned}$$
- $\{\alpha_n\} \subset [0, 1]$ and $\{s_n\} \subset (0, \min\{\frac{1}{2}, \|A\|^{-1}\})$

We note that, since $\{\lambda_{i,n}\} \subset [a_i, b_i] \subset (0, 2\eta_i)$, it is not difficult to see that $\Lambda_n^N : C \to C$ is a nonexpansive mapping for all $n \geq 0$. On the other hand, since $\{r_{j,n}\} \subset [c_j, d_j] \subset (0, 2\mu_j)$, we also have that $\Delta_n^M : C \to C$ is a nonexpansive mapping for all $n \geq 0$.

For arbitrarily given $x_0 \in C$, we propose a multistep relaxed explicit extragradient-like scheme, which generates a sequence $\{x_n\}$ in an explicit way:

$$\left\{ \begin{array}{l} u_n = \Lambda_n^N(x_n) \\ v_n = \Delta_n^M(u_n) \\ y_n = \alpha_n \gamma V(x_n) + (I - \alpha_n \mu F)T_n(v_n), \\ x_{n+1} = P_C[(I - s_n A)T_n(v_n) + s_n\, y_n], \quad \forall n \geq 0, \end{array} \right. \tag{6}$$

where $\{\alpha_n\}$ and $\{s_n\}$ satisfy the following conditions:

(C1) $\{\alpha_n\} \subset [0, 1]$, $\{s_n\} \subset (0, \frac{1}{2})$ and $\alpha_n \to 0$, $s_n \to 0$ as $n \to \infty$;

(C2) $\sum_{n=0}^{\infty} s_n = \infty$.

Then we can establish the strong convergence of $\{x_n\}$ as $n \to \infty$ to the point $\tilde{x} \in \Omega := \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{VI}(C, A_i) \cap \Xi$, which is also the unique solution to the VIP

$$\langle (A - I)\tilde{x}, p - \tilde{x} \rangle \geq 0, \quad \forall p \in \Omega. \tag{7}$$

Theorem 2 *Assume that the set*

$$\Omega := \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{VI}(C, A_i) \cap \Xi$$

is nonempty. Let $\{x_n\}$ be the sequence generated by the explicit scheme (6). If $\{x_n\}$ is weakly asymptotically regular (i.e., the sequence $\{x_{n+1} - x_n\}$ converges weakly to 0), then $\{x_n\}$ converges strongly to a point $\tilde{x} \in \Omega$, which is the unique solution of the VIP (7).

3 Concluding Remarks

We introduced a multistep relaxed explicit extragradient-like scheme for finding a common element of the set of solutions of CMP (3), the set of solutions of finitely many GMEPs and the set of solutions of finitely many VIPs by virtue of Korpelevich's extragradient method [24], the general composite explicit schemes for a nonexpansive mapping $T : H \to H$ (see [9]) and the general composite explicit one for a strict pseudocontraction $T : H \to H$ (see [23]). Our Theorem 2 improve and extend results in [9, 23] in the following aspects.

- The problem of finding a common solution $\tilde{x} \in \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{VI}(C, A_i) \cap \Xi$ in our Theorem 2 is more general and more flexible than the one of finding a fixed point of a nonexpansive mapping $T : H \to H$ in [9], and the one of finding a fixed point of a strictly pseudocontractive mapping $T : H \to H$ in [23]. It is worth pointing out that the problem of finding $\tilde{x} \in \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{VI}(C, A_i) \cap \Xi$ includes as a special case the one of finding $\tilde{x} \in \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{Fix}(T_i) \cap \Xi$ for finitely many strict pseudocontractions $T_i : C \to C, i = 1, ..., N$ and that the one of finding $\tilde{x} \in \cap_{j=1}^{M} \mathrm{GMEP}(\Theta_j, \varphi_j, B_j) \cap \cap_{i=1}^{N} \mathrm{Fix}(T_i) \cap \Xi$ for finitely many strict pseudocontractions $T_i : C \to C, i = 1, ..., N$ generalizes the fixed point problems in [9, 23] from the domain H of the nonexpansive or strictly pseudocontractive mapping to the domain C and from one nonexpansive or strictly pseudocontractive mapping to finitely many strictly pseudocontractive mappings, and extends the fixed point problems in [9, 23] to the setting of CMP (3) and finitely many GMEPs.

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