Proceedings of the International MultiConference of Engineers and Computer Scientists 2016 Vol II,
IMECS 2016, March 16 - 18, 2016, Hong Kong

A Multi-agent Simulation Study for End-of-period Discount Clearance with Consideration of Reference Price Effect

Takeshi Koide and Hiroaki Sandoh, *Member, IAENG*

***Abstract*— A multi-agent simulation study is conducted with respect to an end-of-period clearance pricing for daily perishable products. The clearance pricing could increase the profit of the day and at the same time consumer's reference prices are declined by the pricing. The decline of the reference price causes to decrease of future sales and future profit as well. The authors executed a mathematical analysis in our previous study by formulating the end-of-period inventory clearance problem for a single period as an end-of-period. The study derived a procedure to seek optimal pricing according to consumer's reaction for price preference and, however, the mathematical model focused on the end-of-period and did not contain the influence of consumer's purchase behavior before the end-of-period.**

This study considers the consumer's purchase behavior in a day as two stages. The demand in the first stage influences the number of remaining goods in the second stage. A supplying policy for the target goods is also an essential factor to improve profit. The proposed simulation model estimates long-term profit and distribution of consumers' reference prices in order to assess a discount policy in the end-of-period and a flexible supply policy at the beginning of every days.

***Index Terms*—inventory control, multi-agent simulation, optimal pricing, reference price effect, revenue management**

I. Introduction

REVENUE management is one of the most successful research fields in operations research for practical applications in the last two decades [1]. Revenue management involves many theories and techniques and the pricing on perishable products has been played a leading role in revenue management.

Revenue management was primarily applied to the service industry, such as airlines, hotels, car rental, cruises, and concerts, and subsequently applied to the retailing industry. In revenue management for the retailing industry, markdown strategy for perishable products has been the one of the hottest topic. When the target period is a season, target products are seasonal ones, such as Christmas goods, ski implements, and fur coat. When the target period is a day, target products are daily perishable goods, such as fresh foods and ready-made dishes.

It is important to consider the reference price effect on consumers to execute markdown for daily perishable products. The reference price effect is defined as the effect on the demand changed by consumer's pivot price for a target product. The pivot price, referred to as reference price [2], was originally introduced by Kahneman and Tversky in the prospect theory [3]. In the research field of marketing sciences, some studies were made to derive optimal pricing policies with considering the reference price effect [4, 5, 6]. In the retailing industries, it is also important to control inventory. Petruzzi and Dada conducted the first research to treat the inventory control as well as a discount pricing with considering the reference price effect [7]. Since then, some research discussed optimal pricing, inventory control, and reference price effect simultaneously [8, 9, 10]. The authors also executed a mathematical analysis for both optimal pricing in an end-of-period and inventory control with considering reference price effects [11]. The proposed model focused on the end-of-period and adopted a single period model. Moreover, the model assumed the homogeneity of consumers and consumers' reference prices are assumed common to derive some findings from the model.

This study constructs a multi-agent model for the same situation in order to treat the diversity of consumer's reference price. Each consumer has his/her own reference price and it changes through daily purchase behavior. The model considers two periods and consumer's purchase behaviors before the end-of-period. Simulation studies discuss an effectiveness of a flexible supply policy at the beginning of every days as well as a discount policy in the end-of-period in order to improve the long-term profit.

II. Multi-agent Model

A. Firm

Consider a firm under the monopoly and the firm operates a store. This study focuses on inventory control and pricing for a single type of products at the store. The product is assumed daily perishable and unsold goods are disposed after the closing hour. The firm determines the supply quantity of the day q with considering the amounts of recent sales. It is assumed that the firm can prepare the determined number of goods before the opening hour. The goods are sold at a regular price p_1 every day. Additional supply during business hours is not considered in this model.

The business hours consist of two time stages, named stage 1 and stage 2. Stage 2 is defined as the period before closing

Manuscript received December 31, 2015; revised January 18, 2016. This work was supported in part by Grand-in-Aid for Scientific Research (B), No. 25285131 and MEXT, Japan.

T. Koide is with Department of Intelligence and Informatics, Konan University, Kobe 658-8501, Japan (corresponding author; e-mail: koide@konan-u.ac.jp).

H. Sandoh is with Department of Management Science and Business Administration, Graduate School of Economics, Osaka University, Toyonaka, Osaka 650-0043, Japan (e-mail: sandoh@econ.osaka-u.ac.jp).

time and stage 1 is the remaining business hours except for stage 2. This model does not clarify the time length of stage 2 and the start time of stage 2 is fixed throughout the simulation executions. At the beginning time of stage 2, which is also the end time of stage 1, the firm can mark down the sales price p_2. According to the amount of unsold goods at the beginning time of stage 2, the firm selects an operation among the following three options; (i) continuing to sell the goods at the regular price; $p_2 = p_1$, (ii) cutting the sales price of the goods; $p_2 = p_2' < p_1$ (iii) large marking down the goods; $p_2 = p_2'' < p_2'$. The raw price of a good is denoted by c and it is assumed that $c < p_2''$.

B. Consumers

This study considers homogeneous consumers in a market and focuses on their purchasing activity for the target item. The consumers are segmented with regard to their arrival time to the store. The consumers who arrive at the store in stage 1 and stage 2 are respectively named type 1 consumers and type 2 consumers. The segment sizes of type 1 and type 2 are denoted by N_1 and N_2, respectively.

Type 1 consumers purchase a single item at the regular sales price p_1 with a probability, denoted by a, every day. On the contrary, the actions of type 2 consumers are dependent on the asking price p_2 of the item. Their purchase probability is expressed by $a + b(r - p_2)$ where b implies the price sensitivity coefficient and r is the reference price of the consumer. Consumers have their own reference price r for the target item. The reference price is initially set to the same value of the regular sales price p_1 and then continuously changed by daily asking sales prices p_2. Type 1 consumers come to the store during stage 1 when the item is sold at the regular price p_1 and their reference prices are never changed. Type 2 consumers, however, have a chance to purchase the item at a discounted price and their reference prices are declined. The updating process of reference price follows the following equation:

$$r' = \alpha r + (1 - \alpha) p \tag{1}$$

where p is the asking price and r' is the updated reference price. The parameter α denotes the memory coefficient where $0 < \alpha < 1$.

C. Optimal Supply Quantity

In this model, the firm conducts two types of decision-makings. The firm firstly determines the supply quantity of the day before opening hour and then determines the sales price offered in stage 2 before its start time. For both decision-makings, the optimal supply quantities for target consumers serve an essential role as useful information.

Assume that the firm supplies q target goods and sells them at the price p. Then, the expected profit $\Pi(p, q)$ gained in a stage of the day is expressed as follows:

$$\Pi(p, q) = \int_0^q pxf(x)dx + \int_q^\infty pqf(x)dx - cq$$

$$= p\left\{\int_0^q xf(x)dx + q(1 - F(q))\right\} - cq. \tag{2}$$

In equation (2), $f(x)$ and $F(x)$ are respectively the probability density function and the probability distribution function of a random variable for the sales amount. Differentiating $\Pi(p, q)$ with respect to q induces the optimal supply quantity q^*:

$$q^* = F^{-1}\left(\frac{p-c}{p}\right). \tag{3}$$

The resulting equation (3) is a version of the optimal inventory volume for the well-known newsvendor problem [12].

The sales amount follows the binomial distribution and it could be approximated by the normal distribution if the segment size N is large. The mean and the variance of the normal distribution are $N\pi$ and $N\pi(1 - \pi)$ where π is the purchase probability of a consumer.

This study defines four types of optimal quantities. The first optimal quantity q_1^* is derived for type 1 consumers, where the segment size is N_1, the sales price is equal to the regular price p_1, and the purchase probability is commonly a. The other optimal quantities are defined as the optimal quantities for type 2 consumers whose segment size is N_2. The quantities q_2^*, $q_2^{*'}$, $q_2^{*''}$ are respectively the optimal quantities in selling the goods in stage 2 at the price p_2, p_2', and p_2''. Note that each type 2 consumer has his/her own reference price but here it is assumed that all type 2 consumers have a common reference price equal to regular price p_1. The assumption means that all of type 2 consumer have the same purchase probability.

D. Firm's Decision-Makings

The decision-making on the supply quantity is conducted as follows. On the first day, the firm supplies $q_1^* + q_2^*$ goods before the opening hour for type 2 consumers. In the following days, the supply quantity is determined based on the sale on the previous day. If all goods were sold out in stage 1 on the previous day, the supply quantity is increased by q_2^* to the supplied quantity on the previous day. If all goods were not sold out in stage 1 but in stage 2, the increment of the supply is $q_2^* - q_2$, where q_2 is the quantity of the goods at the start time of stage 2 on the previous day. Finally, if some goods were still unsold at the end of stage 2, the supply quantity is reduced by the same volume as the unsold goods.

As the decision-making on the discount sale in stage 2, the firm has to select an option among (i), (ii), and (iii) explained in subsection A. The selection is conducted according to the volume of goods at the beginning of stage 2, denoted by q_2, and the optimal quantities for type 2 consumers q_2^*, $q_2^{*'}$, $q_2^{*''}$ defined in the previous subsection. If $q_2 < \beta q_2^* + (1 - \beta) q_2^{*'}$ holds, the firm select option (i): to hold the sales price at the regular price p_1. Otherwise if $q_2 < \beta q_2^{*'} + (1 - \beta) q_2^{*''}$ holds, option (ii) is selected to mark down the sales price to p_2'. Otherwise, namely $\beta q_2^{*'} + (1 - \beta) q_2^{*''} < q_2$, option (iii) is selected to decline the sales price to p_2''. The parameter β indicates the willingness to the markdown and the firm with greater value of β conducts price cut in the case of less quantity of unsold goods at the beginning of stage 2.

TABLE I
AVERAGE OF DAILY PROFIT

	fixed supply quantity			flexible supply quantity		
β / b	0.001	0.005	0.009	0.001	0.005	0.009
0.0	12,878	13,296	13,372	12,900	13,169	13,305
0.2	12,806	13,244	13,456	12,945	13,111	13,290
0.4	12,936	13,197	13,349	12,847	13,087	13,187
0.6	12,929	13,060	13,268	12,933	13,044	13,183
0.8	12,941	12,962	13,188	12,933	13,059	13,079
1.0	12,939	12,853	13,137	12,929	13,010	13,065

TABLE II
STANDARD DEVIATION OF DAILY PROFIT

	fixed supply quantity			flexible supply quantity		
β / b	0.001	0.005	0.009	0.001	0.005	0.009
0.0	1,236	940	871	865	789	764
0.2	1,207	917	792	810	785	743
0.4	1,249	886	802	845	773	772
0.6	1,210	894	747	823	716	764
0.8	1,235	934	772	797	755	824
1.0	1,242	940	743	814	777	783

III. SIMULATION EXPERIMENTS

A. Simulation Settings

The proposed multi-agent model has been applied in simulation experiments to assess the effects of firm's decision-makings to improve long-term profit. An execution of simulation is composed of a thousand days.

The parameters in the experiments are set as follows: the regular sales price $p_1 = 100$, the raw cost $c = 10$, the segment size of stage 1 $N_1 = 900$, that of stage 2 $N_2 = 100$, the fundamental purchase probability $a = 0.2$, the memory coefficient $\alpha = 0.8$. The discounted prices p_2' and p_2'' are respectively set to 80 and 50, which means 20%-off price and the half price. The price sensitivity b and the discount willingness β are set to several values to estimate their effects on the long-term profit.

Moreover, the simulation study has been settled to have an option if the decision-making for the supply quantity. When the option not to conduct the decision-making, the daily supply quantity is fixed for all days and the quantity is determined as $q_1^* + q_2^*$.

B. Long-term Profit

The execution of simulation computes daily profits for the thousand days with different values of the parameters b and β. Tables I and II indicate the average and the standard deviation of the daily profit. The three columns with label "fixed supply quantity" represent the results obtained with the fixed daily supply quantity $q_1^* + q_2^*$. The three columns with label "flexible supply quantity" represent the results with the decision-making for daily supply quantity.

For the consumers with $b = 0.001$, the parameter β does not influence daily profits so much. This is because the consumers are slow to react to price variation and the decision-making for end-period discount depends on parameter β. Another decision-making for daily supply quantity control does not influence the average profit for the consumer so much but the standard deviation.

For the consumers with $b = 0.005$ and 0.009, the flexible supply quantity reduce both the average and the standard deviation of the daily profits in most cases. The result shows that the decision-making for daily supply adjusts the supply quantity to consumer's native demand and mitigates the profit deviation but does not contribute to increase average long-term profit.

For the same consumers with $b = 0.005$ and 0.009, the average profits decreases with respect to β in most cases. The parameter β means discount willingness and the store mark down more actively when β has a greater value. The standard deviations of daily profits are independent of the parameter β. The obtained results show that the decision-making for markdown does not contribute to increment of long-term profit if another decision making for supply adjustment is conducted. The decision-making for markdown is sometimes effective to increase profit with the fixed supply quantity policy.

TABLE III
DISTRIBUTION OF REFERENCE PRICES

	β	lower limit of each classification						ave.	s.d.
		64.3	71.4	78.6	85.7	92.9	100		
fixed	0.0	-	-	-	9.6	90.2	0.1	88.6	2.1
	0.2	-	-	0.5	18.6	80.9	0.1	87.9	2.9
	0.4	-	-	2.6	41.4	55.9	0.1	86.0	3.9
	0.6	-	0.1	11.2	64.1	24.6	0.0	83.1	4.2
	0.8	0.1	2.7	30.3	57.2	9.7	0.0	80.3	4.8
	1.0	0.4	11.2	47.5	38.6	2.2	0.0	77.2	5.1
flexible	0.0	-	-	-	3.7	95.7	0.6	89.1	1.5
	0.2	-	-	0.1	14.8	84.0	1.1	88.3	2.7
	0.4	-	-	0.3	22.7	77.0	0.0	87.6	3.1
	0.6	-	-	2.6	43.9	53.1	0.4	85.8	4.0
	0.8	-	0.1	9.1	56.2	34.6	0.0	84.0	4.4
	1.0	-	1.3	21.2	58.2	19.3	0.0	81.8	4.8

C. Reference Price Distribution

The developed simulation model can derive more information than mathematical model. Here, the distribution of consumers' reference prices is focused as an example of the information. Table III indicates the distribution when the price sensitivity b is 0.009. The higher and lower rows show the results when daily supply quantity is fixed and determined by the decision-making, respectively. The distribution consists of six classifications and lower limits of each classification are labeled in Table III. The average and the standard deviation of consumers' reference prices are also shown.

In both cases of the fixed supply and the flexible supply, parameter β with a greater value declines consumer's reference prices more intensively and raises the deviation of the reference prices among consumers. Compared with the difference by the decision-making for flexible supply quantity, Table III reveals that the decision-making keeps the average reference price higher and suppresses the deviation of reference prices among consumers.

IV. CONCLUSION

This paper proposed a multi-agent simulation model to assess the effectiveness of some policies in an end-of-period discounting and inventory control. In the numerical experiments, the end-of-period discount could be effective with fixed daily supply quantity and the effectiveness is deteriorated with flexible daily supply. The discount operation increases the profit of the day. At the same time, it declines consumers' reference price and the deviation of profits in a long term becomes greater.

The proposed simulation model can be extended to tackle a more realistic situation. Some parameters for consumers, such as the memory coefficient α and price sensitivity b, are identical for all consumers in this model but they can be different as consumer's preferences. The model can integrate other factors, such as consumer's arrival time, the store operating hours, substitute goods and complementary goods.

Simulation models have an advantage easy to extend but it is hard to derive generality from simulation studies. It is essential to treat mathematical models as well as simulation models to obtain knowledge of practical situations. The combination of mathematical models and simulation models will be discussed in the forthcoming paper.

ACKNOWLEDGMENT

The authors thank Yuya Suzuki for his research support to construct the proposed multi-agent simulation model.

REFERENCES

[1] K. T. Talluri and G. J. van Ryzin, *The Theory and Practice of Revenue Management*, Springer, New York, 2005.

[2] T. Mazumdar, S. Raj, and I. Sinha "Reference price research: review and propositions", J. Market., vol. 69, no. 4, 2005, pp. 84–102

[3] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econometrica*, vol. 47, pp. 263–291, 1979.

[4] E. A. Greenleaf, "The impact of reference price effects on the profitability of price promotions," *Marketing Science*, vol. 14, pp. 82–104, 1995.

[5] P. K. Kopalle, A. G. Rao, and J. L. Assuncao, "Asymmetric reference price effects and dynamic pricing policies," *Marketing Science*, vol. 15, 1996, pp. 60–85.

[6] I. Popescu and Y. Wu, "Dynamic pricing strategies with reference effects," *Operations Research*, vol. 55, 2007, pp. 413–429.

[7] N. C. Petruzzi and M. Dada, "Pricing and the newsvendor problem: A review with extensions," *Operations Research*, vol. 47, 1999, pp. 183–194.

[8] L. Gimpl-Heersink, "Joint pricing and inventory control under reference price effects", Doctoral thesis, WU Vienna University of Economics and Business, Wien, 2008.

[9] T. L. Urban, "Coordinating pricing and inventory decisions under reference price effects", *International Journal of Manufacturing Technology and Management*, vol. 13, no. 1, 2008, pp. 78–94.

[10] A. Taudes and C. Rudloff, "Integrating inventory control and a price change in the presence of reference price effects: a two-period model", *Mathematical Methods of Operations Research*, vol. 75, no. 1, 2012, pp. 29–65.

[11] T. Koide and H. Sandoh, "A study on multi-period inventory clearance pricing in consideration of consumer's reference price effect", *Industrial Engineering & Management Systems*, vol. 12, no. 2, 2013, pp. 95-102.

[12] N. C. Petruzzi and M. Dada, "Pricing and the newsvendor problem: A review with extensions," *Operations Research*, vol. 47, 1999, pp. 183–194.