Multi-vendor and Multi-buyer Collaborative Planning Forecasting and Replenishment Model

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***Abstract*—This article proposes a two-echelon integrated production-inventory supply chain model. We assume that the production process is unreliable. The defective items will be send back to the upstream (vendors) and the buyers will receive the repaired products after the reworking. In order to forecast buyer's demand more precisely, we design a linear regression equation, which based on past years demand. The objective is to minimize the joint total cost, and present the proposed model and optimal solution by numerical examples.**

***Index Terms*—supply chain; inventory; integer-multiplier policies; defective items;**

I. Introduction

In today's competitive market, an enterprise has to be able to develop sustainably and stay competitive, finding partner and alliance has become a trend nowadays, forming a vertical supply chain which integrates from up to down in order to optimize the profit of the whole supply chain (or minimize the cost). After forming the relationship, the strategy of management is vital, one of the sectors is the strategy of inventory management. Since the cost of the inventory accounts for a high proportion of supply chain, many experts has been making research on inventory management for many years. Harris [1] was the first researcher who developed economic order quantity (EOQ) model, this model was consisted by a single company's ordering costs and holding costs. In the viewpoint of the supply chain management (SCM), collaborating closely with the members of supply chain is certainly necessary, so Harris' research didn't meet the actual situation. In order to meet the SCM's status, Goyal [2] was the first researcher to develop a single-vendor single-buyer's integrated inventory model. Banerjee[3] proposed Joint Economic Lot Size (JELS), assumedorder quantity will not be shipped in batch, but will be shipped to the buyer after production run. Goyal [4] extended Banerjee's [3]research, discussion lot size per production is an integer multiple of the buyer's optimal order quantity. Lu [5] developed single-vendor multi-buyer's integrated inventory model, and assumed each shipping size were equal. Goyal [6] modified Lu's [5] point of view, proved that different delivery policies can get better results. Hill [7] extended Goyal [6] and Lu's [5] model, proposed the lot size of each shipment will between 1 ~ P/ D multiplier during production run. Ha and Kim [8] assumed lot size per production run and buyer's order quantity are equal, and deliver to buyer divided into N times. Goyal and Nebebe [9] extended Goyal [6] and Hill's [7] research, proposed a new model and got a better optimal order quantity. Pan and Yang [10] developed a model which concerned controllable lead time and the size of each shipment are equal. Kelle et al. [11] proposed a model which the lot size per production run is m times greater than shipment size, m is an integer multiplier. The vendor will divided the shipment size into n times and deliver to buyer, n is not necessarily equal to m. Pan and Yang [12] who developed an integrated inventory model which used fuzzy theory, assumed the demand rate and production rate were not a fixed value, and then derived the optimum order quantity. Yang and Lo [13] discussed a single-vendor multi-buyer's system, and lead time is variable. Subrata Mitra [14] proposed two-echelon close-loop supply chain, and discussed variation with traditional supply chain. J.K. Jha [15] added service level constraints in integrated inventory model, and considered the lead time was not fixed.

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Most of the researchers considered the process is very reliable, and assumed the inventory would not deteriorate and defective. In order to get closer to realistic situation, the first concept of defective rate was proposed by Ghare and Schader's [16] research, and the defective rate as a fixed constant. Covert and phillip [17] extended Ghare and Schader's [16] research, assumed the defective rate must comply with Weibull distribution. Chen [18] followed Covert and phillip's [17] research, considered demand rate was depended on time, and allowed backorders. Wee and Law [19] proposed price was depended on demand, and defective rate complied with weibull distribution. Yang and Lin [20] developed multi-echelon integrated inventory model, and used Particle Swarm Optimization (PSO) to obtain the optimal quantity per shipment and number of shipment under unreliable process and uncertain lead time. Yang, Lo, and Lu [21] investigated single-vendor multi-buyers under imperfect work environment's replenishment cycle and production cycle.

Our purpose in this article is to minimize the expected joint total cost. We will develop multi-vendor multi-buyer inventory model with defective rate. We set that the lot produced at each stage is delivered in equal shipments to the downstream (buyers), and shipments are delivered as soon as they are produced, not to wait until a whole lot is produced.

We first defined the parameters and assumptions in Section II, and then we started to develop the integrated inventory model in Section III. In Section IV, we solved the model to get the optimal solution and showed numerical examples in Section V. In the end, we summarized the conclusions in Section VI.

II. Notations and Assumptions

To develop a two-echelon integrated inventory model, the notations and assumptions are used in developing the model are as follow.

A. Notations

T	The basic cycle time of a buyer.
K_v	Integer multiplier for the cycle time at vendor's stage.
$D_{v,g}$	Demand rate for vendor g.
$P_{v,g}$	Production rate for vendor g.
n_v	Number of vendors.
h_s	Materials unit stocking-holding cost per unit per year for vendor.
h_v	Finished product unit stocking-holding cost per unit per year for vendor.
S_v	Set up cost per production run for vendor.
Q_r	Manufacturing cost per unit for vendor.
$D_{b,j}$	Demand rate for buyer j. $D_{b,j}=\hat{E}_{b,j}=\hat{\alpha}_{b,j}+\hat{\beta}_{b,j}Q_n$
n_b	Number of buyers.
h_b	Finished product unit stocking-holding cost per unit per year for buyer.
S_b	Ordering cost for vendor.
R	Recycling cost for vendor.
L	Repairing cost for vendor.
d	Unit screening cost for buyer.
X	Screening rate for buyer.
B	Unit purchasing cost for buyer.
σ	Discount rate (1- $mkY, m=5$) for buyer.
k	Percentage of defective products can not be repaired.
Y	Percentage of defective products.
$\hat{\alpha}_{b,j}$	Slope of population regression function.
$\hat{\beta}_{b,j}$	Intercept of population regression function.
$\hat{E}_{b,j}$	Forecasted demand.
$TC_{v,g}$	Total cost for vendor g.
$TC_{b,j}$	Total cost for buyer j.
$JTC_{(k_v,T)}$	Joint total cost of whole supply chain.

B. Assumptions

(i) This paper is based on multi-vendor and multi-buyer for single item.
(ii) The production rate is finite.
(iii) Shortages are not allowed
(iv) Because of shortages are not allowed, non-defective product's production rate must higher than buyer's demand.
(v) Quantity discount and defective rate has direct relation.
(vi) Returned defective production will be repaired, but not fully repaired.
(vii) Demands are equal at all echelons. Based on pulling production $\sum_{j=1}^{n_b}\left(D_{b,j}\right)=\sum_{g=1}^{n_v}\left(D_{v,g}\right)$
(viii) Each stage of the supply chain's cycle time are equal and the cycle time at each stage is an integer multiplier of the cycle time at the adjacent downstream stage. The cycle time of vendor is $T_v=K_v*T$

III. Model Formulation

In this section, we joint vendor and buyer's total cost into integrated inventory model. In the Fig. I, the start of a cycle the inventory level equal to $TD_{v,g}^2/P_{v,g}$. Then the inventory level starts increasing at the rate $P_{v,g}-D_{v,g}$ until it reaches its maximum level $\left[\left(P_{v,g}-D_{v,g}\right)k_vTD_{v,g}\right]/P_{v,g}$. Then it is consumed at the rate $D_{v,g}$ until the end of the cycle. The vendor of the cycle length is k_vT as mentioned earlier.

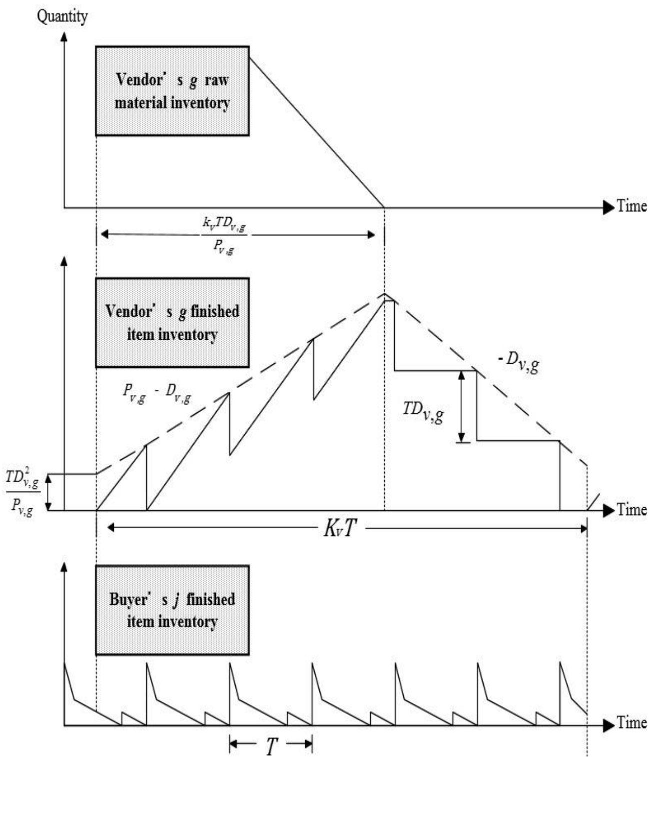

Fig. I. The vendor and buyer inventory level

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A. The vendor's annual total cost

In each production run, the vender's cost includes set up cost, manufacturing cost, holding cost, recycling cost, repairing cost. The vendor's total annual cost consists of the following elements.

(i) Set up cost $= \dfrac{S_v}{K_v T}$

(ii) Manufacturing cost $= Q_r P_{v,j}$

(iii) Recycling cost $= RKYP_{v,g}$

(iv) Repairing cost $= LYD_{v,g}$

(v) Holding cost

$$= \frac{h_s T D_{v,g}^2}{2P_{v,g}} + \frac{h_v T}{2}\left[(2-k_v)\frac{D_{v,g}^2}{P_{v,g}} + (k_v - 1)D_{v,g}\right]$$

B. The buyer's annual total cost

In each production run, the vender's cost includes ordering cost, screening cost, purchasing cost, holding cost. The buyer's total annual cost consists of the following elements.

(i) Set up cost $= \dfrac{S_b}{T}$

(ii) Screening cost $= \dfrac{dD_{b,j}}{(1-kY)}$

(iii) Purchasing cost $= D_{b,j}\beta(1-\sigma)$

(iv) Holding cost is given by:

$$= \frac{h_b}{T}\left[\frac{(D_{b,j}T)^2 Y}{2(X)(1-kY)^2} + \frac{D_{b,j}(T(1-Y))^2}{2(1-kY)} + \frac{D_{b,j}((1-k)YT)^2}{2(1-kY)^2}\right]$$

$$= \frac{h_b D_{b,j} T}{2(1-kY)}\left[\frac{D_{b,j} Y}{(X)(1-kY)} + (1-Y)^2 + \frac{(Y-kY)^2}{(1-kY)}\right]$$

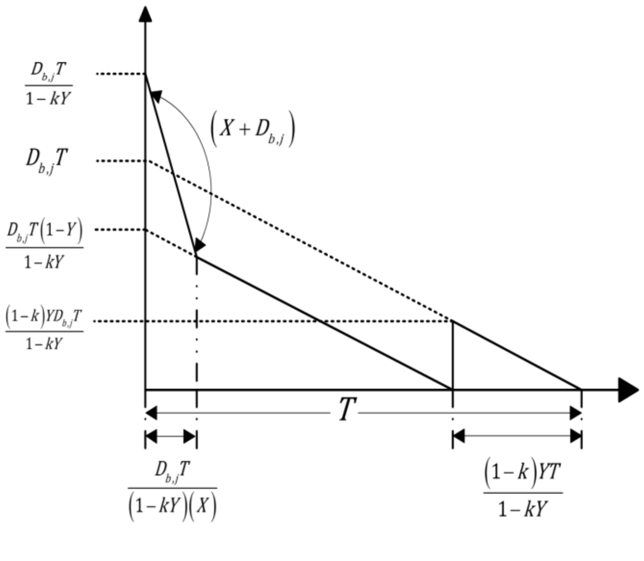

Fig. II. The buyer's inventory level

C. The joint annual total cost

(i) The vendor's annual cost is given by:

$$TC_v = \frac{S_v}{K_v T} + Q_r P_{v,g} + RKYP_{v,g} + LYD_{v,g} + \frac{h_s T D_{v,g}^2}{2P_{v,g}} + \frac{h_v T}{2}\left[(2-k_v)\frac{D_{v,g}^2}{P_{v,g}} + (k_v - 1)D_{v,g}\right] \tag{1}$$

(ii) The buyer's annual cost is given by:

$$TC_b = \frac{S_b}{T} + \frac{dD_{b,j}}{(1-kY)} + D_{b,j}\beta(1-\sigma) + \frac{h_b D_{b,j} T}{2(1-kY)} \times \left[\frac{D_{b,j} Y}{(X)(1-kY)} + (1-Y)^2 + \frac{(Y-kY)^2}{(1-kY)}\right] \tag{2}$$

(iii) The joint annual total cost for the whole supply chain $JTC_{(k_v,T)} = \sum_{g=1}^{n_v} TC_{v,g} + \sum_{j=1}^{n_b} TC_{b,j}$, is given by:

$$JTC_{(k_v,T)} = \sum_{g=1}^{n_v}\left(\begin{array}{l}\dfrac{S_v}{K_v T} + Q_r P_{v,j} + RKYP_{v,g} + LYD_{v,g}\dfrac{h_s T D_{v,g}^2}{2P_{v,g}} \\ + \dfrac{h_v T}{2}\left[(2-k_v)\dfrac{D_{v,g}^2}{P_{v,g}} + (k_v - 1)D_{v,g}\right]\end{array}\right)$$

$$+ \sum_{j=1}^{n_b}\left(\begin{array}{l}\dfrac{S_b}{T} + \dfrac{dD_{b,j}}{(1-kY)} + D_{b,j}\beta(1-\sigma) + \dfrac{h_b D_{b,j} T}{2(1-kY)} \\ \times\left[\dfrac{D_{b,j} Y}{(X)(1-kY)} + (1-Y)^2 + \dfrac{(Y-kY)^2}{(1-kY)}\right]\end{array}\right) \tag{3}$$

IV. Solution Procedure

For given K_v, clearly $JTC_{(k_v,T)}$ is convex function in T for T > 0.Therefore, minimum cost at a unique value of T.To determine the minimum cost, we taking the derivative of $JTC_{(k_v,T)}$ with respect to T, let $\partial JTC(k_v,T)/\partial T = 0$, which yields

$$T^* = \sqrt{2}\sqrt{\frac{(S_v n_v + S_b n_b K_v)(Yk - 1)}{K_v(\pi h_b - 2\theta(1 - Yk))}} \tag{4}$$

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where

$$\pi=\sum_{j=1}^{n_b}\left(D_{b,j}\left(\frac{Y^2(k-1)^2}{(Yk-1)}-(Y-1)^2+\frac{D_{b,j}Y}{(Yk-1)(X)}\right)\right)$$

$$\theta=\sum_{g=1}^{n_v}\left(\frac{-D_{v,g}}{2}\left(h_v\left(\frac{D_{v,g}(K_v-2)}{P_{v,g}}-K_v+1\right)-\frac{D_{v,g}h_s}{P_{v,g}}\right)\right)$$

Because to forecast buyer's annual demand, the $D_{b,j}$ will be substituted by $\hat{E}_{b,j}$,

and

$$\hat{E}_{b,j}=\hat{\alpha}_{b,j}+\hat{\beta}_{b,j}Q_n \tag{5}$$

$$\hat{\beta}_{b,j}=\frac{\sum_{t=1}^{n}\left(Q_{t,j}-\bar{Q}_j\right)\left(F_{t,j}-\bar{F}_j\right)}{\sum_{t=1}^{n}\left(Q_{t,j}-\bar{Q}_j\right)^2}\qquad \forall j=1\sim n_b \tag{6}$$

$$\hat{\alpha}_{b,j}=\bar{F}_j-\hat{\beta}_{b,j}\qquad \forall j=1\sim n_b \tag{7}$$

Algorithm

In order to obtain the minimum values of $JTC_{(k_v,T)}$, we following these steps:

Step 1. Determine $D_{b,j}$ by Eq.(5).

Step 2. Calculate the value $T^{(kv)}$ of using Eq.(4).

Step 3. Compute $JTC(k_v,T^{(k_v)})$ using Eq.(3).

Step 4. Let $k_V = k_V+1$, repeat steps 1 to 3 to find $JTC(k_v,T^{(k_v)})$.

Step 5. If $JTC(k_v, T^{(k_v)}) \le JTC(k_v-1, T^{(k_v-1)})$, returned to Step 4. Otherwise, the optimal solution is $\left(k_v^*,T^*\right)=\left(k_v-1,T^{(k_v-1)}\right)$

V. Numerical Example

The proposed analytic solution procedure is applied to solve the following numerical example. We set number of vendor and buyer is 3, $h_s=\$0.9$/units, $h_v=\$2$/units, $S_v=\$400$/cycle, $S_b=\$150$/cycle, $k=0.7$, $Y=0.03$, $Q_r=\$10$/units, $R=\$3$/units, $L=\$6$/units, $d=\$0.5$/units, $X=6000$ units/year, $B=\$9$/units, $\sigma=1-mkY=0.937$, and buyer's demand in past 7 years is shown in Table I.

Table I.
The data of buyer's demand (units/year) in past 7 years.

j	2009	2010	2011	2012	2013	2014	2015
1	1380	1450	950	1780	1900	1840	1760
2	860	970	800	1230	1450	1500	1600
3	650	630	780	985	1060	1240	1400

Before forecasting buyer's demand, we calculate Eq.(5), and get the value $\hat{\beta}_{b,j},\hat{\alpha}_{b,j}$ by using Eq.(6) and Eq.(7).Therefore, we obtain $D_{b,1}=1727$units/year, $D_{b,2}=1649$units/year, $D_{b,3}=1588$units/year, $D_{v,g}$ and $P_{v,g}$ is shown in Table II.

Table II.
The data of vendor's $D_{v,g}$ and $P_{v,g}$.

g	$D_{v,g}$ (units/year)	$P_{v,g}$ (units/year)
1	1550	5500
2	1640	4500
3	1774	3000

Applying the algorithm, we obtain the optimal solution as follows: buyer's cycle time is $T^*=0.273$(year), $k_v^*=6$, the vendor's cycle time is $T_v=1.636$(year), and the joint total cost is \$ 201,204.14.

We do sensitivity analysis to observe the JTC^* under different values of T, y and k, the results were shown in Table III and Table. IV.

Table. III.
Sensitivity analysis for Y and k.

	JTC*	***Y***		***k***	
		-25%	+25%	-25%	+25%
JTC	\$201,204	\$200,777	\$201,630	\$200,975	\$201,433
Compare with JTC*		-\$426	+\$426	-\$228	+\$229

Table. IV.
Sensitivity analysis for replenishment cycle T.

	JTC*	***T***	
		-25%	+25%
JTC	\$201,204.14	\$199,807.74	\$209,918
Compare with JTC*		-\$1,396.4	+\$8,713.86

VI. Conclusion

In this paper, we develop two-echelon integrated inventory model with unreliable process. Defective rate and repaired rate are important factors to affect the inventory policy. The goal of this paper is to optimize the joint total cost of the supply chain network with coordinating decision-making.

From Table. III, we can know the joint total cost will grow with increased *k* and *Y*, so defective products cause additional time and cost on purchasing and production. In this model, production patterns is pulling production, so back end demand of supply chain is the most important. Base on collaborative planning, forecasting and replenishment **(CPFR)**, we design a linear regression to forecast buyer's demand. According to forecast result, the decision-makers or managers can do a right decision to avoid enterprise operating loss.

Through Table. IV, we can know the joint total cost will increase along with longer replenishment cycle time, representing its affect the overall cost significantly. As mentioned above, we derive buyer's replenishment cycle time (T) from this model, which is the optimum value to whole supply chain, and vendor can predict manufacturing cycle time by multiply an integer number(k_v), thus enhance efficiency and reduce cost.

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Finally, we will improve our further research in more real-world complexities, such as add more factors into linear regression function to make forecasting result more accurate, and cooperate real case to get the actual data that can illustrate real numerical examples to our future research.

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