# Integrated Assembled Production Inventory Model

M.C. Lo, M.F. Yang*, T.S. Hung and S.F. Wang

**Abstract—Suppliers and manufacturers recognize the importance of interactions between financial and inventory decisions in the development of effective supply chains. Moreover, achieving effective coordination among the supply chain players has become a pertinent research issue. This paper considers a two-echelon model, consisting of multi-suppliers and one manufacturer, coordinating their situations to maximize the total supply chain profits. Each supplier supplies one or more components required in the final product produced. In the proposed inventory level model, the permissible delay in payments is coordinated to order quantity between two echelons.**

**Index Terms—Assemble, Permissible delay in payment, Two-echelon model**

## I. INTRODUCTION

A supply chain consists of different facilities where raw materials, intermediate products, or finished products are purchased, produced, or stored. In today's economy, many companies do not have all technical and organizational skills to efficiently satisfy the demand of customers. Therefore, they try to identify the business processes they can conduct efficiently. To manage these facilities like one company, the products, cash, or information flow should be integrated.

In assemble-to-order systems, suppliers send assembled items to the manufacturer when they receive order forms. Hillier[1] indicated that replacing some specific components by a smaller number of common components can reduce safety stock levels due to the benefits of risk pooling. He developed a model to consider the assemble-to-order environment where components were replenished according to a (Q,r) policy. Ervolina *et al.*[2] proposed a novel availability management process called Available-to-Sell (ATS) that drives a better supply chain efficiency. The substitution of higher-class components for lower one was often applied when the latter are stock-out.However, the decision for substitution should be made in advance. Iravani *et al.*[3] considerd an assemble-to-order system where each customer order consists of a mix of key and non-key items. Reiman and Wang[4] introduced a multi-stage stochastic program that provides a lower bound on the long-run average inventory cost. The stochastic program also motivates a replenishment policy for these systems. Recently, Chang *et al.*[5] considerd a two-stage assembly system with imperfect processes. Danilovic *et al.*[6] proposed a new optimization approach to address a multi-period, inventory control problem under stochastic environment. Elhafsi et al.[7] studied a assembled model serving both the demand of end products and the individual components.

Permissible delay in payment is a brand-new issue. The different between a traditional model and a new one is that the buyer must pay immediately when the vendor delivers products to the buyer in a traditional EOQ model. And in the model with permissible delay in payment, the vendor usually gives a fixed period to reduce the stress of capital. During the period, the buyer can keep products without paying to the vendor and earns extra interest from the sale. Jaber and Osman[8] proposed a centralized model where players in a two-level supply chain coordinate their orders to minimize their local costs. Pal *et al.*[9] investigated the optimal replenishment lot size of supplier and optimal production rate of manufacturer under three levels of trade credit policy. In 2013, Chiu and Yang *et al.*[10] developed an improved inventory model which helps the enterprises to advance their profit increasing and cost reduction in a single vendor-single buyer environment depending on the ordering quantity and imperfect production. For more closely conforming to the actual inventories and responding to the factors that contribute to inventory costs, they proposed model can be the references to the business applications. Das *et al.*[11] developed a multi-item inventory model with deteriorating items for multi-secondary warehouses and one primary warehouse. Items were sold from the primary warehouse which is located at the main market. If the stock level were numerous that there are insufficient space of the existing primary warehouse, then excess items will store at multi-secondary warehouses of finite capacity. Sarkar *et al.*[12] assumed a policy along with the production of defective items where the order quantity and lead time are considered as decision variables. Chen and Teng [13] proposed an EOQ model for a retailer when: (1) her/his product deteriorates continuously, and has a maximum lifetime, and (2) her/his supplier offers a permissible delay in payments. Yang and Tseng[14] proposed a three-echelon inventory model with permissible delay in payments under

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controllable lead time and backorder consideration to find out the suitable inventory policy to enhance profit of the supply chain. In the next year, Yang *et al*.[15] added defective production and repair rate to the proposed model and discussed how these factors may affect profits. In addition, holding cost, ordering cost, and transportation cost will also be considered as they develop the integrated inventory model with price-dependent payment period under the possible condition of defective products. Finally, this research consists of multi-suppliers and one manufacturer in the ATO system under the permissible delay in payment which coordinates their situations to minimize the total supply chain costs.

## II. NOTATIONS AND ASSUMPTIONS

In order to develop the two levels inventory model with assemble system and permissible delay in payment. We divide some notations of the expected joint annual inventory model in two parts which are the annual profit of multi-suppliers and one manufacturer. The notations and assumptions as below are used in this two levels inventory model:

### A. Notations

Q = Manufacturer's economic quantity, a decision variable.

$n_s$ = The number of lots delivered in a production cycle from the $s^{th}$ supplier to the manufacturer, a positive integer, a decision variable.

Supplier side

$Q_s$ = Economic delivery component quantity of each supplier, where $Q_s = Q * \sum_i u_{si}$.

$P_s$ = The $s^{th}$ Supplier's production rate.

$D_s$ = Average annual demand per unit time of each supplier.

$C_{si}$ = Supplier's purchasing cost for item i per unit.

$A_s$ = Supplier's ordering cost per order.

$F_{si}$ = Supplier's transportation cost for item i per order.

$h_{si}$ = Supplier's holding cost for item i per unit.

$I_{sp}$ = Supplier's opportunity cost per dollar per year.

$I_{se}$ = Supplier's interest earned per dollar per year.

$T_s$ = Supplier's cycle time.

$u_{si}$ = number of units required in one unit of the finished product which supplied by the $s^{th}$supplier.

m = number of suppliers, where s=1,2,…,m.

$k_s$ = number of different types of items supplied by supplier s to the manufacturer, where i=1,2,…, $k_s$.

k = number of different types of items supplied by m suppliers. Note that $k = \sum_{s=1}^{m} k_s$ and each supplier supply unique items.That is supplier specific and never identical amongst suppliers.

Manufacturer side

$P_m$ = Manufacturer's production rate
= Manufacturer's assembling rate.

D = Average annual demand per unit time.

$C_{mi}$ = Manufacturer's purchasing cost for item i per unit.

$C_p$ = Manufacturer's selling price per unit.

$A_m$ = Manufacturer's ordering cost per order.

B = Manufacturer's assembling cost per unit.

$h_{mi}$ = Manufacturer's holding cost for the item i per unit.

$h_m$ = Manufacturer's holding cost for finished product per unit.

$I_{mp}$ = Manufacturer's opportunity cost per dollar per year.

$I_{me}$ = Manufacturer's interest earned per dollar per year.

X = Manufacturer's permissible delay period.

$n_m$ = The number of lots delivered in a production cycle from the manufacturer to a retailer, a positive integer.

$TP_{sj}$ = Supplier's total annual profit in case j, where j=1,2.

$CTP_{sj}$ = Collective the total annual profit of all suppliers in case j, where j=1,2.

$TP_{mj}$ = Manufacturer's total annual profit in case j, where j=1,2.

$EJTP_j$ = The expected joint total annual profit in case j, where j=1,2.

"j" represents two different cases to the relationship of the supplier's cycle time and permissible payment period of the manufacturer. The detail will be discussed in the Section 3.

### B. Assumptions

In this paper, we assume:

(i) This supply chain system consists of multi-suppliers and a manufacturer.
(ii) The finished product requires k items.
(iii) Demand is deterministic and constant over time.
(iv) Economic delivery quantity multiplies by the number of delivery per production run is economic order quantity (EOQ).
(v) Shortages are not allowed.
(vi) The sale price must not be less than the purchasing cost at any echelon, $C_p > \sum_s \sum_i C_{mi} * u_{si} > \sum_s \sum_i C_{si} * u_{si}$
(vii) The time horizon is infinite.

## III. MODEL FORMULATION

In this section, we discuss and develop the supplier and manufacturer's model and combine them into an integrated joint inventory model.

### A. The supplier's total annual profit

Supplier *s* supplies $i^{th}$ item to the manufacturer and each supplier supplies one or more unique items. We divide a few parts in the supplier's model which are sales revenue, purchasing cost, ordering cost, transportation cost, holding cost, opportunity cost and interest income. The supplier's total annual profit consists of the following elements:

(1)Sales revenue $= D * \sum_i C_{mi} * u_{si}$

(2)Purchasing cost $= D * \sum_i C_{si} * u_{si}$

(3)Ordering cost $= \frac{A_s D}{Q}$

(4)Transportation cost $= \sum_i \frac{n_s F_{si} * D}{Q}$

(5)Holding cost $= \sum_i (h_{si} * \frac{D_s Q_s}{2P_s})$

The inventory level of supplier is the black area in Fig1 which can be calculated as follows:

$$\text{Area}_s = \frac{1}{2} * Q_s * \frac{Q_s}{P_s} = \frac{{Q_s}^2}{2P_s}$$

$$H_s = \sum_i h_{si} * \frac{D_s}{Q_s} * \text{Area}_s = \sum_i (h_{si} * \frac{D_s Q_s}{2P_s})$$


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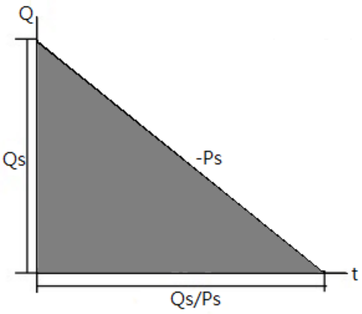


Fig I The supplier's inventory level

Due to the conditions of permissible delay in payments, there are two cases we have to investigate. In Fig 2, when the payment time X was longer than the cycle time $T_s$, it would bring additional interest income to the manufacturer which is paid by the supplier. In other side(Fig 3), if the payment time X was shorter than the cycle time $T_s$, it would bring additional opportunity cost and fewer interest income to manufacturer, and the supplier would earn interest income and pay the fewer opportunity cost. Owing to the fact that the supplier's profit function has two cases, based on length of cycle time $T_s$ and payment time X, the two different parts between two possible cases are as follows:

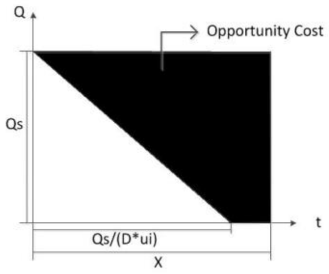


Fig II Q/D>X

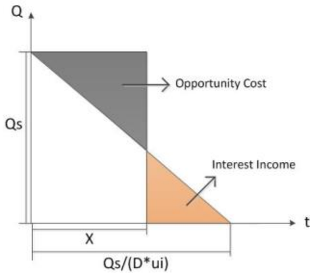


Fig III Q/D<X

***Case 1*** $(\boldsymbol{T_s} < X)$

(6)Opportunity cost $= \sum_i C_{si} * I_{sp} * \left(D_s X - \frac{D_s}{2}\right)$

***Case 2*** $(\boldsymbol{T_s} \geq \mathbf{X})$

(7)Opportunity cost $= \sum_i C_{si} * I_{sp} * \frac{(D_s * X)^2}{2Q_s}$

(8)Interest income $= \sum_i C_{mi} * I_{se} * \frac{(Q_s - D_s * X)^2}{2Q_s}$

In case 1, the collective total annual cost for *m* suppliers can be expressed as follows:

$$\boldsymbol{TP_{s1}} = \sum_s \left[ \mathrm{D} * \sum_i (C_{mi} - C_{si}) * u_{si} - \frac{A_s D}{Q} - \sum_i \frac{n_s F_{si} * D}{Q} - \sum_i \left(h_{si} * \frac{D_s Q_s}{2P_s}\right) - \sum_i C_{si} * I_{sp} * \left(D_s X - \frac{D_s}{2}\right) \right] \quad (1)$$

In case 2, the collective total annual cost for *m* suppliers can be expressed as follows:

$$\boldsymbol{TP_{s2}} = \sum_s \left[ \mathrm{D} * \sum_i (C_{mi} - C_{si}) * u_{si} - \frac{A_s D}{Q} - \sum_i \frac{n_s F_{si} * D}{Q} - \sum_i (h_{si} * \frac{D_s Q_s}{2P_s}) - \sum_i C_{si} * I_{sp} * \frac{(D_s * X)^2}{2Q_s} + \sum_i C_{mi} * I_{se} * \frac{(Q_s - D_s * X)^2}{2Q_s} \right] \quad (2)$$

*B. The manufacturer's total annual profit*

In each production run, we divide a few parts in the manufacturer's model which are sales revenue, purchasing cost, ordering cost, assembling cost, holding cost for items, holding cost for finished products, opportunity cost and interest income. The manufacturer's total annual profit consists of the following elements:

(1)Sales revenue $= \mathrm{D} * C_p$

(2)Purchasing cost $= \mathrm{D} * \sum_s \sum_i (C_{mi} * u_{si})$

(3)Ordering cost $= \frac{A_m D}{Q}$

(4)Assemble cost $= BP_m$

(5)Holding cost of items $= \sum_s \sum_i h_{mi} * n_s D_s Q_s (\frac{2 - n_s}{2p_s} + \frac{n_s - 1}{2D_s})$

The gray area in Fig 4 represents the manufacturer's inventory in one period which can be calculated as follows:

$$Area_m = n_s Q_s \left[\frac{Q_s}{P_s} + \frac{(n_s - 1)Q_s}{D_s}\right] - \frac{1}{2} * n_s Q_s * n_s \frac{Q_s}{P_s} - Q_s * \frac{Q_s}{D_s} * \frac{(n_s - 1)n_s}{2} = n_s {Q_s}^2 \left(\frac{2 - n_s}{2P_s} + \frac{n_s - 1}{2D_s}\right)$$

$$\mathrm{H}_m = h_{mi} * \frac{D_s}{Q_s} * Area_m = h_{mi} * n_s D_s Q_s (\frac{2 - n_s}{2p_s} + \frac{n_s - 1}{2D_s})$$

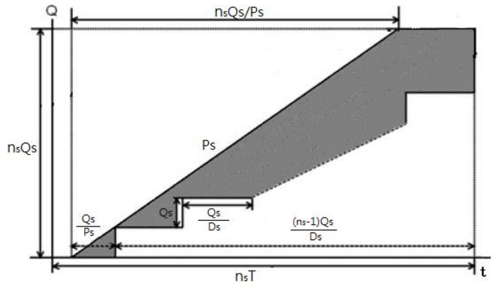


Fig IV The manufacturer's inventory level

After the manufacturer receives the items from multi-suppliers, the manufacturer will starts assembling the products. And the holding cost of the finished products can be revealed as follows:

(6)Holding cost of the finished products $= h_m * n_m DQ(\frac{2 - n_m}{2p_m} + \frac{n_m - 1}{2D})$

Going on the last section, we talk about the relationship between the payment time and the cycle time. There are also two cases we have to investigate in the manufacturer's model which is similar as the supplier's model. Owing to the fact that the manufacturer's profit function has two cases, based on length of cycle time $T_s$ and payment time X, the two different parts between two possible cases are as follows:

***Case 1***

(7)Interest income $= \sum_s \sum_i \left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} \left(D_s X - \frac{D_s}{2}\right)\right]$

***Case 2***

(8)Opportunity cost $= \sum_s \sum_i \left[C_{mi} * I_{mp} * \frac{(Q_s - D_s * X)^2}{2Q_s}\right]$

 

(9)Interest income $= \sum_s \sum_i \left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} * \frac{(D_s * X)^2}{2Q_s}\right]$

Thus, $\boldsymbol{TP_{m1}}$ and $\boldsymbol{TP_{m2}}$ are given by:

$$\boldsymbol{TP_{m1}} = \mathrm{D} * \left[C_p - \sum_s \sum_i (C_{mi} * u_{si})\right] - \frac{A_m D}{Q} - BP_m - \sum_s \sum_i h_{mi} * n_s D_s Q_s \left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right) - h_m * n_m DQ\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right) + \sum_s \sum_i \left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me}\left(D_s X - \frac{D_s}{2}\right)\right] \quad (3)$$

$$\boldsymbol{TP_{m2}} = \mathrm{D} * \left[C_p - \sum_s \sum_i (C_{mi} * u_{si})\right] - \frac{A_m D}{Q} - BP_m - \sum_s \sum_i h_{mi} * n_s D_s Q_s \left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right) - h_m * n_m DQ\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right) - \sum_s \sum_i \left[C_{mi} * I_{mp} * \frac{(Q_s - D_s * X)^2}{2Q_s}\right] + \sum_s \sum_i \left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} * \frac{(D_s * X)^2}{2Q_s}\right] \quad (4)$$

*C. The expected joint total annual profit:*

With the suppliers and manufacturer's total annual profit model, the expected joint total annual profit function, EJTP can be expressed as follows:

$$\mathrm{EJPT}(n_s, Q_s) = \begin{cases} \boldsymbol{EJTP_1}(n_s, Q) = TP_{s1} + TP_{m1} \\ \boldsymbol{EJTP_2}(n_s, Q) = TP_{s2} + TP_{m2} \end{cases},$$

where

$$\boldsymbol{EJTP_1}(n_s, Q) = \mathrm{D} * \left[C_p - \sum_s \sum_i (C_{si} * u_{si})\right] - \frac{D}{Q}\left(\sum_s A_s + A_m + \sum_s \sum_i n_s F_{si}\right) - BP_m - \sum_s \sum_i \left(h_{si} * \frac{D_s Q_s}{2P_s}\right) - \sum_s \sum_i h_{mi} * n_s D_s Q_s \left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right) - h_m * n_m DQ\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right) + \left(D_s X - \frac{D_s}{2}\right)\sum_s \sum_i \left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} - C_{si} * I_{sp}\right] \quad (5)$$

$$\boldsymbol{EJTP_2}(n_s, Q) = \mathrm{D} * \left[C_p - \sum_s \sum_i (C_{si} * u_{si})\right] - \frac{D}{Q}\left(\sum_s A_s + A_m + \sum_s \sum_i n_s F_{si}\right) - BP_m - \sum_s \sum_i \left(h_{si} * \frac{D_s Q_s}{2P_s}\right) - \sum_s \sum_i h_{mi} * n_s D_s Q_s \left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right) - h_m * n_m DQ\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right) + \sum_s \sum_i \frac{(D_s * X)^2}{2Q_s}\left[C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} - C_{si} * I_{sp}\right] + \sum_s \sum_i \frac{(Q_s - D_s * X)^2}{2Q_s}\left[C_{mi} * I_{se} - C_{mi} * I_{mp}\right] \quad (6)$$

## IV. SOLUTION PROCEDURE

This is decentralized decision-making process, which involves multiple decision-maker, where each decision-maker tends to optimize its own performance to maximize the expected joint total annual profit. In order to maximize $EJPT_j(n_s, Q)$, we set $[\partial EJTP_1(n_s, Q)/\partial Q] = 0$ and obtain the economic value of $Q = Q^*_{EJPT1}, Q^*_{EJPT2}$. To prevent the equations are too long to read, we set some notations as follows:

$H_{si} = \sum_s \sum_i \left(h_{si} * \frac{D_s * u_{si}}{2P_s}\right)$, $H_m = h_m * n_m D\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right)$

$H_{mi} = \sum_s \sum_i h_{mi} * n_s D_s * u_{si}\left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right)$

and after calculating we can know that:

$$Q^*_{EJTP1} = \sqrt{\frac{D\left(\sum_s A_s + A_m + \sum_s \sum_i n_s F_{si}\right)}{H_{si} + H_{mi} + H_m}} \quad (7)$$

$$Q^*_{EJTP2} = \frac{\sqrt{\sum_i u_{si}\left[2D \sum_i u_{si}\left(\sum_s A_s + A_m + \sum_s \sum_i n_s F_{si}\right) - \sum_s \sum_i \left(C_p * \frac{u_{si}}{\sum_s \sum_i u_{si}} * I_{me} - C_{si} * I_{sp}\right)(D_s * X)^2 - \sum_s \sum_i \left(C_{mi} * I_{se} - C_{mi} * I_{mp}\right)(D_s * X)^2\right]}}{2\sum_i u_{si}(H_{si} + H_{mi} + H_m) - \sum_i {u_{si}}^2 (C_{mi} * I_{se} - C_{mi} * I_{mp})} \quad (8)$$

*Algorithm*

In order to obtain the optimal values of $EJPT_j(n_s, Q)$, we follow these steps:

***Step 1.*** Choose $s$ supplier where *s=1,2,3…,m.*

***Step 2.*** Set $\mathrm{n} = n_{sj} = 1$ where *j*=1,2 and substitute into (7) and (8) to obtain $Q_{EJPT1}$ and $Q_{EJPT2}$.

***Step 3.*** Find $EJTP_j$ by substituting $n_{sj}$ and $Q_{EJPTj}$, into (5) and (6), where j=1,2.

***Step 4.*** Let $n_i = n_i + 1$ and repeat step2 to step3 until $EJTP_j(n_{sj}) > EJTP_j(n_{sj} + 1)$. The optimal $n^*_{sj} = n_{sj}$, where j=1,2

***Step 5.*** Since there are multi-suppliers, we repeat step1 to step4 until finding all $n^*_{sj}$; $Q^*_j = \mathrm{Q}(\text{all } n^*_{sj})$ where *j*=1,2 and *s=1,2,3…,m*.

***Step 6.*** Compare with payment period and do the sensitivity analysis to observe the economic ordering policies under different values of X.

## V. NUMERICAL EXAMPLE

A numerical example is used to demonstrate the proposed models in this section. Consider a two-level model with three suppliers, a manufacturer, and four items. The suppliers(*s*=1,2,3) have the following input parameters:

Table Ⅰ The data of each cost

| **Suppliers(*s*)** | $P_s$ | $A_s$ | $D_s$ | $I_{sp}$ | $I_{se}$ |
|---|---|---|---|---|---|
| **1** | 1300 | 50 | 2000 | 0.035 | 0.03 |
| **2** | 3000 | 40 | 5000 | 0.03 | 0.025 |
| **3** | 1400 | 55 | 3000 | 0.04 | 0.03 |
| **Manufacturer** | $P_m$ | $A_m$ | $D$ | $I_{mp}$ | $I_{me}$ |
| | 1200 | 70 | 1000 | 0.04 | 0.035 |

Each unit of finished product requires 4 items(*i*=1,2,3,4) with the following input parameters:

Table Ⅱ The data of items

| $s$ | **Items(*i*)** | $u_{si}$ | $C_{si}$ | $C_{mi}$ | $F_{si}$ | $h_{si}$ | $h_{mi}$ |
|---|---|---|---|---|---|---|---|
| **1** | **1** | 2 | 8 | 20 | 50 | 3 | 3 |
| **2** | **2** | 1 | 5 | 15 | 50 | 5 | 5 |
| | **3** | 4 | 2 | 10 | 40 | 1 | 2 |
| **3** | **4** | 3 | 10 | 20 | 60 | 3 | 4 |

The other notations are given $C_p = 300$, $\mathrm{B} = 50$, $\mathrm{X} = 0.205479(\text{i.e } 75\text{days})$, $n_m = 3$. Following the equation and algorithm already given in this paper, the economic


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ordering policy is shown in Table Ⅲ.

Table Ⅲ The economic ordering policy

| | **Case 1** | **Case 2** |
|---|---|---|
| $n_1^*$ | 1 | 1 |
| $n_2^*$ | 2 | 1 |
| $n_3^*$ | 2 | 1 |
| $Q^*$ | 124 | 14 |
| $EJTP_j^*$ | 158131.2 | 201117.5* |

Finally, sensitivity analysis which calculates the $EJTP_j$ under different values of X is shown in TableⅣ.

Table Ⅳ The inventory policy under different X

| X(days)→ | | 65 | 75 | 85 |
|---|---|---|---|---|
| **Case 1** | $n_1^*$ | 1 | 1 | 1 |
| | $n_2^*$ | 2 | 2 | 2 |
| | $n_3^*$ | 2 | 2 | 2 |
| | $Q^*$ | 124 | 124 | 124 |
| | $EJTP_1^*$ | 157115 | 158131.2 | 159147.4 |
| Case 2 | $n_1^*$ | 1 | 1 | 1 |
| | $n_2^*$ | 1 | 1 | 1 |
| | $n_3^*$ | 1 | 1 | 1 |
| | $Q^*$ | 12 | 14 | 17 |
| | $EJTP_2^*$ | 190908.5 | 201117.5 | 209896.2 |

## VI. CONCLUSION

Two-echelon models with ATO system and permissible delay in payment are few and far between in the literature. Most of these works consider only a single situation. This paper is therefore a contribution along this line of research and develops a new model formulated a two-echelon integrated inventory model with multi-suppliers and a manufacturer. From Table 3 and Table 4, we can know that

(i) In this model, case 2 ($T_s \geq X$) can earn more profit than case 1 ($T_s < X$). That means the cycle time of suppliers ($T_s$) should be longer than the credit period (X).
(ii) As the credit period (X) increases, there is a marginal increase in expected joint total profit.
(iii) In case 1, as the credit period (X) increases, the value of ordering quantity (Q) doesn't change.
(iv) There are little correlation between the credit period (X) and $n_s$.
(v) The order quantity in case 1 is much more than case 2.

According to the points we put forward, some arguments are sorted out. First, although offering the credit period to a manufacturer leads additional cost to suppliers, it can reduce the burden of cost for manufacturer. If the manufacturer can control its sale revenue well, it'll enhance the performance to the whole supply chain effectively.

Second, from managerial point of view, it'll be more profitable to run the case 2 than case 1. But the order quantity in case 1 is much more than case 2. That means the number of orders and carriages are quite large. If the burst of economic bubbles makes economic downturn or the oil price increases, the decision may be changed.

Finally, this model can be extended in several directions including extension to systems with multiple-retailers or defective situation. In this paper, we expect the optimal policy, although maybe more complex, will retain the same structure. Another extension would be to the model, where the demand of finished product maybe backordered rather than cash of individual items maybe backordered as well.

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