Interactive Decision Making for Multiobjective Fuzzy Random Simple Recourse Programming Problems and Its Application to Rainfed Agriculture in Philippines

Hitoshi Yano and Rongrong Zhang

***Abstract*—In this paper, we formulate a multiobjective fuzzy random simple recourse programming problem, in which fuzzy random variables coefficients are involved in equality constraints. In the proposed method, equality constraints with fuzzy random variables are defined on the basis of a possibility measure and and a two-stage programming method. For a given permissible possibility level specified by the decision maker, a Pareto optimality concept is introduced. An interactive decision making method is proposed to obtain a satisfactory solution from among a Pareto optimal solution set. The proposed method is applied to a farm planning problem in the Philippines, in which it is assumed that an amount supplied of water resource in dry season is represented as a fuzzy random variable.**

***Index Terms*—multiobjective programming, simple recourse programming, a possibility measure, fuzzy random variables, a satisfactory solution.**

I. Introduction

During the past six decades, various types of stochastic programming approaches have been proposed to deal with mathematical programming problems with random variable coefficients. Such approaches can be classified into two groups, one is two-stage programming methods and the other is chance constraints methods [1], [2], [3], [6], [14], [16], [18]. In two-stage programming problems [1], [2], the first-stage is to minimize the penalty cost for the violation of the equality constraints under the assumption that the decision variables are fixed, and the second-stage is to minimize the original objective function and the corresponding penalty cost. For chance constraint programming problems [13], [14], a probability maximization model and a fractile optimization model were proposed. In a probability model, the probability that the objective function is smaller than a certain target value is maximized. A fractile optimization model can be regarded as a complementary to the corresponding probability maximization model, in which a target variable is optimized under the condition that the probability that the objective function is smaller than the target variable is larger than a given value.

Two-stage programming methods have been applied to various types of water resource allocation problems with random inflow in future [15], [17]. However, if probability density functions of random variables are unknown or the problem is a large scale one with random variables, it may be extremely hard to solve the corresponding two-stage programming problem. From such a point of view, inexact two-stage programming methods have been proposed [5], [10].

As an extension of two-stage programming methods for multiobjective programming problems, Sakawa et al. [12] proposed an interactive fuzzy decision making method for multiobjective stochastic programming problems with simple recourse. However, in the real world decision making situations, it seems to be natural to consider that the uncertainty is expressed by not only fuzziness but also randomness simultaneously. From such a point of view, interactive decision making methods for multiobjective fuzzy random programming problems have been proposed [7], [8], in which chance constraint methods and a possibility measure are applied to deal with fuzzy random variable coefficients [9].

In this paper, we focus on multiobjective fuzzy random simple recourse programming problems, where the coefficients of equality constraints are defined by fuzzy random variables [9], and propose an interactive decision making method to obtain a satisfactory solution from among a Pareto optimal solution set. In section II, using a possibility measure [4] and a two-stage programming method, multiobjective programming problems with fuzzy random variable coefficients are transformed into multiobjective fuzzy random simple recourse programming problems, and a corresponding Pareto optimality concept is introduced. To obtain a candidate of a satisfactory solution from among a Pareto optimal solution set, an interactive algorithm is developed. In section III, we further consider a generalized multiobjective fuzzy random simple recourse programming problem, where not only the coefficients of equality constraints are fuzzy random variables but also the coefficients of the objective functions are random variables. To deal with such a problem, both a two-stage programming method and a chance constraint method are applied simultaneously. In section IV, to show the efficiency of the proposed method, we apply the proposed method to a farm planning problem in the Philippines [20], in which it is assumed that an amount supplied of water resource in dry season is represented as a fuzzy random variable.

II. Multiobjective Fuzzy Random Simple Recourse Programming Problems

In this section, we focus on multiobjective programming problems involving fuzzy random variable coefficients in the

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right-hand sides of the equality constraints.

$$\min_{\boldsymbol{x}\in X}\ (\boldsymbol{c}_1\boldsymbol{x},\cdots,\boldsymbol{c}_k\boldsymbol{x}) \tag{1}$$

subject to

$$A\boldsymbol{x}=\tilde{\bar{\boldsymbol{d}}} \tag{2}$$

where $\boldsymbol{c}_\ell=(c_{\ell 1},\cdots,c_{\ell n}),\ell=1,\cdots,k$ are n dimensional coefficient row vectors of objective function, $\boldsymbol{x}=(x_1,\cdots,x_n)^T\geq\boldsymbol{0}$ is an n dimensional decision variable column vector, X is a linear constraint set with respect to $\boldsymbol{x}$. A is an $(m\times n)$ dimensional coefficient matrix, $\tilde{\bar{\boldsymbol{d}}}=(\tilde{\bar{d}}_1,\cdots,\tilde{\bar{d}}_m)^T$ is an m dimensional coefficient column vector whose elements are fuzzy random variables [9] (The symbols "-" and "~" mean randomness and fuzziness respectively).

In order to deal with fuzzy random variables efficiently, Katagiri et al. [7], [8] defined an LR-type fuzzy random variable, which is a special type of a fuzzy random variable. Under the occurrence of each elementary event ω, $\tilde{\bar{d}}_i(\omega)$ is a realization of an LR-type fuzzy random variable $\tilde{\bar{d}}_i$, which is an LR fuzzy number [4] whose membership function is defined as follows.

$$\mu_{\tilde{\bar{d}}_i(\omega)}(s)=\begin{cases} L\left(\frac{\bar{b}_i(\omega)-s}{\alpha_i}\right), & s\leq\bar{b}_i(\omega)\\ R\left(\frac{s-\bar{b}_i(\omega)}{\beta_i}\right), & s>\bar{b}_i(\omega)\end{cases} \tag{3}$$

where the function $L(t)\stackrel{\rm def}{=}\max\{0,l(t)\}$ is a real-valued continuous function from $[0,\infty)$ to $[0,1]$, and $l(t)$ is a strictly decreasing continuous function satisfying $l(0)=1$. Also, $R(t)\stackrel{\rm def}{=}\max\{0,r(t)\}$ satisfies the same conditions. $\alpha_{ij}(>0)$ and $\beta_{ij}(>0)$ are called left and right spreads [4]. The mean value $\bar{b}_i$ is a random variable, whose probability density function and cumulative distribution function are defined as $f_i(\cdot)$ and $F_i(\cdot)$ respectively. It is assumed that random variables $\bar{b}_i,i=1,\cdots,m$ are independent each other.

Since it is difficult to deal with multiobjective fuzzy random simple recourse programming problems (1) directly, we introduce a permissible possibility level $\gamma(0<\gamma\leq 1)$ based on a concept of a possibility measure [4] for the equality constraints (2),

$$\mathrm{Pos}(\boldsymbol{a}_i\boldsymbol{x}=\tilde{\bar{\boldsymbol{d}}}_i(\omega))\geq\gamma,i=1,\cdots,m, \tag{4}$$

where $\boldsymbol{a}_i=(a_{i1},\cdots,a_{in}),i=1,\cdots,m$ are n-dimensional row vectors of A. From the property of LR fuzzy numbers, the i-th inequality condition (4) can be transformed into the following two inequalities.

$$\bar{b}_i(\omega)-L^{-1}(\gamma)\alpha_i\leq\boldsymbol{a}_i\boldsymbol{x}\leq\bar{b}_i(\omega)+R^{-1}(\gamma)\beta_i \tag{5}$$

For the above two inequalities (5), we introduce two vectors

$$\boldsymbol{y}^+=(y_1^+,\cdots,y_m^+)^T\geq\boldsymbol{0},\boldsymbol{y}^-=(y_1^-,\cdots,y_m^-)^T\geq\boldsymbol{0},$$

where (y_i^+,y_i^-) represent the shortage and the excess for the interval (5), and the following relations hold [19].
(1) For the case $\bar{b}_i(\omega)-L^{-1}(\gamma)\alpha_i>\boldsymbol{a}_i\boldsymbol{x}$, it holds that $y_i^+=\bar{b}_i(\omega)-L^{-1}(\gamma)\alpha_i-\boldsymbol{a}_i\boldsymbol{x}>0,y_i^-=0$.
(2) For the case $\bar{b}_i(\omega)+R^{-1}(\gamma)\beta_i<\boldsymbol{a}_i\boldsymbol{x}$, it holds that $y_i^+=0,y_i^-=\boldsymbol{a}_i\boldsymbol{x}-(\bar{b}_i(\omega)+R^{-1}(\gamma)\beta_i)>0$.
(3) For the case $\bar{b}_i(\omega)-L^{-1}(\gamma)\alpha_i\leq\boldsymbol{a}_i\boldsymbol{x}\leq\bar{b}_i(\omega)+R^{-1}(\gamma)\beta_i$, it holds that $y_i^+=0,y_i^-=0$.

Yano [19] has already formulated fuzzy random simple recourse programming problems using $(\boldsymbol{y}^+,\boldsymbol{y}^-)$. In this paper, as a extension of [19], we formulated a multiobjective fuzzy random simple recourse programming problem (1) as follows.

$$\left.\begin{array}{l} \min\limits_{\boldsymbol{x}\in X}\ \boldsymbol{c}_1\boldsymbol{x}+E\left[\min\limits_{\boldsymbol{y}^+,\boldsymbol{y}^-}\ (\boldsymbol{q}_1^+\boldsymbol{y}^++\boldsymbol{q}_1^-\boldsymbol{y}^-)\right]\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ \min\limits_{\boldsymbol{x}\in X}\ \boldsymbol{c}_k\boldsymbol{x}+E\left[\min\limits_{\boldsymbol{y}^+,\boldsymbol{y}^-}\ (\boldsymbol{q}_k^+\boldsymbol{y}^++\boldsymbol{q}_k^-\boldsymbol{y}^-)\right]\end{array}\right\} \tag{6}$$

subject to

$$\begin{array}{l}\boldsymbol{a}_i\boldsymbol{x}+y_i^+\geq\bar{b}_i(\omega)-L^{-1}(\gamma)\alpha_i,i=1,\cdots,m\\ \boldsymbol{a}_i\boldsymbol{x}-y_i^-\leq\bar{b}_i(\omega)+R^{-1}(\gamma)\beta_i,i=1,\cdots,m\\ \boldsymbol{x}\in X,\boldsymbol{y}^+\geq\boldsymbol{0},\boldsymbol{y}^-\geq\boldsymbol{0}\end{array}$$

where

$$\boldsymbol{q}_\ell^+ = (q_{\ell 1}^+,\cdots,q_{\ell m}^+)\geq\boldsymbol{0},\ell=1,\cdots,k \tag{7}$$
$$\boldsymbol{q}_\ell^- = (q_{\ell 1}^-,\cdots,q_{\ell m}^-)\geq\boldsymbol{0},\ell=1,\cdots,k \tag{8}$$

are m dimensional weighting row vectors for $\boldsymbol{y}^+$ and $\boldsymbol{y}^-$ respectively. For the ℓ-th objective function of (6), the second term can be transformed into follows [19].

$$\begin{aligned} & E\left[\min_{\boldsymbol{y}^+,\boldsymbol{y}^-}\ (\boldsymbol{q}_\ell^+\boldsymbol{y}^++\boldsymbol{q}_\ell^-\boldsymbol{y}^-)\right]\\ = & \sum_{i=1}^m q_{\ell i}^+\Big(E[\bar{b}_i]-\boldsymbol{a}_i\boldsymbol{x}-L^{-1}(\gamma)\alpha_i\Big)\\ & +\sum_{i=1}^m q_{\ell i}^+\left\{(\boldsymbol{a}_i\boldsymbol{x}+L^{-1}(\gamma)\alpha_i)F_i(\boldsymbol{a}_i\boldsymbol{x}+L^{-1}(\gamma)\alpha_i)\right.\\ & \left.-\int_{-\infty}^{\boldsymbol{a}_i\boldsymbol{x}+L^{-1}(\gamma)\alpha_i} b_if_i(b_i)db_i\right\}\\ & +\sum_{i=1}^m q_{\ell i}^-\left\{(\boldsymbol{a}_i\boldsymbol{x}-R^{-1}(\gamma)\beta_i)F_i(\boldsymbol{a}_i\boldsymbol{x}-R^{-1}(\gamma)\beta_i)\right.\\ & \left.-\int_{-\infty}^{\boldsymbol{a}_i\boldsymbol{x}-R^{-1}(\gamma)\beta_i} b_if_i(b_i)db_i\right\}\\ \stackrel{\rm def}{=} & d_\ell(\boldsymbol{x},\gamma)\end{aligned} \tag{9}$$

In the following, we define the objective functions as:

$$z_\ell(\boldsymbol{x},\gamma)\stackrel{\rm def}{=}\boldsymbol{c}_\ell\boldsymbol{x}+d_\ell(\boldsymbol{x},\gamma),\ell=1,\cdots,k. \tag{10}$$

Then, a multiobjective programming problem (1) can be reduced to a multiobjective fuzzy random simple recourse programming problem, in which a permissible possibility level γ is a parameter specified by the decision maker.

$$\min_{\boldsymbol{x}\in X}\ (z_1(\boldsymbol{x},\gamma),\cdots,z_k(\boldsymbol{x},\gamma)) \tag{11}$$

Now, we can define a Pareto optimal solution concept for (11).

Definition 1.
$\boldsymbol{x}^*\in X$ is said to be a γ-Pareto optimal solution to (11), if and only if there does not exist another $\boldsymbol{x}\in X$ such that $z_\ell(\boldsymbol{x},\gamma)\leq z_\ell(\boldsymbol{x}^*,\gamma),\ell=1,\cdots,k$ with strict inequality holding for at least one ℓ.

For generating a candidate for a satisfactory solution which is also a γ-Pareto optimal solution, the decision maker is asked to specify a permissible possibility level γ and

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the reference objective values $\hat{z}_\ell, \ell = 1, \cdots, k$ [11]. Once a permissible possibility level γ and reference objective values $\hat{z}_\ell, \ell = 1, \cdots, k$ are specified, the corresponding γ-Pareto optimal solution, which is in a sense close to his/her requirement or better than that if the reference objective values are attainable, is obtained by solving the following minimax problem [11].

[MINMAX1($\hat{\boldsymbol{z}}$,γ)]

$$\min_{\boldsymbol{x} \in X, \lambda \in \mathrm{R}^1} \lambda \tag{12}$$

$$\text{s.t.} \quad z_\ell(\boldsymbol{x}, \gamma) - \hat{z}_\ell \leq \lambda, \ell = 1, \cdots, k \tag{13}$$

The relationships between the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of MINMAX1($\hat{\boldsymbol{z}}$,γ) and γ-Pareto optimal solutions can be characterized by the following theorem.

Theorem 1.

(1) If $\boldsymbol{x}^* \in X, \lambda^* \in \mathrm{R}^1$ is a unique optimal solution of MINMAX1($\hat{\boldsymbol{z}}$,γ), then $\boldsymbol{x}^* \in X$ is a γ-Pareto optimal solution to (11).

(2) If $\boldsymbol{x}^* \in X$ is a γ-Pareto optimal solution, then $\boldsymbol{x}^* \in X$ $\lambda^* \stackrel{\text{def}}{=} z_\ell(\boldsymbol{x}^*, \gamma) - \hat{z}_\ell, \ell = 1, \cdots, k$ is an optimal solution of MINMAX1($\hat{\boldsymbol{z}}$,γ) for some reference objective values $\hat{\boldsymbol{z}} = (\hat{z}_1, \cdots, \hat{z}_k)$.

(Proof)

(1) Assume that $\boldsymbol{x}^* \in X$ is not a γ-Pareto optimal solution. Then, there exists $\boldsymbol{x} \in X$ such that $z_\ell(\boldsymbol{x}, \gamma) \leq z_\ell(\boldsymbol{x}^*, \gamma), \ell = 1, \cdots, k$ with strict inequality holding for at least one ℓ. This means that $z_\ell(\boldsymbol{x}, \gamma) - \hat{z}_\ell \leq z_\ell(\boldsymbol{x}^*, \gamma) - \hat{z}_\ell \leq \lambda^*, \ell = 1, \cdots, k$, which contradicts the fact that $\boldsymbol{x}^* \in X$ is a unique optimal solution to MINMAX1($\hat{\boldsymbol{z}}$,γ).

(2) Assume that $\boldsymbol{x}^* \in X, \lambda^* \in \mathrm{R}^1$ is not an optimal solution to MINMAX1($\hat{\boldsymbol{z}}, \gamma$) for any reference objective values $\hat{\boldsymbol{z}} = (\hat{z}_1, \cdots, \hat{z}_k)$, which satisfy the equalities $\lambda^* = z_\ell(\boldsymbol{x}^*, \gamma) - \hat{z}_\ell, \ell = 1, \cdots, k$. Then, there exists some $\boldsymbol{x} \in X, \lambda < \lambda^*$ such that $z_\ell(\boldsymbol{x}, \gamma) - \hat{z}_\ell \leq \lambda, \ell = 1, \cdots, k$. This means that $z_\ell(\boldsymbol{x}, \gamma) < z_\ell(\boldsymbol{x}^*, \gamma), \ell = 1, \cdots, k$, which contradicts the fact that $\boldsymbol{x}^* \in X$ is a γ-Pareto optimal solution.

Unfortunately, it is not guaranteed that the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ to MINMAX1($\hat{\boldsymbol{z}}$,γ) is γ-Pareto optimal, if $(\boldsymbol{x}^*, \lambda^*)$ is not unique. In order to guarantee γ-Pareto optimality, we solve a γ-Pareto optimality test problem for $(\boldsymbol{x}^*, \lambda^*)$.

Theorem 2.

Let $\boldsymbol{x}^* \in X$, $\lambda^* \in \mathrm{R}^1$ be an optimal solution to MINMAX1($\hat{\boldsymbol{z}}$,γ), in which $\lambda^* = z_\ell(\boldsymbol{x}^*, \gamma) - \hat{z}_\ell, \ell = 1, \cdots, k$. Corresponding to the optimal solution $\boldsymbol{x}^* \in X$, solve the following γ-Pareto optimality test problem.

$$\max_{\boldsymbol{x} \in X, \boldsymbol{\epsilon} = (\epsilon_1, \cdots, \epsilon_k) \geq \boldsymbol{0}} \sum_{\ell=1}^{k} \epsilon_\ell \tag{14}$$

subject to

$$z_\ell(\boldsymbol{x}, \gamma) - \hat{z}_\ell + \epsilon_\ell \leq \lambda^*, \ell = 1, \cdots, k$$

Let $\check{\boldsymbol{x}} \in X, \check{\epsilon}_\ell \geq 0, \ell = 1, \cdots, k$ be an optimal solution to (14). If $\sum_{\ell=1}^{k} \check{\epsilon}_\ell = 0$, then $\boldsymbol{x}^* \in X$ is a γ-Pareto optimal solution to (11).

On the other hand, the partial differentiation of $z_\ell(\boldsymbol{x}, \gamma), \ell = 1, \cdots, k$ for $x_s, s = 1, \cdots, n$ and $x_t, t = 1, \cdots, n$ can be calculated as follows.

$$\begin{aligned} & \frac{\partial z_\ell(\boldsymbol{x}, \gamma)}{\partial x_s \partial x_t} \\ = & \sum_{i=1}^{m} q_{\ell i}^{+} a_{is} a_{it} f_i(\boldsymbol{a}_i \boldsymbol{x} + L^{-1}(\gamma)\alpha_i) \\ & + \sum_{i=1}^{m} q_{\ell i}^{-} a_{is} a_{it} f_i(\boldsymbol{a}_i \boldsymbol{x} - R^{-1}(\gamma)\beta_i) \end{aligned} \tag{15}$$

The Hessian matrix for $z_\ell(\boldsymbol{x}, \gamma)$ can be written as:

$$\begin{aligned} & \nabla^2 z_\ell(\boldsymbol{x}, \gamma) \\ = & \sum_{i=1}^{m} q_{\ell i}^{+} f_i(\boldsymbol{a}_i \boldsymbol{x} + L^{-1}(\gamma)\alpha_i) \cdot A_i \\ & + \sum_{i=1}^{m} q_{\ell i}^{-} f_i(\boldsymbol{a}_i \boldsymbol{x} - R^{-1}(\gamma)\beta_i) \cdot A_i, \end{aligned} \tag{16}$$

where $A_i, i = 1, \cdots, m$ are $(n \times n)$-dimensional matrices defined as follows.

$$A_i \stackrel{\text{def}}{=} \begin{pmatrix} a_{i1}^2 & \cdots & a_{i1}a_{in} \\ \vdots & \ddots & \vdots \\ a_{in}a_{i1} & \cdots & a_{in}^2 \end{pmatrix}, i = 1, \cdots, m \tag{17}$$

Because of the property of the Hessian matrix for $z_\ell(\boldsymbol{x}, \gamma), \ell = 1, \cdots, k$, the following theorem holds.

Theorem 3.

MINMAX1($\hat{\boldsymbol{z}}, \gamma$) is a convex programming problem.

(Proof)

From the definition (17), it holds that $A_i = \boldsymbol{a}_i^T \cdot \boldsymbol{a}_i$. Therefore, the following relation holds for any n-dimensional column vector $\boldsymbol{y} \in \mathrm{R}^1$.

$$\begin{aligned} \boldsymbol{y}^T A_i \boldsymbol{y} &= \boldsymbol{y}^T \cdot (\boldsymbol{a}_i^T \cdot \boldsymbol{a}_i) \cdot \boldsymbol{y} \\ &= (\boldsymbol{y}^T \cdot \boldsymbol{a}_i^T) \cdot (\boldsymbol{a}_i \cdot \boldsymbol{y}) \\ &= (\boldsymbol{a}_i \cdot \boldsymbol{y})^T \cdot (\boldsymbol{a}_i \cdot \boldsymbol{y}) \geq 0 \end{aligned}$$

This means that matrices $A_i, i = 1, \cdots, m$ are positive semidefinite. Because of the assumptions that probability density functions $f_i(\cdot) \geq 0, i = 1, \cdots, m$, and $q_{\ell i}^{+} \geq 0, q_{\ell i}^{-} \geq 0, \ell = 1, \cdots, k, i = 1, \cdots, m$, the following relation holds for each of the Hessian matrices $\nabla^2 z_\ell(\boldsymbol{x}, \gamma), \ell = 1, \cdots, k$.

$$\begin{aligned} & \boldsymbol{y}^T \nabla^2 z_\ell(\boldsymbol{x}, \gamma) \boldsymbol{y} \\ = & \sum_{i=1}^{m} q_{\ell i}^{+} f_i(\boldsymbol{a}_i \boldsymbol{x} + L^{-1}(\gamma)\alpha_i) \cdot \boldsymbol{y}^T A_i \boldsymbol{y} \\ & + \sum_{i=1}^{m} q_{\ell i}^{-} f_i(\boldsymbol{a}_i \boldsymbol{x} - R^{-1}(\gamma)\beta_i) \cdot \boldsymbol{y}^T A_i \boldsymbol{y} \geq 0 \end{aligned}$$

This means that MINMAX1($\hat{\boldsymbol{z}}, \gamma$) is a convex programming problem.

The relationship between a permissible possibility level γ and the optimal objective function value $z_\ell(\boldsymbol{x}^*, \gamma)$ can be characterized by the following theorem.

Theorem 4.

For the optimal solution $\boldsymbol{x}^* \in X$ to (11), the following relation holds.

$$\begin{aligned}&\frac{\partial z_\ell(\boldsymbol{x}^*,\gamma)}{\partial\gamma}\\ =\ & -\sum_{i=1}^m q_{\ell i}^+\frac{\partial L^{-1}(\gamma)}{\partial\gamma}\alpha_i\\ &+\sum_{i=1}^m q_{\ell i}^+\frac{\partial L^{-1}(\gamma)}{\partial\gamma}\alpha_i F_i(\boldsymbol{a}_i\boldsymbol{x}^*+L^{-1}(\gamma)\alpha_i)\\ &-\sum_{i=1}^m q_{\ell i}^-\frac{\partial R^{-1}(\gamma)}{\partial\gamma}\beta_i F_i(\boldsymbol{a}_i\boldsymbol{x}^*-R^{-1}(\gamma)\beta_i)\end{aligned} \tag{18}$$

Now, following the above discussions, we can present an interactive algorithm to derive a satisfactory solution from among a γ-Pareto optimal solution set to (11).

[An interactive algorithm 1]

Step 1: Set a permissible possibility level $\gamma = 1$.

Step 2: The decision maker sets the initial reference objective values $\hat{z}_\ell$ for $z_\ell(\boldsymbol{x},\gamma), \ell = 1,\cdots,k$.

Step 3: Solve MINMAX1($\hat{\boldsymbol{z}},\gamma$) and obtain the corresponding optimal solution $(\boldsymbol{x}^*,\lambda^*)$. For the optimal solution $\boldsymbol{x}^*$, a γ-Pareto optimality test problem is solved.

Step 4: If the decision maker is satisfied with the current value of the γ-Pareto optimal solution $z_\ell(\boldsymbol{x}^*,\gamma), \ell = 1,\cdots,k$, then stop. Otherwise, the decision maker updates his/her reference objective values $\hat{z}_\ell, \ell = 1,\cdots,k$, and/or a permissible possibility level γ, and return to Step 3.

III. Generalized Multiobjective Fuzzy Random Simple Recourse Programming Problems

In this section, we further consider generalized multiobjective fuzzy random simple recourse programming problems, in which not only fuzzy random variable coefficients but also random variables ones are involved in the constraints and objective functions.

$$\min_{\boldsymbol{x}\in X}\ (\bar{\boldsymbol{c}}_1\boldsymbol{x},\cdots,\bar{\boldsymbol{c}}_k\boldsymbol{x}) \tag{19}$$

subject to

$$A\boldsymbol{x} = \tilde{\bar{\boldsymbol{d}}} \tag{20}$$

where a decision variable vector $\boldsymbol{x}$, a constraint set X, a coefficient matrix A, an LR-type fuzzy random variable vector $\tilde{\bar{\boldsymbol{d}}}$ are already defined in the previous section, and $\bar{\boldsymbol{c}}_\ell = (\bar{c}_{\ell 1},\cdots,\bar{c}_{\ell n}), \ell = 1,\cdots,k$ is an n dimensional random variable coefficient row vectors of the objective function $\bar{\boldsymbol{c}}_\ell\boldsymbol{x}$. Let us assume that the each element $\bar{c}_{\ell j}$ is a Gaussian random variable, *i.e.*, $\bar{c}_{\ell j} \sim \mathrm{N}(E[\bar{c}_{\ell j}],\sigma_{\ell jj})$, and the positive definite variance covariance matrices $V_\ell, \ell = 1,\cdots,k$ between Gaussian random variables $\bar{c}_{\ell j}, j = 1,\cdots,n$ are given as:

$$V_\ell = \begin{pmatrix}\sigma_{\ell 11} & \sigma_{\ell 12} & \cdots & \sigma_{\ell 1n}\\ \sigma_{\ell 21} & \sigma_{\ell 22} & \cdots & \sigma_{\ell 2n}\\ \cdots & \cdots & \cdots & \cdots\\ \sigma_{\ell n1} & \sigma_{\ell n2} & \cdots & \sigma_{\ell nn}\end{pmatrix}, i = 1,\cdots,k. \tag{21}$$

We denote the vectors of the expectation for the random variable row vector $\bar{\boldsymbol{c}}_\ell$ as $\boldsymbol{E}[\bar{\boldsymbol{c}}_\ell] = (E[\bar{c}_{\ell 1}],\cdots,E[\bar{c}_{\ell n}]), \ell = 1,\cdots,k$. Then, using the variance covariance matrix V_ℓ, the objective function $\bar{\boldsymbol{c}}_\ell\boldsymbol{x}$ becomes a Gaussian random variable.

$$\bar{\boldsymbol{c}}_\ell\boldsymbol{x} \sim \mathrm{N}(\boldsymbol{E}[\bar{\boldsymbol{c}}_\ell]\boldsymbol{x}, \boldsymbol{x}^T V_\ell\boldsymbol{x}), \ell = 1,\cdots,k \tag{22}$$

According to the discussion in the previous section, for a permissible possibility level, a generalized multiobjective fuzzy random simple recourse programming problem (19) can be reduced to the following multiobjective stochastic programming problem.

$$\min_{\boldsymbol{x}\in X}\ (\bar{\boldsymbol{c}}_1\boldsymbol{x}+d_1(\boldsymbol{x},\gamma),\cdots,\bar{\boldsymbol{c}}_k\boldsymbol{x}+d_k(\boldsymbol{x},\gamma)) \tag{23}$$

where $d_\ell(\boldsymbol{x},\gamma), \ell = 1,\cdots,k$ are penalty costs defined by (9). If the decision maker specifies permissible probability levels $\hat{p}_\ell, \ell = 1,\cdots,k$ for $\bar{\boldsymbol{c}}_\ell\boldsymbol{x}$, the multiobjective stochastic problem (23) can be transformed into the following multiobjective programming problem through a fractile optimization model [13], [14].

$$\min_{\boldsymbol{x}\in X}(f_1(\boldsymbol{x},\gamma,\hat{p}_1),\cdots,f_k(\boldsymbol{x},\gamma,\hat{p}_k)) \tag{24}$$

where $f_\ell(\boldsymbol{x},\gamma,\hat{p}_\ell)$ is defined as follows.

$$\begin{aligned}f_\ell(\boldsymbol{x},\gamma,\hat{p}_\ell) \overset{\text{def}}{=}\ & \boldsymbol{E}[\bar{\boldsymbol{c}}_\ell]\boldsymbol{x}+\Phi^{-1}(\hat{p}_\ell)\cdot\sqrt{\boldsymbol{x}^T V_\ell\boldsymbol{x}}\\ & +d_\ell(\boldsymbol{x},\gamma)\end{aligned} \tag{25}$$

where $\Phi^{-1}(\cdot)$ is an inverse function of a cumulative standard normal distribution. It should be noted here that the problem (24) can be regarded as a generalized version of (11), since $f_\ell(\boldsymbol{x},\gamma,0.5)$ is equivalent to $z_\ell(\boldsymbol{x},\gamma)$ if $\boldsymbol{E}[\bar{\boldsymbol{c}}_\ell]$ is replaced by $\boldsymbol{c}_\ell$.

Similar to Definition 1, we can define a Pareto optimal solution concept to (24).

Definition 2.

$\boldsymbol{x}^* \in X$ is said to be a (γ, $\hat{\boldsymbol{p}}$)-Pareto optimal solution to (24), if and only if there does not exist another $\boldsymbol{x} \in X$ such that $f_\ell(\boldsymbol{x},\gamma,\hat{p}_\ell) \le f_\ell(\boldsymbol{x}^*,\gamma,\hat{p}_\ell), \ell = 1,\cdots,k$ with strict inequality holding for at least one ℓ.

For the reference objective values $\hat{f}_\ell, \ell = 1,\cdots,k$ specified by the decision maker, the corresponding (γ, $\hat{\boldsymbol{p}}$)-Pareto optimal solution is obtained by solving the following minimax problem [11].

[MINMAX2($\hat{\boldsymbol{f}}$, γ, $\hat{\boldsymbol{p}}$)]

$$\min_{\boldsymbol{x}\in X,\lambda\in\mathrm{R}^1}\ \lambda \tag{26}$$

$$\text{s.t.}\qquad f_\ell(\boldsymbol{x},\gamma,\hat{\boldsymbol{p}})-\hat{f}_\ell \le \lambda, \ell = 1,\cdots,k \tag{27}$$

Similar to Theorem 3, MINMAX2($\hat{\boldsymbol{f}}$, γ, $\hat{\boldsymbol{p}}$) become a convex programming problem, we can present the interactive algorithm to obtain a satisfactory solution from among a (γ, $\hat{\boldsymbol{p}}$)-Pareto optimal solution set.

[An interactive algorithm 2]

Step 1: Set a permissible possibility level $\gamma = 1$ and permissible probability levels $\hat{p}_\ell, \ell = 1,\cdots,k$.

Step 2: The decision maker sets the initial reference objective values $\hat{f}_\ell, \ell = 1,\cdots,k$ for $f_\ell(\boldsymbol{x},\gamma,\hat{\boldsymbol{p}}), \ell = 1,\cdots,k$.

Step 3: Solve MINMAX2($\hat{\boldsymbol{f}}$, γ, $\hat{\boldsymbol{p}}$) and obtain the corresponding optimal solution $(\boldsymbol{x}^*,\lambda^*)$. For the optimal solution $\boldsymbol{x}^*$, a (γ, $\hat{\boldsymbol{p}}$)-Pareto optimality test problem is solved.

Step 4: If the decision maker is satisfied with the current value $f_\ell(\boldsymbol{x}^*,\gamma,\hat{\boldsymbol{p}}), \ell = 1,\cdots,k$ then stop. Otherwise, the decision maker updates his/her reference objective values $\hat{f}_\ell, \ell = 1,\cdots,k$, a permissible possibility level γ, and/or permissible probability levels $\hat{p}_\ell, \ell = 1,\cdots,k$ and return to Step 3.

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TABLE I
PROFIT COEFFICIENTS $c_{tj}, t = 1, \cdots, 5, j = 1, \cdots, 7$

	j	1	2	3	4	5	6	7
t	year	c_{t1}	c_{t2}	c_{t3}	c_{t4}	c_{t5}	c_{t6}	c_{t7}
1	1989	4.5	32.6	4.0	72.6	7.3	2.7	10.9
2	1990	5.7	22.7	29.5	13.6	6.3	4.3	24.5
3	1991	3.8	26.3	42.3	42.9	5.1	1.2	13.3
4	1992	3.5	21.3	20.1	35.7	5.2	2.5	26.1
5	1993	4.4	26.2	39.3	22.5	8.4	2.2	26.6

IV. A SECOND CROP PLANNING PROBLEM OF PADDY FIELDS IN THE PHILIPPINES

In this section, we formulate a second crop planning problem of paddy fields in the Philippines [20] as a multiobjective simple recourse programming problem, in which the water availability constraint in the dry season is expressed as a equality constraint with a fuzzy random variable. In the model farm, only rice (x_1) is grown in the wet season between May and October, and in the dry season between November and April, tobacco(x_2), tomatoes(x_3), garlic(x_4), mungbeans(x_5), corn(x_6) and sweet peppers(x_7) are grown, where x_j means the cultivation area (unit: 1 ha) for each crop $j = 1, \cdots, 7$. It is assumed that the farmer has one person of available family labor, but does not have access to hired labor, and he/she must decide the planting ratio among seven kinds of crops ($x_j, j = 1, \cdots, 7$) in his/her farmland to maximize his/her total income and minimize total work hours.

Table I shows the profit coefficients c_{tj} of seven crops j ($j = 1, \cdots, 7$) in each year [20]. From Table I, we can compute the expected values as

$$\begin{aligned} & (E[\bar{c}_1], E[\bar{c}_2], E[\bar{c}_3], E[\bar{c}_4], E[\bar{c}_5], E[\bar{c}_6], E[\bar{c}_7]) \\ = \; & (4.38, 25.82, 27.04, 37.46, 6.46, 2.58, 20.28). \end{aligned}$$

Then, the first objective function (total profit, unit: 1000 pesos) can be defined as $\sum_{j=1}^{7} E[\bar{c}_j]x_j$. The second objective function is total working hours. Table II shows the required working hours $L_{\ell j}$ for each crop ($j = 1, \cdots, 7$) and each period (from the middle ten days in May to the last ten days in April, $\ell = 1, \cdots, 27$) [20]. Then, the second objective function (a total number of working hours, unit: 1 hour) can be expressed as $\sum_{\ell=1}^{27} \sum_{j=1}^{7} L_{\ell j} x_j$. Since the upper limit of the working hours for each period ($\ell = 1, \cdots, 27$) can be computed as 8 (hours) $\times$ 1 (person) $\times$ 10 (days) = 80 (hours), the constraints $\sum_{j=1}^{7} L_{\ell j} x_j \leq 80, \; \ell = 1, \cdots, 27$ must be satisfied. As two land area constraints (unit: 1 ha) for the wet and dry season, $x_1 \leq 1, \sum_{j=2}^{7} x_j \leq 1, x_j \geq 0, j = 1, \cdots, 7$ must be satisfied. We assume that the water availability constraint in the dry season is expressed as

$$\sum_{j=1}^{7} w_j x_j = \tilde{\bar{d}},$$

where the water demand coefficients w_j for the crops ($j = 2, \cdots, 7$) are set as $(w_2, w_3, w_4, w_5, w_6, w_7) = (264.6, 232.3, 352.8, 88.2, 44.1, 220.5)$ [20], and the water supply possible amount is defined as a following LR-type fuzzy random variable $\tilde{\bar{d}}$ (unit : 1000 gallons).

$$\mu_{\tilde{\bar{d}}(\omega)}(s) = \begin{cases} L\left(\frac{\bar{b}(\omega) - s}{\alpha}\right), & s \leq \bar{b}(\omega) \\ R\left(\frac{s - \bar{b}(\omega)}{\beta}\right), & s > \bar{b}(\omega) \end{cases}$$

TABLE II
THE REQUIRED WORKING HOURS FOR EACH PERIOD $L_{\ell j}$

period : ℓ	$L_{\ell 1}$	$L_{\ell 2}$	$L_{\ell 3}$	$L_{\ell 4}$	$L_{\ell 5}$	$L_{\ell 6}$	$L_{\ell 7}$
2-May : 1	26						
1-Jun : 2	16						
2-Jun : 3	160						
1-Jul : 4	16						
2-Jul : 5	6						
2-Aug : 6	8						
3-Sep : 7	140						
1-Oct : 8	32	8					
3-Oct : 9		46			6		
1-Nov : 10		36		174		8	
2-Nov : 11		100	10	44	12	54	50
3-Nov : 12			22	16	12	8	20
1-Dec : 13		8	38	16	10	16	108
2-Dec : 14		16	94	16			72
3-Dec : 15		8	32	16			24
1-Jan : 16		8	14				64
2-Jan : 17		36	14		12		16
3-Jan : 18		70	6				
1-Feb : 19		70	6	180			48
2-Feb : 20		36	14			60	56
3-Feb : 21		36	6				48
1-Mar : 22			36				56
2-Mar : 23			30		32		
3-Mar : 24			30				
1-Apr : 25			38		56		
2-Apr : 26			30				
3-Apr : 27			26				

where $\bar{b} \sim \mathrm{N}(300, 5)$, $\alpha = \beta = 100$, and $L(t) = R(t) = 1 - t, 0 \leq t \leq 1$.

Since the penalty cost arises only for the shortage of water resource, it is assumed that $q_1^+ = 0, q_1^- = 10$ and $q_2^+ = q_2^- = 0$. Then, for the reference objective value $\hat{z}_\ell, \ell = 1, 2$ specified by the decision maker, the corresponding γ-Pareto optimal solution is obtained by solving the following minimax problem.

$$\begin{aligned} & \max_{\boldsymbol{x} \in X, \lambda \in \mathrm{R}^1} \lambda \\ \text{s.t.} \quad & -\sum_{j=1}^{7} E[\bar{c}_j]x_j + d(\boldsymbol{x}, \gamma) - \hat{z}_1 \leq \lambda \\ & \sum_{\ell=1}^{27} \sum_{j=1}^{7} L_{\ell j} x_j - \hat{z}_2 \leq \lambda \end{aligned}$$

where $d(\boldsymbol{x}, \gamma)$ is defined as follows.

$$\begin{aligned} d(\boldsymbol{x}, \gamma) \stackrel{\text{def}}{=} \; & q_1^- \left\{ (\sum_{j=2}^{7} w_j x_j - R^{-1}(\gamma)\beta) \right. \\ & \cdot \Phi(\sum_{j=2}^{7} w_j x_j - R^{-1}(\gamma)\beta) \\ & \left. - \int_{-\infty}^{\sum_{j=2}^{7} w_j x_j - R^{-1}(\gamma)\beta} b\phi(b)db \right\} \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ mean the probability density function and the cumulative distribution one for $\mathrm{N}(300, 5)$.

For comparison, we show two kinds of tables, Table III for $\gamma = 1$ and Table IV for $\gamma = 0.5$, where the interactive processes under the hypothetical decision maker are shown according to An interactive algorithm 1. In each table, a satisfactory solution is obtained at the third iteration. Each γ-Pareto optimal solution is obtained by applying Mathematica

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TABLE III
INTERACTIVE PROCESSES FOR $\gamma = 1$

	1	2	3	4
μ	300	300	300	300
σ^2	5	5	5	10
$\hat{z}_1$	-33	-33	-30	-30
$\hat{z}_2$	680	620	620	620
$z_1(\boldsymbol{x}^*)$	-34.1648	-33.5212	-33.4891	-32.3451
$z_2(\boldsymbol{x}^*)$	678.835	619.479	616.511	617.655
x_1^*	0.557996	0.411074	0.403728	0.410482
x_2^*	0.000	0.000	0.000	0.000
x_3^*	0.537202	0.537202	0.537202	0.636229
x_4^*	0.462798	0.462798	0.462798	0.363771
x_5^*	0.000	0.000	0.000	0.000
x_6^*	0.000	0.000	0.000	0.000
x_7^*	0.000	0.000	0.000	0.000

TABLE IV
INTERACTIVE PROCESSES FOR $\gamma = 0.5$

	1	2	3	4
μ	300	300	300	300
σ^2	5	5	5	10
$\hat{z}_1$	-33	-33	-30	-30
$\hat{z}_2$	680	620	620	620
$z_1(\boldsymbol{x}^*)$	-35.4266	-34.7831	-34.7509	-33.6070
$z_2(\boldsymbol{x}^*)$	677.574	618.217	615.249	616.393
x_1^*	0.549943	0.403021	0.395675	0.402428
x_2^*	0.000	0.000	0.000	0.000
x_3^*	0.41272	0.41272	0.41272	0.511748
x_4^*	0.58728	0.58728	0.58728	0.488252
x_5^*	0.000	0.000	0.000	0.000
x_6^*	0.000	0.000	0.000	0.000
x_7^*	0.000	0.000	0.000	0.000

to solve the corresponding minimax problem and the γ-Pareto optimality test problem. By comparing Table III for $\gamma = 1$ with Table IV for $\gamma = 0.5$, it is clear that any γ-Pareto optimal solution for $\gamma = 0.5$ is superior to γ-Pareto optimal solution for $\gamma = 1$ because of the definition of a possibility measure. In any γ-Pareto optimal solution of Table III and Table IV, only tomatoes (x_3) and garlic (x_4) in the dry season and rice (x_1) in the wet season are cultivated. The larger value of a permissible possibility level γ gives the larger planting ratio of tomatoes (x_3) and the smaller one of garlic (x_4) because of the difference of the water demand coefficients of tomatoes (x_3) and garlic (x_4). At the fourth iteration Table III and Table IV, γ-Pareto optimal solutions are shown under the assumption that the variance σ^2 of the random variable $\bar{b}$ is changed to 10. Then, the objective function $z_1(\boldsymbol{x}^*)$ at the fourth iteration become worse than the one at the third iteration, because of the corresponding penalty cost $d(\boldsymbol{x}^*, \gamma)$.

V. CONCLUSIONS

In this paper, we formulate a multiobjective fuzzy random simple recourse programming problem, in which fuzzy random variables coefficients are involved in equality constraints. In the proposed method, equality constraints with fuzzy random variables are defined on the basis of a possibility measure and and a two-stage programming method. For a given permissible possibility level and reference objective values specified by the decision maker, corresponding minimax problem is solved to obtain a Pareto optimal solution. The proposed method is applied to a farm planning problem in the Philippines, in which it is assumed that an amount supplied of water resource in dry season is represented as a fuzzy random variable. However, in such a farm planning problem, we assume that the expected values are adopted as profit coefficients. If we consider the correlation between seven kinds of profit coefficients, we have to deal with the profit coefficients as random variables. Such decision making situations are discussed in section III, and we will apply a generalized version of the proposed method to a farm planning problem in the near future.

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