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Integrated Supply Chain Inventory Model with Progressive Carbon Taxation

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***Abstract*—Mitigating carbon emissions is a considerable issue in the global supply chain nowadays. This paper presents an integrated inventory model with progressive carbon taxation which minimizes total cost and carbon emissions. Results analysis of the numerical example indicate that excess progressive cost of the second gap and transport lot significant impact on the integrated inventory policy.**

***Index Terms*—Integrated inventory, progressive carbon taxation, supply chain**

I. INTRODUCTION

Facing global warming and climate change, the government and enterprise effort to reduce carbon emissions. The United Nations, the European Union and many governments have enacted legislations or designed mechanisms such as carbon taxes, carbon offset, clean development, cap and trade, carbon caps and joint implementation to curb the total amount of carbon emissios [1]. Firms develops a series of reforms and connects a green supply chain. They are undertaking initiatives to reduce their carbon footprints in response to mechanisms or legislations, expectation of customer and environmental responsibility.

Greenhouse Gases, which is the culprit of global warming and climate change, includes carbon dioxide (CO2), methane (CH4), nitrous oxide (N2O), Chlorofluorocarbons (CFCs) and sulfur hexafluoride (SF6). According to the investigation of Intergovernmental Panel on Climate Change (IPCC) found that carbon dioxide causes the greenhouse effect occupied the highest proportion. Further, the amount of carbon dioxide emissions is related to the carbon content of fuels. As a result, many countries achieve carbon reduction targets with carbon taxes.

Carbon taxes is a kind of price policy, and is calculated based on carbon-containing of the general common energy which such as oil, coal, electricity and natural gas. Carbon tariff achieve rational allocation of environmental costs, internalizing the externality cost and also known as Pigovian taxes. For industry, carbon taxes is a persistent and clear price signals. Corporations can conduct financial plan in accordance with the tax rates, and invest equipment or technology for reduce carbon emissions. A well-designed carbon tax system has double dividend. The first dividend is restrain carbon emission. The second dividend, the government has the carbon tax revenue which can reduce the levy to other taxable items (the income tax) or inject research and development in low-carbon technology as well as social welfare spending. Carbon taxes have the effect of tax circulation; hence, carbon taxes possess revenue neutral feature [2].

Chen et al. [3] note that firms have focused for the most part on reducing emissions through innovations of the physical processes involved, for example by redesigning products and packaging, deployment and use of less polluting sources of energy, or replacing energy in efficient equipment and facilities. Bonney and Jaber [4] found out increasing the amount of products transported and reducing the frequency of delivery compared to those proposed by the traditional EOQ model gives better results in terms of ordering costs and carbon emissions. Benjaafar et al. [5] showed that how important insights could be drawn by integrating carbon emissions parameters into traditional and widely used lot-sizing models.

However currently firms put more emphasis productivity and customer satisfaction, which leads firms to focus on their supply chain and integrated logistics [6]. Yang et al. [7] developed a model which is useful particularly for integrated inventory systems where the vendor and the buyer form a strategic alliance for profit sharing. El Saadany et al. [8] studied a simple two-echelon supply chain model in which demand depends on the environmental quality of the systems(measured using 30 criteria) and the associated costs. Wahab et al. [9] offer an approach to optimally define the delivery/production policy to minimize the total cost of supply in a global supply chain.

Ghosh and Shah [10] examine some supply chain coordination with players initiating product “greening”. Cooperation between stakeholders does lead to higher greening levels but also to higher retail prices. In some coordination cases, the retailer has to provide suitable incentives to the manufacturer for him to participate in the bargaining process [11]. Swami and Shash [12] develop a model with a manufacturer and a retailer that coordinate their operations(wholesale price and green effort for the manufacturer, market price and green effort for the retailer). They propose a two-part tariff contract to produce channel coordination and ‘greener’ efforts.Further, Chiu et al. [13] suggest the fuzzy multi-objective integrated logistics model with the transportation cost and demand fuzziness to solve

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green supply chain problems in the uncertain environment. Teck-Koon [2] presented four EOQ inventory management models, respectively the carbon tax, carbon emissions permissions, progressive taxation and regressive taxation.

The purpose of this research is develop a novel model that takes into account the link between inventory policy, carbon taxes and green supply chain. The construction of model is based on EOQ model with progressive carbon taxation [2] and integrated inventory model [14]; then to minimize both the total costs and carbon emissions.

This paper is organized as follows. Section II defined the parameters and assumptions. Next, Section III develops the integrated inventory model with carbon taxes. After that, Section IV solved the model to get the optimal solution and showed numerical examples in Section V. Finally, this leads over the discussion of the findings and future research opportunities.

II. NOTATIONS AND ASSUMPTIONS

In order to develop an integrated inventory model with progressive carbon tax, this research adopted progressive carbon tax. Progressive carbon tax is a levy in a tax system where the tax rate increases as the taxable base (carbon emission) increases. Excess progressive rate is determined by the excess part of carbon emissions as the progressive basis. The following notations and assumptions below are used to build the model:

A. Notations

To establish the mathematical model, the following notations and assumptions are used.

Q= order quantity of the purchaser, a decision variable

Q^*= the optimal order quantity

m= an integer representing the number of lots in which the items are delivered from the vendor to the purchaser, a decision variable

m^*= the optimal transport lots

y= the first gap ceiling of excess progressive rate

β_1= excess progressive tariff of the first gap per unit carbon emission

β_2= excess progressive tariff of the second gap per unit carbon emission

r = annual inventory holding cost per dollar invested in stocks

Purchaser side

D = average annual demand per unit time

A = purchaser's ordering cost per order

L = length of lead time

C_p= purchaser's purchasing cost per unit

e_0 = carbon emissions of empty trucks generate by purchaser

e = variable carbon emissions factor per transport unit

g_0 = fixed carbon emissions of holding inventory generate by purchaser

g = variable carbon emissions factor per holding unit

CO_2^p= carbon emissions of the purchaser

TEC_p= purchaser's total expected annual cost

Vendor side

P = vendor's production rate

S= vendor's set-up cost per set-up

C_v= vendor's purchasing cost per unit

f_0= carbon emissions of empty trucks generate by vendor

f = variable carbon emissions factor per transport unit

h_0= fixed carbon emissions of holding inventory generate by vendor

h = variable carbon emissions factor per holding unit

CO_2^v= carbon emissions of the vendor

TEC_v = vendor's total expected annual cost

$JTEC^*$ = the optimal values of the expected joint total cost

$JTEC_i$ = the expected joint total cost, i = 1, 2, 3, 4*

*"i" represents four different cases
$$Csse\ 1:\ CO_2^p(Q) > y, CO_2^v(Q) > y$$
$$Csse\ 2:\ CO_2^p(Q) > y, CO_2^v(Q) \le y$$
$$Csse\ 3:\ CO_2^p(Q) \le y, CO_2^v(Q) > y$$
$$Csse\ 4:\ CO_2^p(Q) \le y, CO_2^v(Q) \le y$$

B. Assumptions

(i) This supply chain system consists of a single vendor and a single purchaser for a single product.

(ii) The product is manufactured with a finite production rate P, and P > D.

(iii) The government divided the carbon emissions into two gaps, the first gap ceiling is y, and $\beta_1 < \beta_2$.

(iv) The demand X during lead time L follows a normal distribution with mean μ_L and standard deviation $\sigma\sqrt{L}$.

(v) The reorder point (ROP) equals the sum of the expected demand during lead time and the safety stock.

(vi) Shortages are not allowed.

(vii) Inventory is continuously reviewed

III. MODEL FORMULATION

In this section, we discuss the model of purchaser and vendor combined them into an integrated inventory model with progressive carbon tax.

A. The purchaser's total expected cost

Based on the above notations and assumptions, the total expected annual cost for the purchaser $TEC_p = \text{Ordering cost} + \text{Holding cost} + \text{Carbon tax cost}$.

To start with, since A is the ordering cost per order, the expected ordering cost per year is given by:

(i) Ordering cost = $\left(\frac{D}{Q}\right)A$

From assumption (v), the reorder point $\mathrm{ROP} = \mu L + k\sigma\sqrt{L_{ij}}$, where k is known as the safety factor. Then, the average on-hand inventory for the purchaser is shown as $\bar{I}_P \cong \frac{Q}{2} + ROP - \mu_L = \frac{Q}{2} + k\sigma\sqrt{L}$. Hence,

(ii) Holding cost = $rC_p(\frac{Q}{2} + k\sigma\sqrt{L})$

After that, carbon tax cost is calculated with carbon emission of transportation and warehousing, as external costs of production to reflect environmental costs. The carbon emission of the purchaser is presented as

$$CO_2^p(Q) = (e_0 + eQ)\frac{D}{Q} + \left(g_0 + g\frac{Q}{2}\right) = \frac{e_0 D}{Q} + \frac{gQ}{2} + eD + g_0 (1).$$

There are two different situations of the carbon emission. One is more than the first gap ceiling of excess progressive rate, the other is less than or equal; then, multiplied by the corresponding carbon tax. As a result, if $CO_2^p(Q) > y$

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(iii) Carbon tax cost = $y\beta_1 + (\frac{e_0 D}{Q} + \frac{gQ}{2} + eD + g_0 - y)\beta_2$.

If $CO_2^p(Q) \le y$,

(iv) Carbon tax cost = $(\frac{e_0 D}{Q} + \frac{gQ}{2} + eD + g_0)\beta_1$.

B. The vendor's total expected cost

For the vendor's inventory model, its total expected annual cost $TEC_v =$ set − up cost + Holding cost + Carbon tax cost.

First, because S is the vendor's set-up cost per set-up, and the production quantity for the vendor in a lot will be mQ, and

(i) Set-up cost = $(\frac{D}{mQ})$S.

Second, the vendor produces the item in the quantity of mQ, and the purchaser would receive it in m lots, with which each having a quantity of Q. For the vendor, its average inventory can be evaluated as follows: (see Fig I.)

$\bar{I}_v = \left\{\left[mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P}\right] - \left[\frac{Q}{D}\left(1 + 2 + \cdots + (m-1)\right)Q\right]\right\} / (\frac{mQ}{D}) = \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)$, and therefore

(ii) Holding cost = $rC_v\left(\frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right)$

Next, the carbon emission of the vendor is represented as $CO_2^v(Q) = (f_0 + fQ)\frac{D}{mQ} + \left(h_0 + h\frac{Q}{2}\right) = \frac{f_0 D}{mQ} + \frac{hQ}{2} + \frac{fD}{m} + h_0$ (2).

However, if $CO_2^v(Q) > y$

(iii) Carbon tax cost = $y\beta_1 + (\frac{f_0 D}{mQ} + \frac{hQ}{2} + \frac{fD}{m} + h_0 - y)\beta_2$.

If $CO_2^v(Q) \le y$

(iv) Carbon tax cost = $(\frac{f_0 D}{mQ} + \frac{hQ}{2} + \frac{fD}{m} + h_0)\beta_1$.

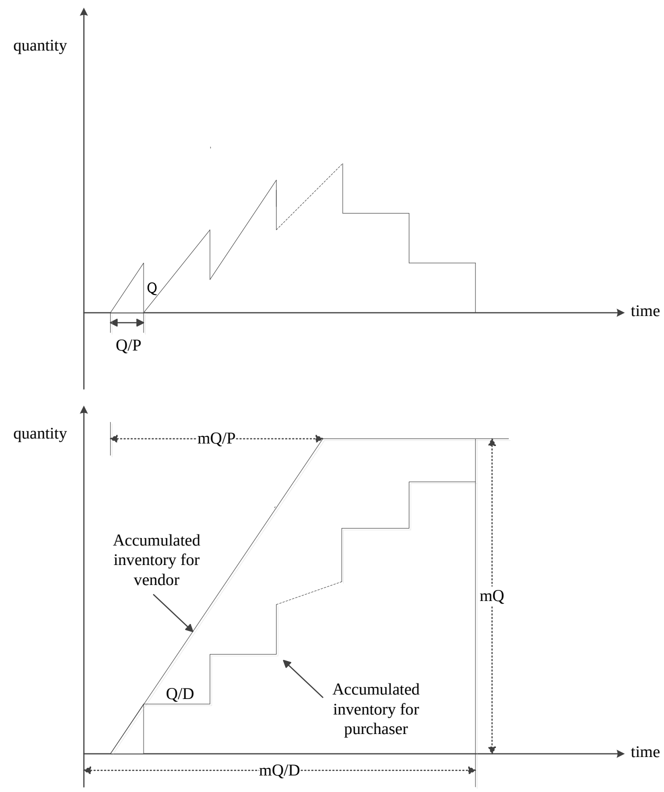

Fig 1. The inventory pattern for the vendor [14]

C. The expected joint total cost

According to the four different conditions, the expected joint total cost function, $JTEC_i(Q,m) = TEC_p + TEC_v$ can be expressed as

$$JTEC_i(Q,m) = \begin{cases} JTEC_1(Q_1,m_1) & for\ Csse\ 1 \\ JTEC_2(Q_2,m_2) & for\ Csse\ 2 \\ JTEC_3(Q_3,m_3) & for\ Csse\ 3 \\ JTEC_4(Q_4,m_4) & for\ Csse\ 4 \end{cases}$$ where

$$JTEC_1(Q,m) = \frac{D}{Q}\left[A + \frac{S}{m} + \beta_2\left(e_0 + \frac{f_0}{m}\right)\right] + \frac{Q}{2}\left\{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + \beta_2(g+h)\right\} + \beta_2\left[D\left(e + \frac{f}{m}\right) + g_0 + h_0\right] + 2y(\beta_1 - \beta_2) + rC_p k\sigma\sqrt{L} \quad (3)$$

$$JTEC_2(Q,m) = \frac{D}{Q}\left[A + \frac{S}{m} + e_0\beta_2 + \frac{f_0\beta_1}{m}\right] + \frac{Q}{2}\left\{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + g\beta_2 + h\beta_1\right\} + \beta_1\left[\frac{fD}{m} + h_0 + y\right] + \beta_2(eD + g_0 - y) + rC_p k\sigma\sqrt{L} \quad (4)$$

$$JTEC_3(Q,m) = \frac{D}{Q}\left[A + \frac{S}{m} + e_0\beta_1 + \frac{f_0\beta_2}{m}\right] + \frac{Q}{2}\left\{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + g\beta_1 + h\beta_2\right\} + \beta_1[eD + g_0 + y] + \beta_2\left(\frac{fD}{m} + h_0 - y\right) + rC_p k\sigma\sqrt{L} \quad (5)$$

$$JTEC_4(Q,m) = \frac{D}{Q}\left[A + \frac{S}{m} + \beta_1\left(e_0 + \frac{f_0}{m}\right)\right] + \frac{Q}{2}\left\{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + \beta_1(g+h)\right\} + \beta_1\left[D\left(e + \frac{f}{m}\right) + g_0 + h_0\right] + rC_p k\sigma\sqrt{L} \quad (6)$$

To minimize $JTEC_i(Q,m)$, this paper set $\frac{\partial JTEC_i(Q_i,m_i)}{\partial Q_i} = 0$ and obtain the value of $Q = Q_1^*, Q_2^*, Q_3^*\ and\ Q_4^*$.

$$Q_1^* = \left(\frac{2D\left[A + \frac{S}{m} + \beta_2\left(e_0 + \frac{f_0}{m}\right)\right]}{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + \beta_2(g+h)}\right)^{0.5} \quad (7)$$

$$Q_2^* = \left(\frac{2D\left(A + \frac{S}{m} + e_0\beta_2 + \frac{f_0\beta_1}{m}\right)}{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + g\beta_2 + h\beta_1}\right)^{0.5} \quad (8)$$

$$Q_3^* = \left(\frac{2D\left(A + \frac{S}{m} + e_0\beta_1 + \frac{f_0\beta_2}{m}\right)}{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + g\beta_1 + h\beta_2}\right)^{0.5} \quad (9)$$

$$Q_4^* = \left(\frac{2D\left[A + \frac{S}{m} + \beta_1\left(e_0 + \frac{f_0}{m}\right)\right]}{r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + C_p\right] + \beta_1(g+h)}\right)^{0.5} \quad (10)$$

IV. SOLUTION PROCEDURE

Summarizing the above arguments, we establish the following algorithm to obtain the optimal values of Q^*, m^* and $JTEC^*$, where the other parameters are known.

Algorithm

Step 1. Set m =1 and substitute into (7), (8), (9) and (10) to obtain Q_1, Q_2, Q_3 and Q_4, ∀i = 1, 2, 3, and 4.

Step 2. Find $CO_2^p(Q_i)$ and $CO_2^v(Q_i)$ by substituting Q_i into (1) and (2), ∀i = 1, 2, 3, and 4.

Step 3. Compare between $CO_2^p(Q_i)$, $CO_2^v(Q_i)$, and y.

According to the different conditions, calculate corresponding JTECi using (3), (4), (5) or (6). ∀i = 1, 2, 3, and 4.

Step 4. Let m = m + 1 and repeat step 1 to step 3 until $JTEC_i$

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$(Q_i, m) < JTEC_i(Q_i, m+1)$. ∀i = 1, 2, 3, and 4.

Step 5. The optimal $m^* = m;\ Q^* = Q_i, and\ JTEC^*(Q^*, m^*) = JTEC_i(Q_i, m_i)$, ∀i = 1, 2, 3, and 4. CO_2^v

The solution procedure of the whole system is shown in the following flowchart (Fig. 2).

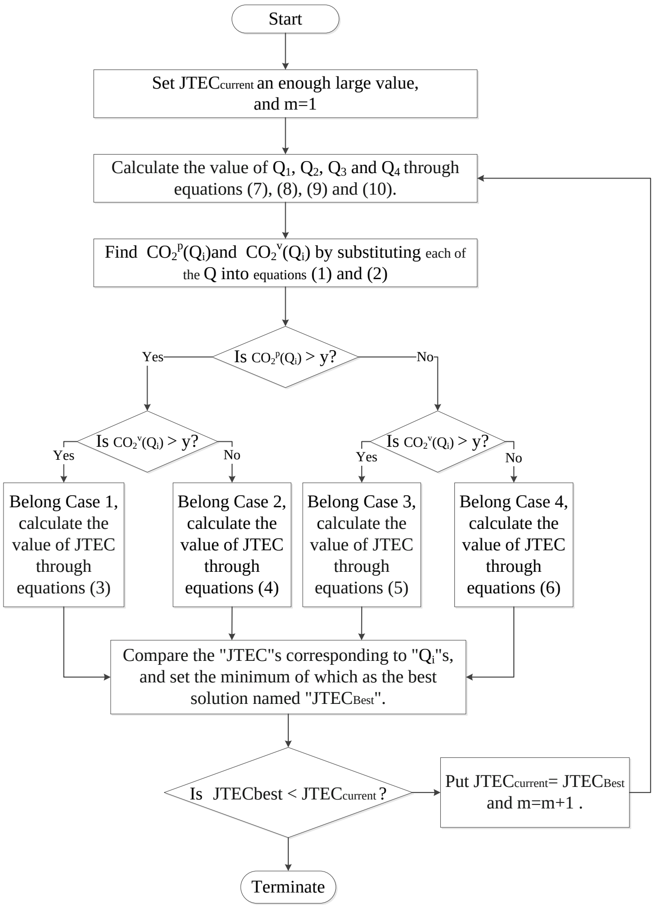

Fig 2. Flowchart of the solution procedure

V. NUMERICAL EXAMPLE

To illustrate the proposed solution procedure, consider an inventory item with parameters tabulated in Table I.

Table I. Parameters of the example

common	purchaser		vendor	
$m \leqq 7$	D	1000	P	3200
R = 0.2	A	25	S	400
L = 42	C_p	25	C_v	20
K = 2.33	Carbon emission parameters			
σ = 7	e_0	65	f_0	180
y = 500	e	0.02	f	0.01
β1 = 0.06	g_0	40	h_0	110
β2 = 0.1	g	1.4	h	0.9

Applying the equation and algorithm already given in this article, the optimal integer policy is shown in Table II.

Table II. The optimal solution for given parameters

m	Q_i	Q^*	CO_2^p	CO_2^v	CO_2^T	$JTEC^*$
1	Q_3	373	495	770	1266	2503
2		228	505	612	1117	2195
3	Q_1	169	563	545	**1108**	2111
4		136	633	505	1138	**2090**
5		114	709	479	1188	2095
6	Q_2	100	783	458	1241	2113
7		89	855	441	1297	2138

The findings of the numerical result indicated that the minimum cost is $2,090 and carbon emission is 1138 tons with the optimal order quantity is 136 and transport lot is 4 in the integrated chain.

VI. CONCLUSION

With the pressure of global warming and climate change recently, reducing carbon emission has been becoming a trend of supply chain management. In this research, we formulated integrated inventory model with progressive carbon taxation. Both order quantity and transport lot are important factor to impact the inventory policy. Furthermore, excess progressive cost of the second gap which play significant role impact the optimal order quantity.

The numerical results showed that progressive carbon taxation is useful curb the carbon emission for integrated inventory systems because the purchaser and vendor made an effort to decrease carbon emission which remains in the vicinity of the standard. However, progressive carbon taxation also obstructs mass production.

REFERENCE

[1] A. Choudhary, S. Sarkar, S. Settur, M.K. Tiwari. "A carbon market sensitive optimization model for integrated forward–reverse logistics" *International Journal of Production Economics*, Vol. 164, pp. 433–444, 2015.

[2] T-K, Ng. "Development and Analysis of Inventory Management Model Considering Carbon Tax", Transportation and Logistics Management Department, National Chiao Tung University. 2012.

[3] X. Chen, S. Benjaafar, A. Elomri, "The carbon-constrained EOQ", *Operations Research Letters*, Vol. 41, pp. 172–179, 2013.

[4] M. Bonney, M. Y. Jaber. " Environmentally responsible inventory models: non-classical models for a non-classical era" *International Journal of Production Economic*, Vol. 133(1), pp. 43–53, 2011.

[5] S. Benjaafar, Y. Li, M. Daskin. " Carbon footprint and the management of supply chains: insights from simple models" *IEEE Transactions on Automation Science and Engineering*, Vol. 10(1), pp. 99–116, 2013.

[6] M.S. Pishvaee, F. Jolai, J.A. Razmi, "Stochastic optimization model for integrated forward/ reverse logistic network design", *Journal of Manufacturing Systems*, Vol. 28, pp.107–114. 2009.

[7] M.-F. Yang, M.- C. Lo, T. -P. Lu. " A vendor-buyers integrated inventory model involving quality improvement investment in a supply chain", *Journal of Marine Science and Technology*, Vol. 21 (5), pp. 586-593, 2013.
[8] El Saadany, A.M.A., M.Y. Jaber, M. Bonney. " Environmental performance measures for supply chains", *Management Research News*, Vol. 34(11), pp. 1202–1221, 2011.
[9] M.I.M. Wahab, S.M.H Mamun, P. Ongkunaruk. "EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact" *International Journal of Production Economic*, Vol. 134(1), pp. 151–158, 2011.
[10] D. Ghosh, J. Shah, "A comparative analysis of greening policies across supply chain structures", *International Journal of Production Economics*, Vol. 135(2), pp. 568–583, 2012.
[11] H. Vincent, B. Laurent, "The carbon-constrained EOQ model with carbon emission dependent demand", *International Journal of Production Economics*, Vol. 164, pp. 285-291, 2015.
[12] S.Swami, J.Shash,"Channel coordination in green supply chain management", *Journal of the Operational Research Society*, Vol. 64, pp. 336–351, 2013.
[13] C.Y. Chiu, Y. Lin, M.F. Yang. "Applying Fuzzy Multi-objective Integrated Logistics Model to Green Supply Chain Problems" *Journal of Applied Mathematics*, Vol. 2014, ID 767095, 12 pages, 2014.
[14] JCH Pan, J.S. Yang. "A study of an integrated inventory with controllable lead time" *International Journal of Production Research*, Vol. 40, no. 5, pp. 1263-1273, 2002.