Performance Analysis of Series Configuration Queueing System with Four Service Stations

Yu-Li Tsai, Daichi Yanagisawa, and Katsuhiro Nishinari

Abstract—**In this paper, we investigate the performance of open queueing networks consisting of four service stations with different service rate for each service station. We assume Poisson arrivals and exponential service times. Steady-state probabilities of the quasi-birth-death process are evaluated by matrix-geometric method. Performance measures including mean number in the system, mean waiting time in in the system and blocking probability of the service stations in front of the terminal service station are defined. We derive the exact formulae of stability conditions for the system with both the same and different service rates. Disposition strategies of service rates for each service station are proposed for the queueing system with the arbitrary number of service stations.**

Keywords—**Disposition Strategy, Matrix-geometric method, Performance Analysis, Simulation**

I. Introduction

Queueing networks with no intermediate waiting space between service stations are very popular in modern automated production system. The series configuration system with four service stations is depicted in **Fig. 1**. In this system, every customer must go through each service station to finish operations. A complete service is defined as a customer enters to each service station in order and finishes the works in each station. It is obvious that we can apply simulation results of this queueing system to real industrial applications, such as automobile assembly line or other similar systems. Intuitively, it is reasonable to assume that the disposition strategies are all the same for the system with the arbitrary number of service stations. However, we propose general disposition strategies for the series configuration queueing system with different service rate of service stations based on our results.

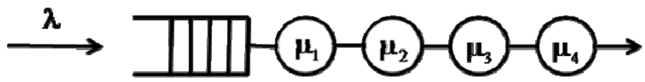

Fig 1. Series configuration queueing system with four service stations.

Traditional theoretical analysis of this kind of queueing system focused on deriving analytic performance measures for the system with same service rate for each service station. However, the problem of the performance analysis for the system becomes very complex when the system consisting more than three service stations and the service rate for each service station is different. Fortunately, we have obtained important insights about disposition strategy for the system with the arbitrary number of service stations in order to increase the operational efficiency of the system (i.e. decreasing mean waiting time in the system) by applying matrix-geometric method. This method not only evaluates the steady-state probabilities, which are very significant quantities for estimating related performance measures, but also provides a systematic way to derive stability conditions for the system in exact formulations. Furthermore, the numerical results show that the upper bound of the stability condition is consistent with the analytic formulae we derived. We expect that our results can provide insights for real industrial applications.

Hunt [1] first studied queueing networks with no intermediate waiting queue between service stations and blocking phenomena. Mathematical analysis and related applications of matrix-geometric method was systematically studied by Neuts [2]. Zhou et al. [3] investigated a two-stage tandem queueing network with Markovian arrival process inputs and buffer sharing. They discovered that the buffer sharing policy is more flexible when the inputs have large variant and are correlated. Hillier [4] studied the optimal design of unpaced assembly lines. He analyzed the joint optimization of both the allocation of workload and the allocation of buffer spaces simultaneously when the objective is to maximize the revenue from throughput minus the cost of work-in-process inventory. Ke et al. [5] first considered the disposition strategy for a self-blocking queueing system consisting of two service stations with different service rate. Hudson et al. [6] gave complete reviews for the topics about unbalanced unpaced serial production lines. Several unanswered questions about the performance of assembly line are described in this work. Tsai et al. [7] compared the disposition strategies for the open queueing networks with two and three service stations. They discovered that the mean waiting time in the system can be reduced significantly by applying appropriate disposition strategy for setting higher service rate for specific service stations.

II. Problem Formulation and Notations

The queueing system consists of independent service stations in series configuration and operates simultaneously. Every customer follows Poisson arrival process with mean arrival rate λ. The time to serve a customer in each service station is exponentially distributed with mean service time

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$\frac{1}{\mu}$. Each customer should enter all of the service stations from the first service station to the terminal station in order. A complete service is defined as a customer enters to each service station in order and finishes the works in each station. There are no intermediate waiting queues between each service station. The distinctive phenomenon so called blocking after service happens in the case that a customer completes the service in a service station, but another customer in the next station has not finished the service yet. The customer who completed the service is blocked by the customer who is still receiving the service located next station. In this system, the blocking phenomenon happens in the station-1, the station-2 and the station-3. A queue with infinite capacity is allowed in front of the first service station. In addition, only a customer can enter each service station at a time and the service rate is independent of the number of customers. The service of the system obeys the first come first serve (FCFS) discipline.

We use P_{n_1,n_2,n_3,n_4,n_5} to denote the steady-state probability P_{n_1,n_2,n_3,n_4,n_5} of n_1 customer in the station-4 and n_2 customer in the station-3 and n_3 customer in the station-2 and n_4 customer in the station-1 and n_5 customer in the queue.

III. Modeling Framework

Let $\mathbf{P} = [\mathbf{P_0}, \mathbf{P_1}, \mathbf{P_2}, ...]$ denote the steady-state probability vector corresponding to the transition matrix Q. The steady-state equation of the quasi-birth-death process is $\mathbf{P}\mathrm{Q} = \mathbf{0}$, with the normalization constraint $\mathbf{P1} = 1$. We can obtain the following set of matrix equations with a finite dimension:

$$\mathbf{P_0}B_{0,0} + \mathbf{P_1}B_{1,0} = \mathbf{0}, \quad (1)$$

$$\mathbf{P_0}B_{0,1} + \mathbf{P_1}A_1 + \mathbf{P_2}A_2 = \mathbf{0}, \quad (2)$$

$$\mathbf{P}_i A_0 + \mathbf{P}_{i+1}A_1 + \mathbf{P}_{i+2}A_2 = \mathbf{0}, \qquad i \geq 1. \quad (3)$$

The following recurrence relation can be constructed with a rate matrix R

$$\mathbf{P}_i = \mathbf{P}_{i-1}R = \mathbf{P}_1 R^{i-1}, \qquad i \geq 1. \quad (4)$$

The unknown rate matrix R can be obtained by substituting (4) into (3), we obtain the following characteristic equation of the recurrence relation

$$A_0 + RA_1 + R^2 A_2 = 0. \quad (5)$$

The simplified matrix equations of (1) and (2) can be represented as

$$\mathbf{P_0}B_{0,0} + \mathbf{P_1}B_{1,0} = \mathbf{0}, \quad (6)$$

$$\mathbf{P_0}B_{0,1} + \mathbf{P_1}(A_1 + RA_2) = \mathbf{0}. \quad (7)$$

According to Bloch et al. [8], the normalization condition equation that only involves $\mathbf{P_0}$ and $\mathbf{P_1}$ is given by

$$\mathbf{P_0 1} + \mathbf{P_1}(I - R)^{-1}\mathbf{1} = 1, \quad (8)$$

where I is the identity matrix with same size as the rate matrix R.

The rate matrix R in (5) is solved by iterative method. Collecting (6), (7) and (8) together, the steady-state probability vector of $\mathbf{P_0}$ and $\mathbf{P_1}$ can be obtained by solving following matrix equation

$$(\mathbf{P_0}, \mathbf{P_1})\begin{pmatrix} B_{0,0} & B_{0,1}^* & \mathbf{1} \\ B_{1,0} & (A_1 + RA_2)^* & (I - R)^{-1}\mathbf{1} \end{pmatrix} = (\mathbf{0}, 1). \quad (9)$$

where $(.)^*$ indicates that the last column of the included matrix is removed to avoid linear dependency.

- Stability Conditions

The stability condition is given by Neuts [2] for the ergodicity of steady-state probabilities:

$$\mathbf{P}_A A_0 \mathbf{1} < \mathbf{P}_A A_2 \mathbf{1}, \quad (10)$$

where $\mathbf{P}_A$ is the steady-state probability vector corresponding to the generator matrix A.

The stability conditions for the system consisting of four service stations are shown in following

(1) For $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

$$\lambda < \frac{N}{D}, \quad (11)$$

where the exact results of the N and the D are shown in the supplementary document.

(2) Special case: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$

$$\lambda < \frac{4024}{7817}\mu. \quad (12)$$

IV. PERFORMANCE METRICS AND DISPOSITION STRATEGY

Performance measures including mean number in the system, mean number in the queue, mean waiting time in the system, mean waiting time in the queue and blocking probability of the service stations in front of the terminal station for the series configuration system consisting of four service stations are defined. In addition, we propose general disposition strategies for the series configuration queueing system consisting of the arbitrary number of service stations based on the numerical results in this section.

- Performance measures

Performance measures for the system consisting of four service stations are defined by

(1) Mean number of customers in the system

$$\begin{aligned}
L = &(P_{0,0,0,1,0} + P_{0,0,1,0,0} + P_{0,1,0,0,0} + P_{1,0,0,0,0} + P_{1,b,0,0,0} + P_{0,0,1,b,0} + P_{0,1,b,0,0} + P_{0,1,b,b,0} + P_{1,b,b,0,0}) \\
&+ 2\,(P_{0,0,0,1,1} + P_{0,0,1,1,0} + P_{0,1,0,1,0} + P_{1,0,0,1,0} + P_{1,0,1,0,0} + P_{1,1,0,0,0} + P_{0,1,1,0,0} \\
&+ P_{1,b,0,1,0} + P_{0,0,1,b,1} + P_{0,1,1,b,0} + P_{1,1,b,0,0} + P_{0,1,b,1,0} + P_{1,0,1,b,0} + P_{1,b,1,0,0}) \\
&+ 3\,(P_{0,0,0,1,2} + P_{0,0,1,1,1} + P_{0,1,0,1,1} + P_{1,0,0,1,1} + P_{1,0,1,1,0} + P_{1,1,0,1,0} + P_{0,1,1,1,0} + P_{1,1,1,0,0}) \\
&+ \sum_{n=4}^{\infty}(P_{0,0,0,1,n-1} + P_{0,0,1,1,n-2} + P_{0,1,0,1,n-2} + P_{1,0,0,1,n-2} + P_{1,0,1,1,n-3} + P_{1,1,0,1,n-3} + P_{0,1,1,1,n-3} + P_{1,1,1,1,n-4}) \cdot n \\
&+ \sum_{n=3}^{\infty}(P_{1,b,0,1,n-2} + P_{0,0,1,b,n-1} + P_{0,1,1,b,n-2} + P_{1,1,b,1,n-3} + P_{0,1,b,1,n-2} + P_{1,0,1,b,n-2} + P_{1,b,1,1,n-3} + P_{1,1,1,b,n-3}) \cdot n \\
&+ \sum_{n=2}^{\infty}(P_{0,1,b,b,n-1} + P_{1,b,b,1,n-2} + P_{1,1,b,b,n-2} + P_{1,b,1,b,n-2}) \cdot n \\
&+ \sum_{n=1}^{\infty}(P_{1,b,b,b,n-1}) \cdot n.
\end{aligned} \quad (13)$$

(2) Mean number of customers in the queue

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$$
\begin{aligned}
L_q =& (P_{0,0,0,1,1} + P_{0,0,1,b,1} + P_{0,0,1,1,1} + P_{0,1,0,1,1} + P_{1,0,0,1,1} + P_{1,0,1,1,1} + P_{1,1,0,1,1} \\
&+ P_{1,b,0,1,1} + P_{0,1,1,1,1} + P_{0,1,1,b,1} + P_{0,1,b,b,1} + P_{0,1,b,1,1} + P_{1,0,1,b,1}) \\
&+2\,(P_{0,0,0,1,2} + P_{0,0,1,1,2} + P_{0,1,0,1,2} + P_{1,0,0,1,2} + P_{0,0,1,b,2}) \\
&+3\,(P_{0,0,0,1,3}) + \sum_{n=4}^{\infty}(P_{0,0,0,1,n})\cdot n + \sum_{n=3}^{\infty}(P_{0,0,1,1,n} + P_{0,1,0,1,n} + P_{1,0,0,1,n} + P_{0,0,1,b,n})\cdot n \\
&+ \sum_{n=2}^{\infty}(P_{1,0,1,1,n} + P_{1,1,0,1,n} + P_{1,b,0,1,n} + P_{0,1,1,1,n} + P_{0,1,1,b,n} + P_{0,1,b,b,n} + P_{0,1,b,1,n} + P_{1,0,1,b,n})\cdot n \\
&+ \sum_{n=1}^{\infty}(P_{1,1,1,1,n} + P_{1,1,b,1,n} + P_{1,b,b,1,n} + P_{1,b,1,1,n} + P_{1,1,1,b,n} + P_{1,b,b,b,n} + P_{1,1,b,b,n} + P_{1,b,1,b,n})\cdot n.
\end{aligned}
\tag{14}
$$

(3) Mean waiting time in the system (Little's Law)

$$W = \frac{L}{\lambda}. \tag{15}$$

(4) Mean waiting time in the queue (Little's Law)

$$W_q = \frac{L_q}{\lambda}. \tag{16}$$

(5) Blocking probability of the customer in the station-1

$$
\begin{aligned}
P_{b,1} =& \sum_{n=0}^{\infty} P_{0,0,1,b,n} + P_{0,1,1,b,n} + P_{0,1,b,b,n} + P_{1,0,1,b,n} \\
&+ P_{1,1,1,b,n} + P_{1,b,b,b,n} + P_{1,1,b,b,n} + P_{1,b,1,b,n}.
\end{aligned}
\tag{17}
$$

(6) Blocking probability of the customer in the station-2

$$
\begin{aligned}
P_{b,2} =& \sum_{n=0}^{\infty} P_{1,1,b,1,n} + P_{0,1,b,1,n} + P_{0,1,b,b,n} \\
&+ P_{1,b,b,1,n} + P_{1,b,b,b,n} + P_{1,1,b,b,n}.
\end{aligned}
\tag{18}
$$

(6) Blocking probability of the customer in the station-3

$$
\begin{aligned}
P_{b,3} =& \sum_{n=0}^{\infty} P_{1,b,0,1,n} + P_{1,b,1,1,n} + P_{1,b,b,1,n} \\
&+ P_{1,b,b,b,n} + P_{1,b,1,b,n}.
\end{aligned}
\tag{19}
$$

Proposition 3.1. Disposition strategies for the series configuration queueing system consisting of the arbitrary number of service stations with different service rates are different.

We propose different disposition strategies for the system based on our previous research Tsai et al. [7] and this work in order to increase the operational efficiency.

(1) Series configuration queueing system with **the odd number** of service stations

It is better to arrange lower service rate for the first service station compared with other service stations in the system in order to obtain the best operational efficiency for the system with the odd number of service stations.

(2) Series configuration queueing system with **the even number** of service stations

We suggest setting higher service rates for the service stations in front of the terminal station as possibly as we can. In this way, the mean waiting time in the system would be the shortest compared with other disposition strategies.

V. NUMERICAL RESULTS

In this section, we illustrate numerical experiments for the queueing system consisting of four stations. Performance metrics of the system with equivalent service rates (i.e. $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$) and different service rates are presented. We will suggest better disposition strategies to increase operational efficiency for the system according to the results of simulations.

- Same service rates for each service station

First, we study the increasing trends of mean number in the system and blocking probabilities as a function of mean arrival rate λ. **Fig. 2.** presents the mean number in the system. It is observed that the upper bound of the stability condition of the mean number in the system approaches to $\frac{4024}{7817}$ (≈ 0.514), which proves the correctness of the exact results we derived in the section 3.2. Blocking probability of the station-1, the station-2 and the station-3 as a function of mean arrival rate of the system consisting of four service stations is shown in **Fig. 3**. Furthermore, it is investigated that the blocking probability of the station-1 is higher than that of the station-2 and of the station-3 in this case.

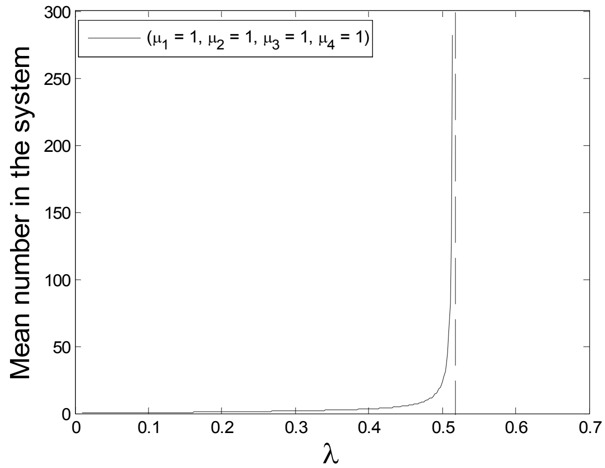

Fig 2. Mean number in the system.

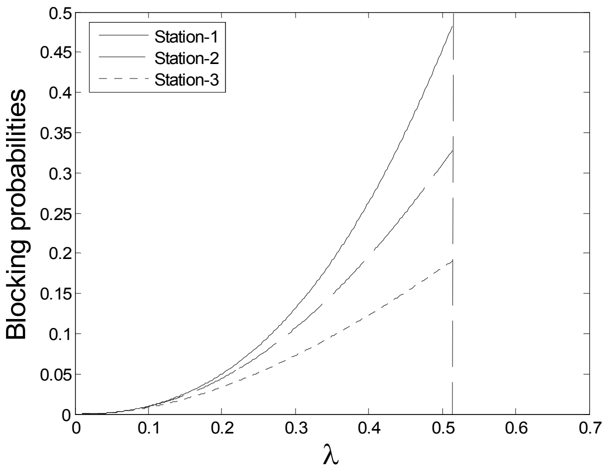

Fig 3. Blocking probability.

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- Controlling the service rates of the three service stations

In the case of different service rates, we study the conditions that we can concurrently control the service rates of three service stations and the service rate of only one service station for the system consisting of four service stations.

First, we investigate the cases that we are able to control three service rates of the service stations in this system. We set $\mu_1 = 2, \mu_2 = 2, \mu_3 = 2, \mu_4 = 1$ and $\mu_1 = 2, \mu_2 = 2, \mu_3 = 1, \mu_4 = 2$ and $\mu_1 = 2, \mu_2 = 1, \mu_3 = 2, \mu_4 = 2$ and $\mu_1 = 1, \mu_2 = 2, \mu_3 = 2, \mu_4 = 2$, then vary the mean arrival rate λ from 0.01 to 0.7. It is suggested to set higher service rates for the station-1, the station-2 and the station-3 in order to obtain the best operational efficiency for the system, as shown in **Fig 4**. This best disposition strategy for the system consisting of four service stations is accordant with the result of the system comprising two service stations indicated by Tsai et al. [7].

Since the mean waiting time in the system of the case $\mu_1 = 2, \mu_2 = 1, \mu_3 = 2, \mu_4 = 2$ is always higher than that of the case $\mu_1 = 2, \mu_2 = 2, \mu_3 = 1, \mu_4 = 2$ for all mean arrival rates. We just compare the cases between $\mu_1 = 1, \mu_2 = 2, \mu_3 = 2, \mu_4 = 2$ and $\mu_1 = 2, \mu_2 = 2, \mu_3 = 1, \mu_4 = 2$. It is investigated that the mean waiting time of the system of the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 2, \mu_4 = 2$ is higher than that of the case $\mu_1 = 2, \mu_2 = 2, \mu_3 = 1, \mu_4 = 2$ when mean arrival rate is lower than 0.65. This result shows that when the mean arrival rate is lower than 0.65, the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 2, \mu_4 = 2$ causes longer time for the customers waiting in the queue as show in **Fig 4**. The mean waiting time in the queue of the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 2, \mu_4 = 2$ becomes shorter than that of the case $\mu_1 = 2, \mu_2 = 2, \mu_3 = 1, \mu_4 = 2$ when the mean arrival rate is greater than 0.65.

We suggest the case $\mu_1 = 2, \mu_2 = 2, \mu_3 = 2, \mu_4 = 1$ as the best disposition strategy, when we are able to control service rates of three service stations for the system.

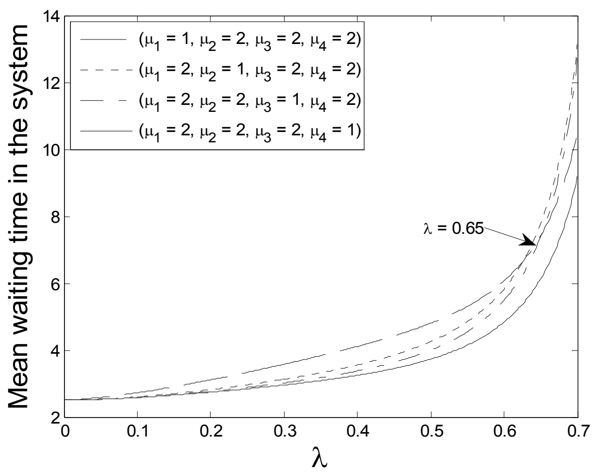

Fig 4. Mean waiting time in the system.
(Controlling the service rates of the three service stations)

- Controlling the service rates of only one service station

Next, we study the case of controlling service rate of one service station, we set $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_4 = 1$ and $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ and $\mu_1 = 1, \mu_2 = 1, \mu_3 = 2, \mu_4 = 1$ and $\mu_1 = 1, \mu_2 = 1, \mu_3 = 1, \mu_4 = 2$ then vary the mean arrival rate λ from 0.01 to 0.5. It is investigated that the mean waiting time is the greatest in the case of $\mu_1 = 1, \mu_2 = 1, \mu_3 = 1, \mu_4 = 2$ compared with other three cases as shown in **Fig 5**. This disposition strategy makes the customers in the queue difficult to enter the service stations, since the mean waiting time in the queue is higher than other three cases.

Similar to the case studies of controlling three service stations in previous section, we note that the mean waiting time in the system in the case of $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ is always lower than that of the case $\mu_1 = 1, \mu_2 = 1, \mu_3 = 2, \mu_4 = 1$. We compare the case $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_4 = 1$ with the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ for discussing the disposition strategies. We consider that the mean waiting time in the system of the case $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_4 = 1$ is lower than that of the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ when the mean arrival rate is lower than 0.42. It is noted the mean waiting time in the queue is almost the same for both cases, so setting higher service rate for the station-1 is better to make customer to enter the service stations when mean arrival rate is lower than 0.42, as shown in **Fig 5**. When the mean arrival is greater than 0.42, it is observed that the mean waiting time in the system in the case of $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_4 = 1$ is larger than that of the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$. While the increasing of the mean arrival rate, the setting of lower service rates in the station-1 and the station-2 and the station-3 makes customers take longer waiting time in the queue.

We suggest that setting $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_3 = 1$ as the best disposition strategy when the mean arrival rate is lower than 0.42. On the other hand, for the case that we can control only one of the service rates for the system consisting of four service stations, we observe that case of $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ is a relatively better disposition strategy compared with the case $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_3 = 1$ when the mean arrival rate becomes larger than 0.42.

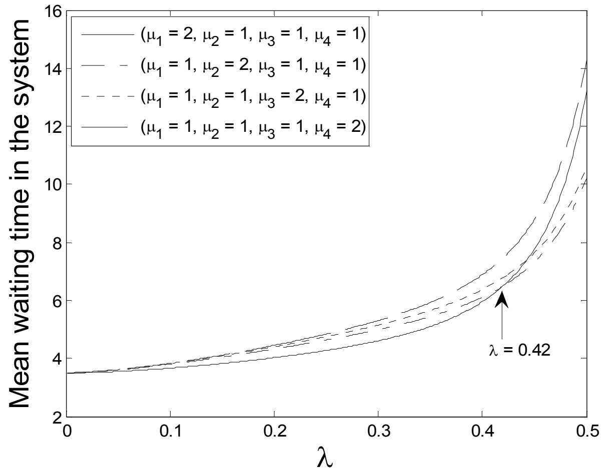

Fig 5. Mean waiting time in the system.

VI. Conclusion

Disposition strategies for the series configuration queueing are well studied through this work and our previous research, Tsai et al. [7]. At first, we suppose that the disposition strategies for the system consisting of the arbitrary number of service stations are all the same. However, our previous works show that there are different disposition strategies for the system comprising two and three service stations. In addition, this work further validates the consistent disposition strategy for the system consisting of the even number of service stations. Therefore, we propose the general disposition strategies for the series configuration queueing system consisting of the arbitrary number of service stations with different service rates based on this series of studies.

The simulations reveal that setting higher service rate for the station-1, and station-2 and the station-3 in order to obtain the best operational efficiency of the system, when we are able to control the service rate of the three service stations. On the other hand, while controlling only one of the service rate of the service stations, the case $\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_4 = 1$ and the case $\mu_1 = 1, \mu_2 = 2, \mu_3 = 1, \mu_4 = 1$ would be the better considerations depending on the conditions of mean arrival rate according to the numerical results. It is obvious for us to perceive the contrast of disposition strategies between the system consisting of the even number of service stations and the odd number of service stations. Consequently, we expect our theoretical analyses provide insights for the wide industrial applications.

We will conduct experiments in real assembly line to validate our theoretical analyses and propositions. Since the works are all based on the assumption of steady-state analysis and mean value analysis, it still needs to inspect the prediction results in real industrial applications. Extensions of statistical analysis and transient analysis about the series configurations queueing system would be considered in the future.

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