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# A Study on Applying Interactive Multi-objective Optimization to Multiagent Systems

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*Abstract*—**Constraint optimization problems in multiagent systems have been studied as fundamental problems of decision making and resource allocation. When each agent has its own specific interest in a problem, that task is defined as a multi-objective optimization problem with asymmetric functions. Since multi-objective problems have a number of Pareto optimal solutions in general cases, a preferred solution is selected using an appropriate criterion. The interactive methods are approaches of solution methods for multi-objective optimization problems, where a solution is repeatedly modified based on user-specified parameters. While interactive methods are developed for single users, such a framework can be considered as the base of analysis or games on multiagent systems by replacing single users with sets of agents. To study such an approach, this paper investigates a framework that resembles the interactive aspiration level methods employing a scalarization criterion that addresses unfairness among agents, and evaluates an example case of interaction.**

*Index Terms*—**multiagent, multi-objective, interactive method**

## I. Introduction

Constraint optimization problems in multiagent systems have been studied as fundamental problems of decision making and resource allocation. When each agent has its own interest in a problem, it is defined as a multi-objective optimization problem with asymmetric functions. In an asymmetric problem, each agent has its own individual evaluation on part of the problem, which is represented as its own objective function. Such classes of problems have been recently studied for decentralized optimization in multiagent systems [1], [2], [3], [4], [5]. While traditional optimization problem optimize the summation of objective functions, multiple objectives should be simultaneously optimized. This formalization is critical for several practical resource allocations such as task allocation with battery consumption, and collaborative team generation considering both the efficiency and the fairness of all members. For multi-objective problems in multiagent systems, each objective corresponds to an individual agent. Since multi-objective problems have a number of Pareto optimal solutions in general cases, a preferred solution is selected using an appropriate criterion [6], [7]. The interactive methods are approaches of the solution methods for multi-objective optimization problems, where a solution is repeatedly modified based on user-specified parameters [8]. While interactive methods are developed for single users, such a framework can be considered as a base of analysis or games on multiagent systems by replacing single users with sets of agents. For example, agents may require information about the current solution to review their positions in the current solution to investigate the possibilities of selecting another solution based on their other preferences. To study such an approach, this paper investigates a framework that resembles interactive aspiration level methods employing a scalarization criterion that considers unfairness among agents, and evaluates an example case of interaction.

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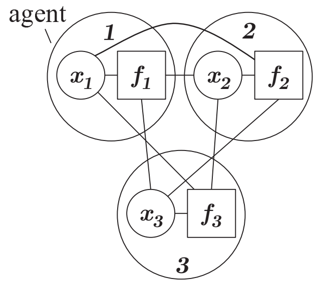


Fig. 1. Asymmetric multi-objective COP

The rest of this paper is organized as follows. In the next section, several preliminaries of this study, including problem settings, the concepts of multi-objective problems, the scalarization criteria, and interactive methods, are presented. In Section III, a framework of an interactive solution method for agents is presented. The proposed approach is experimentally evaluated in Section IV and discussed in Section V. Then the paper is concluded in Section VI.

## II. Preliminaries

### A. Multiobjective problem among agents

Asymmetric multiple objective problems for multiagent systems have been recently studied [1], [2], [3], [4], [5]. This paper investigates such problems assuming centralized solution methods. Here a class of asymmetric multi-objective constraint optimization problems, which resembles an asymmetric multi-objective distributed constraint optimization problem [4], [5], is addressed. An asymmetric multi-objective constraint optimization problem (asymmetric multi-objective COP) is defined as follows.

*Definition 1 (asymmetric multi-objective COP for agents):* An asymmetric multi-objective COP is defined by $(A, X, D, F)$, where $A$ is a set of agents, $X$ is a set of variables, $D$ is a set of the domains of variables, and $F$ is a set of objective functions. Here single variable $x_i \in X$ and single objective function $f_i \in F$ correspond to agent $a_i \in A$. The variable and function represent the agent's decision and utility. Variable $x_i \in X$ takes values from its domain $D_i \in D$, which is a discrete finite set. For set of variable $X_i \subseteq X$, function $f_i \in F$ is defined as $f_i(x_{i,1}, \cdots, x_{i,k}) : D_{i,1} \times \cdots \times D_{i,k} \rightarrow \mathbb{R}_0$, where

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$x_{i,1}, \cdots, x_{i,k} \in X_i$. $f_i(x_{i,1}, \cdots, x_{i,k})$ is simply denoted by $f_i(X_i)$. Although ideally, objective functions should be simultaneously maximized, there are trade-offs among them in general cases. The goal is to find an optimal solution under a criterion.

Figure 1 shows an asymmetric multi-objective COP with three agents, each of which corresponds to its variable and function. In this example, each agent has a ternary function, which represents an objective of the agent.

Since each agent prefers to maximize the utility values of its own objective function, this is a multi-objective problem. A tuple of the utility values of all the objective functions for an assignment is defined as an objective vector.

*Definition 2 (Objective vector):* Objective vector $\mathbf{v}$ is defined as $[v_1, \cdots, v_K]$, where $v_j$ is an objective value for agent $j$. Vector $\mathbf{F}(X)$ of the objective functions is defined as $[f_1(X_1), \cdots, f_K(X_K)]$, where $f_j(X_j)$ is an objective function for agent $j$. For assignment $\mathcal{A}$, vector $\mathbf{F}(\mathcal{A})$ of the functions returns objective vector $[v_1, \cdots, v_K]$. Here $v_j = f_j(\mathcal{A}_{\downarrow X_j})$.

Since there are trade-offs among objective functions in general cases, those objectives cannot be maximized simultaneously. Therefore, the objective vectors are compared with Pareto dominance [6], [7]. The dominance between two objective vectors is defined as follows.

*Definition 3 (Pareto dominance):* Vector $\mathbf{v}$ dominates $\mathbf{v}'$ if and only if $\mathbf{v} \geq \mathbf{v}'$, and $v_k > v'_k$ for at least one objective $k$.

Based on Pareto dominance, Pareto optimality is defined as follows.

*Definition 4 (Pareto optimality):* Assignment $\mathcal{A}^*$ is Pareto optimal if and only if there is no other assignment $\mathcal{A}$, such that $\mathbf{F}(\mathcal{A}) \geq \mathbf{F}(\mathcal{A}^*)$, and $F^k(\mathcal{A}^k) > F^k(\mathcal{A}^{*k})$ for at least one objective $k$.

In general cases, there are multiple Pareto optimal solutions. The set of objective vectors corresponding to the Pareto optimal solutions is called a Pareto front.

When the objective functions represent cost values, the problem is defined as a multi-objective minimization problem. In this case, the comparisons of the values are inverted.

### B. Scalarization and optimization

The order on objective vectors is defined by scalarization or a social welfare criterion. With scalarization, a single objective problem whose optimal solution is Pareto optimal is defined from the original multi-objective problem. There are several scalarization and social welfare criteria [7], [6].

Summation $\sum_{j=1}^{K} f_j(X_j)$ of the objectives is traditional social welfare. The summation is a 'utilitarian' criterion, since it represents the total value of the objectives. Even though the maximization on summation ensures Pareto optimality, it does not capture the equality of these objectives.

*Maximin* maximizes the minimum objective value. Although maximin improves the worst case, it is not Pareto optimal. Maximin is also improved by a summation that breaks the ties of maximin ordering. See the literature [6], [7] for the details of the above criteria.

*Leximin* is a lexicographic order on the objective vectors whose values are sorted in ascending order.

*Definition 5 (Sorted vector):* A sorted vector based on vector $\mathbf{v}$ is a vector where all the values in $\mathbf{v}$ are sorted in ascending order.

*Definition 6 (Leximin):* Let $\mathbf{v} = [v_1, \cdots, v_K]$ and $\mathbf{v}' = [v'_1, \cdots, v'_K]$ respectively denote the sorted vectors of identical length $K$. The relation of leximin ordering $\prec_{leximin}$ is defined as follows. $\mathbf{v} \prec_{leximin} \mathbf{v}'$ if and only if $\exists t, \forall t' < t, v_{t'} = v'_{t'} \wedge v_t < v'_t$.
The leximin is an 'egalitarian' criterion, since it reduces the inequality of the objectives. Maximization on leximin ordering ensures Pareto optimality.

The above scalarization and social welfare criteria can be applied to minimization problems by inverting the comparisons. Maximin is redefined as 'minimax'. In this case, the maximization function is called a Tchebycheff function. Maximization on leximin is redefined as minimization on 'leximax'.

### C. Interactive method based on aspiration levels

Since scalarization approaches only capture a part of the original Pareto front, they are modified as the parametric criteria to find other Pareto optimal solutions. Several parametric criteria cannot fit in part of the original Pareto front. For example, weighted summation $\sum_{j=1}^{K} w_i f_j(X_j)$ cannot fit several non-convex parts of the original Pareto front. Since weighted min/max(Tchebycheff function) $\text{min/max}_{j=1}^{K} w_i f_j(X_j)$ is not Pareto optimal, its variant with a tie-break by summation is Pareto optimal and fits the original Pareto front well. Therefore, this scalarization function is commonly used.

In this approach, the appropriate parameters depend on a user's preferences. To determine the parameters, interactive solution methods are employed, where a problem is repeatedly solved with different user-specified parameters [8]. In interactive methods, ideal points are employed as the base levels of objective values. Ideal point $f_i^*$ of $f_i$ is determined as the ideal value of $f_i$. For a minimization problem[1], the following type of interactive methods is employed.

$$\text{minimize} \quad [\max_{f_i \in F} w_i(f_i(X_i) - f_i^*), \sum_{f_i \in F} w_i f_i(X_i)]. \quad (1)$$

Here the summation part is employed as a tie-break on the maximization part. While the user can modify parameters $w_i$ by referring to the previous results in this framework, weight values $w_i$ are not intuitive.

As an intuitive approach, the aspiration level is employed [8], [9]. Aspiration level $\overline{f}_i^t$ is a desirable value of $f_i$ at the $t$th interaction step. For a minimization problem, an interactive method employs $\overline{f}_i^t$ as follows.

$$\text{minimize} \quad [\max_{f_i \in F} w_i(f_i(X_i) - \overline{f}_i^t), \sum_{f_i \in F} w_i f_i(X_i)] \quad (2)$$

$$\text{where} \quad w_i = 1/(\overline{f}_i^t - f_i^*). \quad (3)$$

In this case, the user directly determines the desirable value of $f_i$, and $w_i$ becomes a normalization value.

[1]Here a simple case of minimization problems is shown to combine 'unsatisfactory' and cost values.

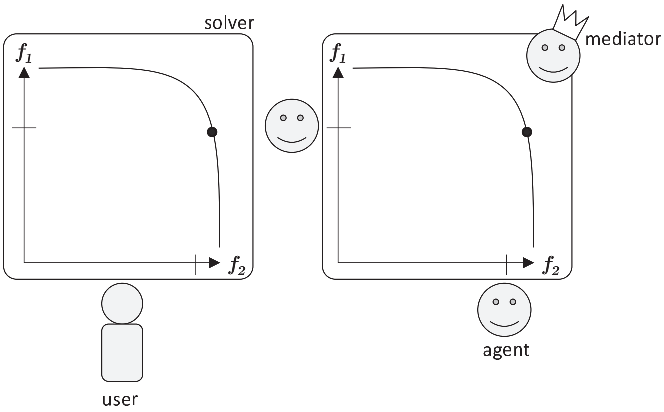


Fig. 2. Interactive method for agents

## III. Framework of interactive solution method for agents

### A. Interactive method for agents

The original interactive methods were developed for single users to provide an intuitive user interface. On the other hand, in multiagent settings, such methods can be considered as interaction between a solver and a set of agents. In such a framework, the solver serves as a mediator among the agents. In the interaction, the agents repeatedly set their aspiration levels and the mediator solves the problem. To determine the next aspiration level, the mediator must reveal some information about the current solution.

Figure 2 shows the concept of the proposed framework, where user and solver correspond to a set of agents and a mediator. In the case of a multiagent, the mediator might hide the details of individual private objectives for all agents. Instead, some statistical information can be provided about the solution quality as well as some suggestions or strategies to choose another solution. Based on such information, agents adjust the aspiration levels for their own objectives, and the mediator solves the problem again.

For the above framework, two major parts of the system must be addressed: (i) the solver should manage the fairness among agents. (ii) some schemes are necessary to choose the next aspiration levels. Below, the leximax criterion for fairness is applied to modify the weighted Tchebycheff function for aspiration levels. Also, an experimental strategy to determine the aspiration levels for the interactions of agents is investigated.

### B. Criteria for agents

Here we focus on the maximization problem. While utility functions can be redefined as cost functions, Eqs. (2) and (3) are modified for maximization problems as follows:

$$\text{minimize} \quad [\max_{f_i \in F} w_i(\overline{f}_i^t - f_i(X_i)), -\sum_{f_i \in F} w_i f_i(X_i)] \tag{4}$$

$$\text{where} \quad w_i = 1/(f_i^* - \overline{f}_i^t). \tag{5}$$

Note that the 'unsatisfactory' values are still minimized. On the other hand, the summation values for tie-breaks are maximized in this case.

While the minimization on the Tchebycheff function optimizes the worst case, it does not capture fairness among objective values. In multiagent settings, it is natural to employ another criterion for fairness. Therefore, we employ leximin-based criteria. Since each $w_i(f_i(X_i) - \overline{f}_i^t)$ should be minimized in such cases, the sorted objective values are compared in descending order. Therefore, minimization on 'leximax' is applied to the sorted vectors:

$$\text{minimize} \quad [v_1, \cdots, v_K] \text{ based on leximax} \tag{6}$$

$$\text{where} \quad v_i = w_i(\overline{f}_i^t - f_i(X_i)) \tag{7}$$

$$w_i = 1/(f_i^* - \overline{f}_i^t). \tag{8}$$

Note that the summation part for the tie-break is now unnecessary, since leximax repeats it between the values of the vectors in descending order.

As shown above, a criterion based on the aspiration level can be applied to interactive methods. On the other hand, in the case of multiagents, the scale of the criterion might be common for all agents, while weight value $w_i$ in the above criterion emphasizes aspiration levels that are closer to ideal points. In some interaction strategies for agents, simpler weight values will be intuitive.

The simplest weight value is a constant value $w_i = 1$. In this case, the criterion directly represents the 'unsatisfactory' values of agents with a common scale.

A different weight value can be defined based on $f_i^\top$ and $f_i^\perp$, which are the maximum and minimum values of $f_i(X_i)$, as follows.

$$w_i = (f_i^\top - \min_j f_j^\perp)/(\max_j f_j^\top - \min_j f_j^\perp). \tag{9}$$

This emphasizes the agents, whose $f_i^\top$ values are relatively large, when the summation of all the objective values is considered.

### C. Interaction among agents

When the interactive solution method is considered to be a mediator, the solution method should provide information of the current solution to determine the next aspiration levels of the agents. Such information includes the number of agents, the summation of the objective values, the minimum objective value, and other statistic values, so that agents are aware of their position in the current solution. Based on the solution's information, the agents modify their aspiration levels to search for another solution. This approach is considered as analysis or games performed by agents. Since there are various strategies and criteria to choose the aspiration levels, here a simple case of trade-offs among agents is addressed as the preliminary study.

When all agents $i$ set their aspiration levels to the maximum value of objective function $f_i(X_i)$, the problem resembles a maximization problem with $f_i(X_i)$ and a leximin criterion. Even though its optimal solution will improve the minimum objective value and fairness, the summation of the objective values will be decreased. Assuming a property transfer, the influences of several agents on the solution can be analyzed to improve the summation of the objective values. Note that an external assumption on property transfers

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TABLE I
PARETO FRONT

| func. | arity $a$ | Pareto opt. slts. | Pareto front sz. |
|---|---|---|---|
| uniform | 3 | 707 | 654 |
| | 5 | 4285 | 4268 |
| gamma | 3 | 1362 | 1297 |
| | 5 | 9057 | 9053 |

TABLE II
SOLUTIONS OF ASPIRATION LEVEL METHODS

| func. | | uniform | | | | gamma | | | |
|---|---|---|---|---|---|---|---|---|---|
| arity $a$ | | 3 | | 5 | | 3 | | 5 | |
| $asplb$ [%] | criteria | P | N | P | N | P | N | P | N |
| 100 | wgttbc | 1 | 0 | 1.2 | 0 | 1 | 0 | 1 | 0 |
| | lexmax | 1 | 0 | 1.1 | 0 | 1 | 0 | 1 | 0 |
| 75 | wgttbc | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | lexmax | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 50 | wgttbc | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | lexmax | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | wgttbc | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | lexmax | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

P: Pareto optimal solutions. N: Non Pareto optimal solutions.

TABLE III
SOLUTIONS IN ASPIRATION LEVEL METHOD (UNIFORM, $a = 3$)

| $asplb$ [%] | criteria | asp. | | | | | org. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | min | max | sum | mTheil | var. | min | max | sum | Theil | var. |
| 100 | wgttbc | 0 | 31.97 | 134.4 | 0.313 | 145.95 | 6.8 | 10 | 85.7 | 0.0103 | 1.425 |
| | lexmax | 0 | 31.97 | 134.4 | 0.313 | 145.95 | 6.8 | 10 | 85.7 | 0.0103 | 1.425 |
| 75 | wgttbc | -0.913 | 2.80 | 4.6 | 0.234 | 1.97 | 4.72 | 10 | 82.25 | 0.0304 | 3.262 |
| | lexmax | -0.903 | 2.80 | 5.1 | 0.233 | 1.87 | 4.7 | 9.99 | 81.61 | 0.0308 | 3.257 |
| 50 | wgttbc | -0.951 | 1.11 | -2.0 | 0.230 | 0.68 | 4.62 | 10 | 82.03 | 0.0312 | 3.343 |
| | lexmax | -0.946 | 1.11 | -1.7 | 0.227 | 0.64 | 4.61 | 9.99 | 81.38 | 0.0321 | 3.375 |
| 0 | wgttbc | -0.977 | 0.13 | -5.7 | 0.231 | 0.20 | 4.55 | 10 | 81.78 | 0.0354 | 3.559 |
| | lexmax | -0.974 | 0.13 | -5.6 | 0.228 | 0.19 | 4.52 | 9.99 | 81.25 | 0.0357 | 3.557 |

is necessary, since the agents already have a Pareto optimal solution.

For a maximization problem with leximin, the agents of the minimum objective value are basically 'bottle-neck' agents that relate to implicit constraints on the criterion. How they affect the solution can be an interest. The bottle-neck agents are aware of that when the solver provides a current partial solution for each agent and the statistic information of the objective values. Then such agents reduce their aspiration levels and the solution method is performed again. Here the influences of the aspiration levels on the solution quality, including Pareto optimality, are mainly addressed with simple cases.

## IV. EVALUATION

### A. Fairness among agents in aspiration level method

In the first experiment, the effects of the criteria on the aspiration level methods are evaluated. The problem consists of ten ternary (i.e., $|D_i| = 3$) variables and ten functions for ten agents. Since the Pareto optimal solutions and the Pareto front were enumerated using a brute-force search, the size of the problems was restricted. Arity $a$ (i.e., the scope sizes) of the functions was either three or five. The following two cases of function values were evaluated:

- **uniform**: an integer value in $[1, 10]$ based on uniform distribution.

TABLE IV
SOLUTIONS IN ASPIRATION LEVEL METHOD (UNIFORM, $a = 5$)

| $asplb$ [%] | criteria | asp. | | | | | org. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | min | max | sum | mTheil | var. | min | max | sum | Theil | var. |
| 100 | wgttbc | 0 | 33 | 152 | 0.396 | 155.8 | 6.7 | 10 | 84.8 | 0.0111 | 1.558 |
| | lexmax | 0 | 33 | 154 | 0.375 | 138.4 | 6.7 | 10 | 84.6 | 0.0098 | 1.384 |
| 75 | wgttbc | -0.8947 | 2.07 | 4.32 | 0.284 | 1.16 | 4.76 | 10 | 81.85 | 0.0304 | 3.304 |
| | lexmax | -0.8863 | 2.07 | 4.51 | 0.285 | 1.13 | 4.79 | 10 | 81.53 | 0.0306 | 3.302 |
| 50 | wgttbc | -0.9297 | 0.63 | -2.18 | 0.279 | 0.31 | 4.71 | 10 | 81.24 | 0.0328 | 3.447 |
| | lexmax | -0.9292 | 0.63 | -2.16 | 0.278 | 0.31 | 4.72 | 10 | 81.25 | 0.0324 | 3.407 |
| 0 | wgttbc | -0.9619 | -0.15 | -5.91 | 0.279 | 0.09 | 4.67 | 10 | 81.18 | 0.0330 | 3.485 |
| | lexmax | -0.9617 | -0.15 | -5.89 | 0.277 | 0.08 | 4.69 | 10 | 81.09 | 0.0329 | 3.464 |

TABLE V
SOLUTIONS IN ASPIRATION LEVEL METHOD (GAMMA, $a = 3$)

| $asplb$ [%] | criteria | asp. | | | | | org. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | min | max | sum | mTheil | var. | min | max | sum | Theil | var. |
| 100 | wgttbc | 4.88 | 52.9 | 326.2 | 0.376 | 254.25 | 17.8 | 31.5 | 236.9 | 0.0150 | 17.66 |
| | lexmax | 4.88 | 52.9 | 330.1 | 0.341 | 225.92 | 17.5 | 31.7 | 236.3 | 0.0153 | 17.86 |
| 75 | wgttbc | -0.76 | 5.8 | 18.8 | 0.259 | 5.80 | 13.58 | 35.28 | 236.01 | 0.0416 | 45.74 |
| | lexmax | -0.69 | 5.8 | 19.7 | 0.250 | 5.27 | 13.25 | 35.08 | 232.31 | 0.0435 | 46.53 |
| 50 | wgttbc | -0.86 | 2.9 | 5.5 | 0.250 | 1.99 | 13.45 | 34.98 | 235.11 | 0.0419 | 45.64 |
| | lexmax | -0.82 | 2.9 | 6.1 | 0.242 | 1.82 | 13.26 | 34.88 | 231.81 | 0.0431 | 45.86 |
| 0 | wgttbc | -0.93 | 1.1 | -1.9 | 0.245 | 0.62 | 13.06 | 35.21 | 235.05 | 0.0445 | 47.82 |
| | lexmax | -0.91 | 1.1 | -1.6 | 0.234 | 0.57 | 13.2 | 35.02 | 231.91 | 0.0440 | 46.69 |

TABLE VI
SOLUTIONS IN ASPIRATION LEVEL METHOD (GAMMA, $a = 5$)

| $asplb$ [%] | criteria | asp. | | | | | org. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | min | max | sum | mTheil | var. | min | max | sum | Theil | var. |
| 100 | wgttbc | 20.66 | 56.2 | 437.5 | 0.447 | 149.70 | 18.7 | 33.5 | 240.2 | 0.0166 | 20.84 |
| | lexmax | 23.63 | 56.2 | 438.8 | 0.423 | 132.41 | 18.8 | 32.8 | 239.9 | 0.0155 | 19.45 |
| 75 | wgttbc | 0.06 | 6.9 | 32.4 | 0.283 | 6.42 | 12.19 | 37.68 | 233.94 | 0.0591 | 64.14 |
| | lexmax | 0.08 | 6.9 | 33.1 | 0.276 | 6.12 | 12.16 | 37.57 | 230.32 | 0.0597 | 63.28 |
| 50 | wgttbc | -0.45 | 3.9 | 13.2 | 0.263 | 2.94 | 11.98 | 38.1 | 233.38 | 0.0621 | 67.08 |
| | lexmax | -0.43 | 3.9 | 13.5 | 0.256 | 2.85 | 11.94 | 38.09 | 229.66 | 0.0632 | 66.89 |
| 0 | wgttbc | -0.70 | 1.8 | 2.3 | 0.248 | 1.07 | 11.86 | 38.31 | 233.01 | 0.0629 | 67.81 |
| | lexmax | -0.68 | 1.8 | 2.5 | 0.241 | 1.03 | 11.59 | 38.22 | 228.79 | 0.0650 | 68.03 |

- **gamma**: a rounded integer value based on a gamma distribution with $(\alpha = 9, \beta = 2)$, similar to [5], [10].

Also, the following criteria were employed[2]:

- **wgttbc**: a weighted Tchebycheff function in Eqs. (4) and (5).
- **lexmax**: a leximax criterion in Eqs. (6), (7), and (8).

Aspiration level $\overline{f}_i^t$ is randomly set from $(f_i^\top - asplb \cdot (f_i^\top - f_i^\perp), f_i^\top]$ with uniform distribution based on parameter $asplb = 100, 75, 50, 0\%$. The results are averaged for ten problem instances with ten random parameters of $asplb$. For multiple optimal solutions, these results are averaged, while a single solution is found in most cases.

Table I shows the Pareto front of the original problems. Also, Table II shows the average number of solutions reported by aspiration level methods. Note that the optimization of solution methods are based on the aspiration levels, while these evaluations are for the Pareto optimality in the original problems. Although the aspiration level methods

[2]Ideal point $f_i^* = f_i^\top + 0.01(f_i^\top - f_i^\perp)$ is used to avoid zero-division, where $f_i^\top$ and $f_i^\perp$ are the maximum and minimum values of $f_i(X_i)$.

TABLE VII
MODIFICATION ON ASPIRATION LEVELS (SUM, UNIFORM, $a = 3$)

| weight | last · first [%] | | | best · first [%] | | best step [%] | | |
|---|---|---|---|---|---|---|---|---|
| | LT | EQ | GT | EQ | GT | first | others | last |
| wsu | 20 | 60 | 20 | 80 | 20 | 80 | 20 | 0 |
| wa1 | 10 | 50 | 40 | 60 | 40 | 60 | 40 | 0 |
| wah | 10 | 50 | 40 | 60 | 40 | 60 | 40 | 0 |

·: comparison. LT: less than. EQ: equal to. GT: greater than.

TABLE VIII
MODIFICATION ON ASPIRATION LEVELS (SUM, UNIFORM, $a = 5$)

| weight | last · first [%] | | | best · first [%] | | best step [%] | | |
|---|---|---|---|---|---|---|---|---|
| | LT | EQ | GT | EQ | GT | first | others | last |
| wsu | 30 | 50 | 20 | 80 | 20 | 80 | 20 | 0 |
| wa1 | 30 | 50 | 20 | 80 | 20 | 80 | 20 | 0 |
| wah | 30 | 50 | 20 | 80 | 20 | 80 | 20 | 0 |

TABLE IX
MODIFICATION ON ASPIRATION LEVELS (SUM, GAMMA, $a = 3$)

| weight | last · first [%] | | | best · first [%] | | best step [%] | | |
|---|---|---|---|---|---|---|---|---|
| | LT | EQ | GT | EQ | GT | first | others | last |
| wsu | 20 | 40 | 40 | 60 | 40 | 60 | 40 | 0 |
| wa1 | 10 | 60 | 30 | 70 | 30 | 70 | 30 | 0 |
| wah | 30 | 40 | 30 | 70 | 30 | 70 | 30 | 0 |

optimized their own criteria, their optimal solutions were on the Pareto front of the original problems because the aspiration level method does not limit the improvement of the solutions even if the aspiration level is satisfied.

Tables III-VI show the statistic information of the solutions both in the aspiration level method and the original problems. Note that here the original problems were not directly solved. The results show the summation/minimum/maximum values, the variance, and the Theil index, a well-known measurement of inequality, which is defined as follows.

$$T = \frac{1}{|A|} \sum_{a_i \in A} \left( \frac{v_i}{\overline{v}} \ln \frac{v_i}{\overline{v}} \right) \tag{10}$$

Here $\overline{v}$ denotes the average value for all $v_i$. For the aspiration level methods, the following modified Theil index 'mTheil' is used so that negative values are allowed.

$$T = \frac{1}{|A|} \sum_{a_i \in A} \left( \frac{v'_i}{\overline{v'}} \ln \frac{v'_i}{\overline{v'}} \right) \tag{11}$$

$$\text{where } v'_i = (\max_i v_i) - v_i \tag{12}$$

These criteria take a minimum value zero when all the values are equal.

The result shows that both methods minimized the maximum value of the aspiration level criteria. The method with leximax also improved the fairness among agents in the aspiration level criteria in most cases. This means that the fairness among agents with similar requirements is relatively well optimized using leximax.

### *B. Modification of aspiration levels*

Next, the influence of modification on part of the aspiration levels was investigated. In this experiment, the simple case shown in Section III-C was employed. In the first step, the problem is solved with the aspiration levels of the maximum

TABLE X
MODIFICATION ON ASPIRATION LEVELS (SUM, GAMMA, $a = 5$)

| weight | last · first [%] | | | best · first [%] | | best step [%] | | |
|---|---|---|---|---|---|---|---|---|
| | LT | EQ | GT | EQ | GT | first | others | last |
| wsu | 70 | 20 | 10 | 90 | 10 | 90 | 10 | 0 |
| wa1 | 30 | 50 | 20 | 80 | 20 | 80 | 20 | 0 |
| wah | 20 | 60 | 20 | 80 | 20 | 80 | 20 | 0 |

values of the objective functions. Then in the following steps, the aspiration levels of the agent, whose objective values are the minimum values in the current solution, are decreased. Based on the new aspiration levels, the problem is solved again. In this experiment, ten interaction steps were performed. To clarify the margin of the trade-offs, each aspiration level is evenly decreased in each step so that the final value is to be the minimum objective value.

Here how the aspiration levels with different weight values affect the solutions was addressed. The leximax was employed as a criterion of the aspiration level and the following weight values $w_i$ shown in Section III-B were compared.

- **wsu**: the weight values for a single user as shown in Eq. (8).
- **wa1**: the weight values for the agents. All are set to 1.
- **wah**: the weight values for the agents. Higher objective values are preferred as shown in Eq. (9).

Tables VII-X show the results of the modifications of the aspiration levels. 'last · first' and 'best · first' denote comparisons among the first, last and best summation values. Since these interaction strategies simply decrease the aspiration levels that are selected based on the results in the first step, a decrement of several aspiration levels decreased the summation of the objective values in several cases. It was also observed that other agents are affected by the modification of some of the aspiration levels. In addition, the summation value can be non-monotonic to the aspiration levels. Such influences were varied in different weight values and problems. While **wsu** was relatively ineffective in the cases of (uniform, $a = 3$) and (gamma, $a = 5$), the results are different in other cases. The results reveal that the interactive approach is affected by both interaction strategies and 'unsatisfactory' values.

The tables also show the information of the step when the best summation value was obtained ('best step'). The ratio of steps are categorized into the first, last and other steps. Since the aspiration level only gives a bias on the objective values, there are thresholds that change the solutions before the last aspiration levels, which are the minimum objective values.

## V. DISCUSSION

This paper proposed a framework based on interactive solution methods for asymmetric constraint optimization problems on multiagent systems. Since this class of problem needs relatively high computational cost for dense problems, partial centralization approaches of solution methods might be practical. In the case of asymmetric multi-objective problems in multiagent systems, agents require information about its position on the current solution and need to know the possibility of another solution. When agents have additional

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hidden preferences, such solution methods resemble a platform of games.

In this work, a brute-force search was employed for the preliminary study, but there are several opportunities of efficient solution methods, including the pruning of search space and the decomposition of problems. Genetic algorithms are another well-known approach for multi-objective problems. On the other hand, the quality of the approximation method might need several investigations.

The aspiration level method was originally developed for a single user. For multiagents, other weighting approaches might be necessary, since the aspiration level can easily affect other objectives. This idea also relates to how Pareto optimality is ensured. Sensitivity analysis is another important issue to provide wider information to agents and reduce redundant computations.

## VI. Conclusion

In this study, the framework of an interactive solution method for asymmetric multi-objective constraint optimization problems on multiagent systems was investigated. This framework is based on an approach of the interactive aspiration level methods. For multiagents, a scalarization criterion leximax that considers unfairness among agents was applied and the proposed approach was experimentally evaluated with an example interaction case.

The future work will include more sophisticated parameterizations/strategies in interactive methods, norms to represent additional preferences of agents, efficient solution methods, and investigations in practical domains.

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