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Posicast PID × (*n*-2) Stage PD Cascade Controllers for Magnetically-Levitation System

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***Abstract*—When a plant to be controlled is third or higher n^{th} order, the PID×(*n*-2) stage PD cascade controllers are very suitable to be applied for control. To verify the advantages of these controllers, the Magnetically-Levitation plant is then selected as an example of unstable plant to be stabilized and controlled. The design technique is based on placing the controller zeros. The overall controlled system can be approximated as a standard second-order system prompt for designing the Posicast controller to obtain the output response with no overshoot in the last step.**

***Index Terms*—Magnetic Levitation system, PID×(*n*-2) stage PD controllers, Posicast Controller**

I. Introduction

Because the PID (Proportional-Integral-Derivative) controller is properly applied to a typical second order plant only not for any *n*th order. In order to control a third or higher *n*th order plant, the PID×(*n*-2) stage PD cascade controller design based on root locus technique is proposed in Continuous-Time (CT) framework [1]. The original design technique known as "Kitti's Method" is aimed to satisfy the desired specifications without trial and error. Then, the forward controller is employed to decrease the undesirable overshoot, the controlled system structure becomes two-degree of freedom (2-DOF) system as shown in Fig. 1.

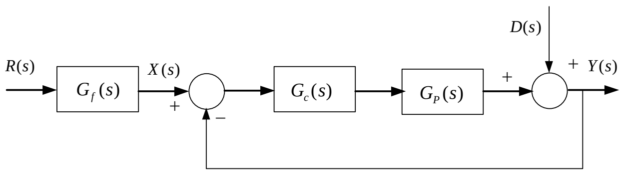

Fig. 1. Structure of the 2-DOF control system.

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By placing the zeros of the designed controller in the way of Kitti's Method, the overall controlled system is approximated as a standard second-order system. From the response to a unit step input, the maximum overshoot occurred at the first peak time t_p with the amplitude of $1+M_p$. If the unit step input is reshaped into two parts. The first part is the step input with amplitude of $1/(1+M_p)$ at $t = 0$. The second part is a stair with amplitude of $M_p/(1+M_p)$ and delay caused by the time t_p (or written by $e^{-t_p s}$) as shown in Fig. 2 [2].

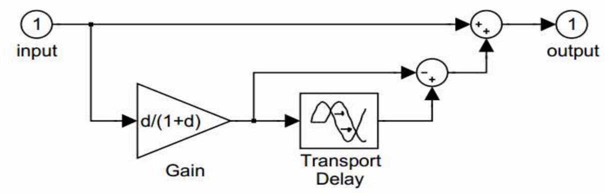

Fig. 2. SIMULINK diagram.

II. METHODOLOGY

Fig. 3 shows the steps to design digital control systems [3]. The major steps are plant modeling and controller design.

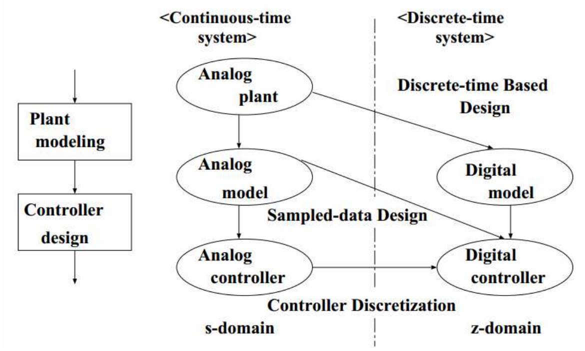

Fig. 3. The steps of the digital control system design.

A. Plant modeling

In Fig. 4 [4], a ball bearing of mass m is placed underneath the electromagnet at distance x. The current flowing into the electromagnetic coil will generate electromagnetic force to attract the ball bearing. The net force between the electromagnetic force and gravitational force will induce an up or down motion of the ball bearing. The photoresistor senses the variation of the position of the ball bearing by the amount of shadow casted on its surface and feeds back this signal to the control circuit and amplifier to regulate the input current i. The ball bearing is kept in a dynamic balance around its equilibrium point.

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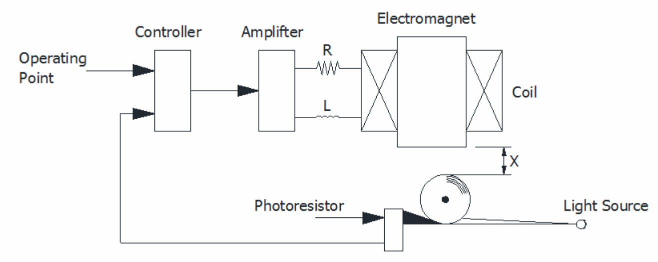

Fig. 4. Magnetic levitation control system.

The system's dynamic equations can be obtained as follows.

$$f = C\left(\frac{i}{x}\right)^2 \quad (1)$$

$$e = Ri + L\frac{di}{dt} \quad (2)$$

$$m\frac{d^2x}{dt^2} = mg - f \quad (3)$$

where

- f is an electromagnetic force.
- i is a coil current.
- x is a distance between electromagnet and ball bearing.
- C is a constant.
- e is a voltage across the coil.
- R is a coil resistance.
- L is a coil inductance.
- m is a mass of ball bearing.
- g is a gravitational acceleration.

The linearization equations describing the variations from the operating point are obtained by using only the linear terms from the Talor series expansion. If the variables of the operating point are expressed with subscript "0" and the variables at the neighborhood of the operating point are represented with subscript "1," then linearlized equations are

$$f_1 = \frac{2Ci_0}{x_0^2}i_1 - \frac{2Ci_0^2}{x_0^3}x_1 \quad (4)$$

$$e_1 = Ri_1 + L\frac{di_1}{dt} \quad (5)$$

$$m\frac{d^2x_1}{dt^2} = -f_1 \quad (6)$$

Laplace transform of (4)-(6) yields

$$F_1(s) = k\left[I_1(s) - \frac{i_0}{x_0}X_1(s)\right] \quad (7)$$

$$E_1(s) = (R + Ls)I_1(s) \quad (8)$$

$$ms^2X_1(s) = -F_1(s) \quad (9)$$

where $k = 2C\dfrac{i_0}{x_0^2}$.

The block diagram of the magnetic levitation system is shown in Fig. 5. The characteristic equation of the control system can be obtained as follows.

$$\begin{aligned} Q(s) = x_0Lms^3 + x_0Rms^2 - ki_0Ls - \\ ki_0R + G_c(s)kx_0B = 0. \end{aligned} \quad (10)$$

Equation (10) can be rearranged as

$$Q(s) = 1 + \frac{G_c(s)kx_0B}{x_0Lms^3 + x_0Rms^2 - ki_0Ls - ki_0R} = 0 \quad (11)$$

and

$$Q(s) = 1 + \frac{G_c(s)\dfrac{kB}{mL}}{\left(s+\sqrt{\dfrac{ki_0}{mx_0}}\right)\left(s-\sqrt{\dfrac{ki_0}{mx_0}}\right)\left(s+\dfrac{R}{L}\right)} = 0. \quad (12)$$

Substituting the parameters into (12), yields

$$Q(s) = 1 + \frac{60990G_c(s)}{(s+49.5)(s-49.5)(s+58)} = 0. \quad (13)$$

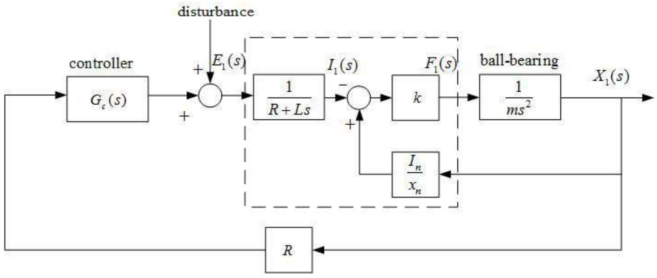

Fig. 5. Block diagram.

TABLE I
NOMENCLATURE

Symbol	Quantity
x_0	0.008 m
m	0.068 kg
R	28 Ω
L	0.483 H
i_0	0.76 A
C	7.39×10^{-5} N·m^2/A^2
k	1.756 N/A
B	1.14×10^3 V/m

B. Controller design

Let the nth order plant $G(s)$ to be controlled by the controllers $K(s)$, their transfer function is assumed to be given by

$$\begin{cases} G(s) = \dfrac{K_n}{s^N(T_1s+1)(T_2s+1)\cdots(T_ps+1)}, \\ \quad = \dfrac{60990}{(s+49.5)(s-49.5)(s+58)}; \quad n = 3,\ N = 0. \end{cases} \quad (14)$$

If the PID controller transfer function is

$$K_{PID}(s) = K_p + \frac{K_i}{s}K_ds = K_{pid}\frac{(s+z_1)(s+z_2)}{s}, \quad (15)$$

where K_p, K_i, and K_d are a proportional gain, an integral gain, a derivative gain, respectively. Hence, the PD controller transfer function can be stated as

$$K_{PD}(s) = K_p + K_d s = K_{pd}(s + z_{pd}). \tag{16}$$

The open-loop transfer function for the PID×(*n*-2) stage PD controllers $K(s)$ and the plant $G(s)$ is

$$KG(s) = \frac{\overbrace{K_{pid}(s+z_1)(s+z_2)}^{\text{PID Controller}} \times \overbrace{K_{pd}(s+z_{pd})\cdots}^{(n-2)\text{ PD}} K_n}{s \cdot \underbrace{s^N (s+p_1)(s+p_2)\cdots(s+p_p)}_{n\text{th order Plant}}}, \tag{17}$$

or,

$$KG(s) = \frac{K \overbrace{(s+49.6)(s+58.1)}^{\text{Pre-assigned}} (s+z_{pd})60990}{s \cdot \underbrace{(s+49.5)(s+58)(s-49.5)}_{\text{3rd order plant}}}. \tag{18}$$

By using Kitti's Method, $z_1 = 49.6$ and $z_2 = 58.1$ are first assigned, then find only z_{pd} and K from the following root locus angle and magnitude conditions.

$$\begin{cases} \measuredangle KG(s) = \pm(2k+1)\pi, \quad k = 0,1,2,\cdots, \\ |KG(s)| = 1. \end{cases} \tag{19}$$

The desired specifications to be designed are usually specified in terms of transient and steady state response characteristics of a control system to a unit-step input, exhibited by a pair of complex-conjugate dominant closed-loop poles $s_{d\pm} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ as follows.

$$\begin{cases} \text{Percent Overshoot } (P.O.) = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100\% = 5\% \\ \text{Settling Time } (t_s) = \dfrac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_n} = 0.1\text{secs.} \\ \qquad (\pm 2\%) \end{cases} \tag{20}$$

From the given desired specification in term of the Percent Overshoot $(P.O.)$, the damping ratio is

$$\zeta = \sqrt{\left[\ln\left(\frac{P.O.}{100}\right)\right]^2 \bigg/ \left\{\pi^2 + \left[\ln\left(\frac{P.O.}{100}\right)\right]^2\right\}} = 0.69, \tag{21}$$

and from the given Settling Time $\{t_s(\pm 2\%)\}$, then the undamped natural frequency is

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta t_s} = 61.373 \text{ rad./sec}. \tag{22}$$

Hence, one of the dominant closed-loop poles is located at

$$s_d = -42.354 + j44.416. \tag{23}$$

The open-loop transfer function without z_{pd} at s_d is

$$\begin{cases} KGwoz_{pd}(s_d) = \\ \dfrac{\overbrace{(s_d+49.6)}^{80.734^\circ}\overbrace{(s_d+58.1)}^{70.48^\circ}60990}{\underbrace{s_d}_{133.639^\circ} \cdot \underbrace{(s_d+49.5)}_{80.86^\circ}\underbrace{(s_d+58)}_{70.595^\circ}\underbrace{(s_d-49.5)}_{154.194^\circ}}, \\ KGwoz_{pd}(s_d) = \underbrace{|3.025 + j9.269|}_{9.75} \measuredangle 71.927^\circ. \end{cases} \tag{24}$$

The angle from the zero z_{pd} to s_d is

$$\begin{cases} \arg(z_{pd}) = \pi - \arg\left(KGwozpd(s_d)\right) = \measuredangle(s_d + z_{pd}), \\ \quad = 108.073^\circ. \end{cases} \tag{25}$$

Now, it is implied that

$$\begin{cases} \measuredangle KG(s_d) = \measuredangle \dfrac{\overbrace{(s_d+49.6)(s_d+58.1)}^{151.214^\circ}\overbrace{(s_d+z_{pd})}^{108.073^\circ}60990}{\underbrace{s_d \cdot (s_d+49.5)(s_d+58)(s_d-49.5)}_{439.287^\circ}}, \\ = 151.214^\circ + 108.073^\circ - 439.287^\circ = -180^\circ. \end{cases} \tag{26}$$

Since, $\measuredangle(s_d + z_{pd})$ is greater than 90°, so the zero z_{pd} is located at the right hand side of s_d as

$$z_{pd} = |\text{Re}(s_d)| - \frac{|\text{Im}(s_d)|}{\tan\left(\pi - \measuredangle(s_d + z_{pd})\right)} = 27.86. \tag{27}$$

The controller gain K can be found from the magnitude condition of the root locus technique as follows.

$$K = \frac{\overbrace{|s_d|}^{61.373}\overbrace{|s_d+49.5|}^{44.987}\overbrace{|s_d+58|}^{47.091}\overbrace{|s_d-49.5|}^{102.029}}{\underbrace{|s_d+49.6|}_{45.003}\underbrace{|s_d+58.1|}_{47.125}\underbrace{|s_d+27.86|}_{46.721}60990} = \frac{2.195}{1000}. \tag{28}$$

Hence, the magnitude of the open-loop transfer function $|KG(s_d)|$ is

$$\frac{\overbrace{\left(\frac{2.195}{1000}\right)}^{\text{Find}}\overbrace{|s_d+49.6||s_d+58.1|}^{\text{Pre-assigned}}\overbrace{|s_d+27.86|}^{\text{Find}}60990}{|s_d||s_d+49.5||s_d+58||s_d-49.5|} = 1. \tag{29}$$

Finally, the open-loop transfer function can be expressed as follows.

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$$\begin{cases} KG(s) = \dfrac{K(\cancel{s+49.6})(\cancel{s+58.1})(s+z_{pd})60990}{s(\cancel{s+49.5})(\cancel{s+58})(s-49.5)}, \\ K = 2.195\times10^{-3}, \quad z_{pd} = 27.86. \end{cases} \tag{30}$$

To decrease the overshoot caused by adding the zero $(s+z_{pd})$ to the open-loop transfer function $KG(s)$, the following forward controller is introduced.

$$K_f(s) = z_{pd}\big/\left(s+z_{pd}\right) \tag{31}$$

The overall system is then approximated as if it is a standard second-order system as follows.

$$\begin{aligned} \frac{Y(s)}{R(s)} &\approx \left(\frac{z_{pd}}{\cancel{s+z_{pd}}}\right)\left(\frac{K(\cancel{s+z_{pd}})60990}{s(s-49.5)+K(s+z_{pd})60990}\right) \\ &\approx \frac{K60990z_{pd}}{s^2+\underbrace{(K60990-49.5)}_{84.383}s+\underbrace{(K60990z_{pd})}_{3.73\times10^3}}, \\ &\approx \frac{3.73\times10^3}{s^2+2\cdot\underbrace{0.691}_{\zeta}\cdot\underbrace{61.073}_{\omega_n}s+\underbrace{3.73\times10^3}_{\omega_n^2}}. \end{aligned} \tag{32}$$

From the response to a unit step input of a standard second order system, the maximum overshoot is

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.05, \quad \zeta = 0.691. \tag{33}$$

This maximum overshoot is occurred at the peak time,

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.071\,\text{secs.}, \quad \omega_n = 61.073\ \text{rad/sec.} \tag{34}$$

In order to achieve the response with no overshoot, the unit step input will be rescaled by the factor $(1+M_p)$ in two parts as follows.

$$\underbrace{\frac{1}{1+M_p}}_{❶}+\underbrace{\frac{M_p}{1+M_p}e^{-t_ps}}_{❷} = \overbrace{1-\underbrace{\frac{M_p}{1+M_p}}_{❶}+\underbrace{\frac{M_p}{1+M_p}e^{-t_ps}}_{❷}}^{\text{Posicast}\,=\,1+P(s)}. \tag{35}$$

III. SIMULATION RESULTS

Fig. 6 shows the root loci of the magnetic levitation system. It is evidently seen that it is unstable system in nature, because there is a real pole in the right half plane corresponds to an exponentially increasing component $Ce^{\sigma t}$ in the homogeneous response, (where the constant C is determined from the given set of initial conditions) as shown in Fig. 7 [5]. After the PID×(n-2) stage PD cascade controllers are applied, the pole from the integral term at the origin then can brings the unstable pole across imaginary axis toward the left half plane as shown in Fig. 8. It can be concluded that the magnetic levitation system is stabilizable by the designed controllers with excellence. In Fig. 9, the unit step response in red solid line is for the PID×(n-2) stage PD cascade controllers in the control loop at the designed value only. The response in green solid line is for after the forward controller is introduced. From placing the controller zeros close to the poles of the plant, the overall system can be then approximated as a standard second-order system with no zero and two poles only. Then, the maximum overshoot is obtained within desired value. Once, the properties of the second order system are known, it is easy to design the feedforward controller, which is a Posicast pre-filter for shaping the reference input using either two or three steps. The blue solid line is a response to a two-step input shown by white dashed line. The last magenta dashed line is the response by increasing the controller gain to ten times of the designed value.

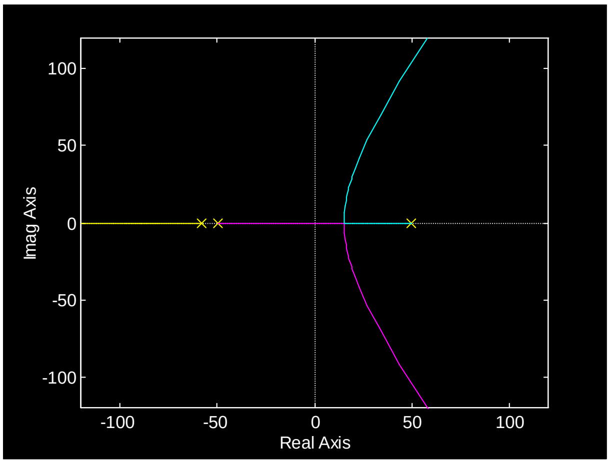

Fig. 6. Root loci for Magnetic Levitation system.

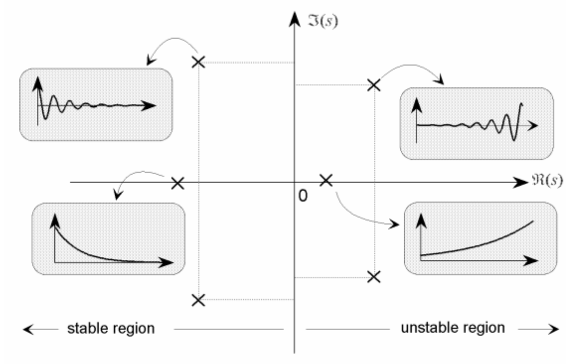

Fig. 7. Stable / Unstable region.

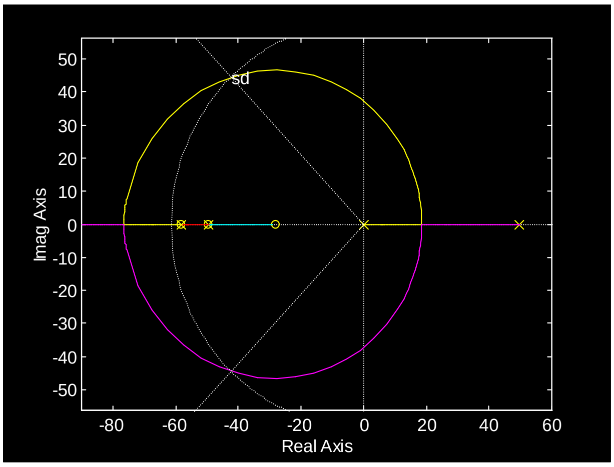

Fig. 8. Root loci for the controlled system.

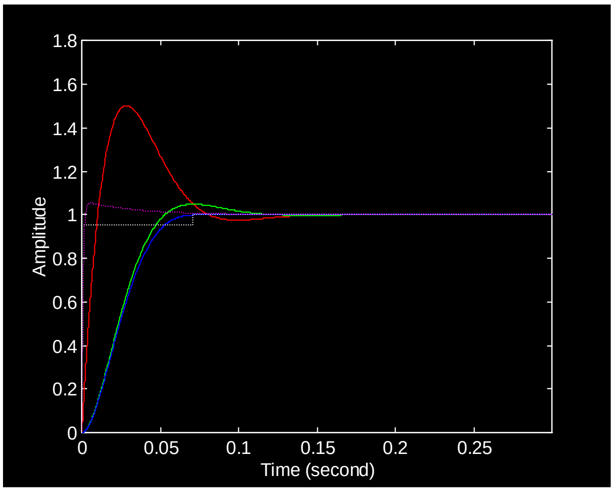

Fig. 9. Unit step responses.

IV. CONCLUSION

An unstable, third-order magnetic levitation plant has been selected as an interesting maglev plant to be controlled by the proposed PID×(*n*-2) stage PD cascade controllers with incorporating Posicast controller. It seems to be that there are three steps to design the controllers in this paper. The first step is the design of the PID×(*n*-2) stage PD cascade controllers. After finish this step, the settling time may be satisfied, but may not for the maximum overshoot. The second step is the design of the forward controller to achieve the maximum overshoot within desired value. Then the overall controlled system is approximated as a standard second-order system that prompts to design the Posicast controller in the last step.

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