

# Design of Guided Rocket Autopilot Based on Frequency Domain Method

Yong-chao Chen, Xin-bao Gao, Min-Gao, Tian-peng Li

**Abstract**—The autopilot has been added in the guided and control system of guided rocket to improve the damping characteristics of rocket and the response speed of control system. Based on the frequency domain characteristic, the inner loop and outside loop has been designed, the influence of rudder inertia and rate gyroscope inertia on the dynamic characteristics of autopilot has been discussed, and the compensation action of corrective network has been analyzed. The results show that the disadvantage of guided rocket as underdamping can be solved by autopilot effectively, and the character of control process can be improved by autopilot.

**Index Terms**—autopilot, guided rocket, frequency domain method

## I. INTRODUCTION

**D**URING the flying of guided rocket, the pitch maneuver and yaw maneuver of guided rocket will be done, which caused by the deflecting of rudders, with the requirement from control system. The inertial measuring units such as accelerometer and gyroscope are added to the guided rocket, and the output information of these inertial measuring units compos the feedback of control system. In general, the control system loop, which composed by projectile dynamic, actuator, sense organ and controller, is defined as autopilot [1]. The autopilot can be divided into three classes: pitch autopilot, yaw autopilot and roll autopilot [2].

The autopilot is an important part of control system, and the loop composed by autopilot and projectile is defined as stable control system or stable loop. In the stable control system, the autopilot is controller, and the projectile is controlled plant. The functions of autopilot are stabilize the rocket's attitude dynamic around the mass center, and control the flight of guided rocket accurately and fleetly with the guided dictate[3]. In this paper, the design philosophies of autopilot are put forward based on the fabric and function of autopilot, and one guided rocket autopilot has been designed.

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## II. DESIGN PHILOSOPHIES OF GUIDED ROCKET AUTOPILOT

In general, the guided rocket is underdamping, and the rocket is oscillating for external disturbance and without autopilot. Then the attack angle will be overlarded, the guided rocket is likely to instability or stall, and the control system can't track the control instruction. Therefore, the main function of autopilot is to improve the damping characteristic of guided rocket control system, and stabilize the flight of guided rocket [4].

The rudder and rate gyroscope are the control component of guided control system, the rocket is the controlled plant, therefor, the inherent frequency of rocket and the bandwidth of control component should be analyzed together. When the bandwidth of rudder is five times higher than the inherent frequency of rocket, the cut-off frequency between rudder's both ends is two or three times higher than the inherent frequency of rocket without corrective network, and the stability of control system is guaranteed [5]. In addition, the corrective network should be design, except the design of dynamic gain coefficient and static gain coefficient, to insure the stability and speedability of control system is enough. In theory, the corrective network can be designed on every characteristic point, and the corrective networks are switched on different characteristic point of flight, but the fixed corrective network is adopted to reduce the complexity of control system. And the corrective network is designed on the point with maximal intrinsic frequency to improve the adaptability of corrective network to other characteristic points.

## III. DESIGN OF GUIDED ROCKET AUTOPILOT

The object of this study is one active guided rocket, and the pith channel autopilot is designed.

### A. Structure of guided rocket autopilot

The structure of guided rocket autopilot is shown in Fig. 1. Where  $G_f(s)$  represents rudder,  $G_g(s)$  represents rate gyroscope.

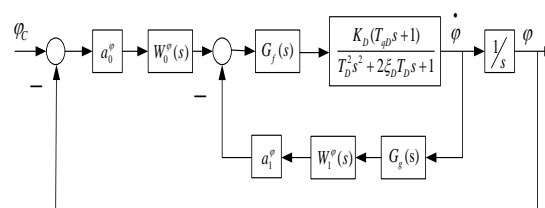


Fig. 1 Structure of guided rocket autopilot

As shown in Fig. 1, the control law of pitch channel attitude can be represented as

$$\delta_\varphi = a_0^\varphi (\varphi_c - \varphi) W_0^\varphi(s) + a_1^\varphi \dot{\varphi} W_1^\varphi(s)$$

Where,  $\varphi_c$  is the inputted pitch attitude angle control signal,  $\dot{\varphi}$  is the pitch attitude angle rate,  $a_0^\varphi$  and  $a_1^\varphi$  are the dynamic gain coefficient and static gain coefficient which need to be designed,  $W_1^\varphi(s)$  and  $W_0^\varphi(s)$  are the corrective network which need to be designed.

The transfer function of rudder is  $G_f(s) = \omega_f^2 / (S^2 + 2\xi_f \omega_f S + \omega_f^2)$ , where,  $\omega_f = 62.8 \text{ rad/s}$ ,  $\xi_f = 0.7$ .

The transfer function of rate gyroscope is  $G_g(s) = \omega_g^2 / (S^2 + 2\xi_g \omega_g S + \omega_g^2)$ , where,  $\omega_g = 200 \text{ rad/s}$ ,  $\xi_g = 0.6$ .

The transfer function of rudder's deflection angle to pitch attitude angle is  $G_\delta^\varphi(s) = K_D(T_{qD}S + 1) / (T_D^2 S^2 + 2\xi_D T_D S + 1)$ .

### B. Selection of characteristic point

The freeze coefficient method is used in the design of guided rocket autopilot usually [3]. The element of this method is that designs the control parameter on representative characteristic point first, and then tests the control characteristic by trajectory simulation. Therefore, the selection of characteristic point is important during the design of autopilot.

There are many representative characteristic points, such as staling control point, tail-off point, maximum dynamic pressure point, and minimum dynamic pressure point. The tail-off point is studied in this paper as an example, and the pitch channel autopilot is designed. As shown in Tab. 1, the parameter values of autopilot are computed with the aerodynamic parameters on tail-off point. The computing method has been study in [6].

TABLE 1

PARAMETER VALUES OF GUIDED ROCKET AUTOPILOT ON TAIL-OFF POINT

Parameter	Value
$K_D$ (1/s)	1.6461
$T_{qD}$ (s)	0.7141
$T_D$ (s)	0.0621
$\xi_D$	0.0930
$\omega_D$ (rad/s)	16.10

### C. Design of guided rocket autopilot inner loop

The guided rocket autopilot inner loop is shown in Fig. 2. The dynamic gain coefficient  $a_1^\varphi$  and corrective network  $W_1^\varphi(s)$  should be design.

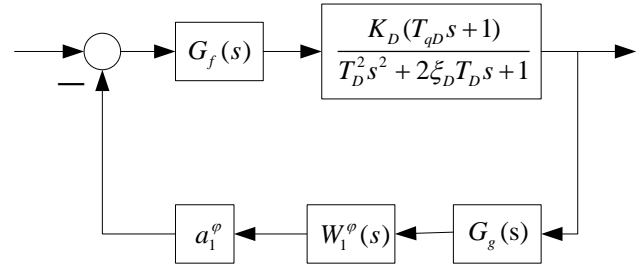


Fig. 2 Inner loop of guided rocket autopilot

In preliminary design, the rudder and rate gyroscope are regard as non-inertia link, and  $G_f(s) = 1$ ,  $G_g(s) = 1$ . Therefore, the close loop transfer function can be represented as

$$\begin{cases} \bar{G}_\delta^\varphi(s) = \frac{\bar{K}_D(T_{qD}S + 1)}{\bar{T}_D^2 S^2 + 2\bar{\xi}_D \bar{T}_D S + 1} \\ \bar{K}_D = \bar{G}_\delta^\varphi(s)|_{s=0} = K_D / (1 + a_1^\varphi K_D) \\ \bar{T}_D = T_D / \sqrt{1 + a_1^\varphi K_D} \\ \bar{\xi}_D = (2\xi_D T_D + a_1^\varphi K_D T_{qD}) / 2T_D \sqrt{1 + a_1^\varphi K_D} \end{cases} \quad (1)$$

As  $a_1^\varphi K_D \ll 1$ , so  $\bar{\xi}_D \approx \xi_D + a_1^\varphi K_D T_{qD} / 2T_D$ , and it can be deduced that:

$$a_1^\varphi \approx 2T_D (\bar{\xi}_D - \xi_D) / K_D T_{qD} \quad (2)$$

Automatic control theory indicates that the inner loop performance is favorable with  $\bar{\xi}_D \approx 0.707$ . Then it can be obtained that  $a_1^\varphi = 0.064875$ ,  $\bar{K}_D = 1.4602$ ,  $\bar{T}_D = 0.0589$ ,  $\bar{\xi}_D = 0.67203$ .

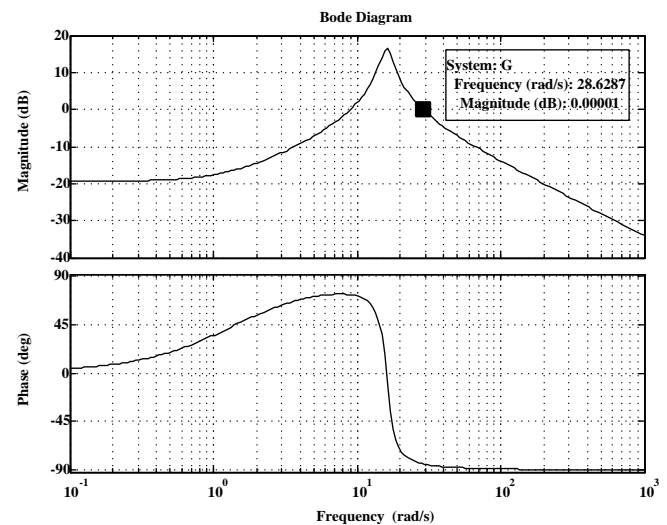


Fig. 3 Bode curves of open loop transfer function

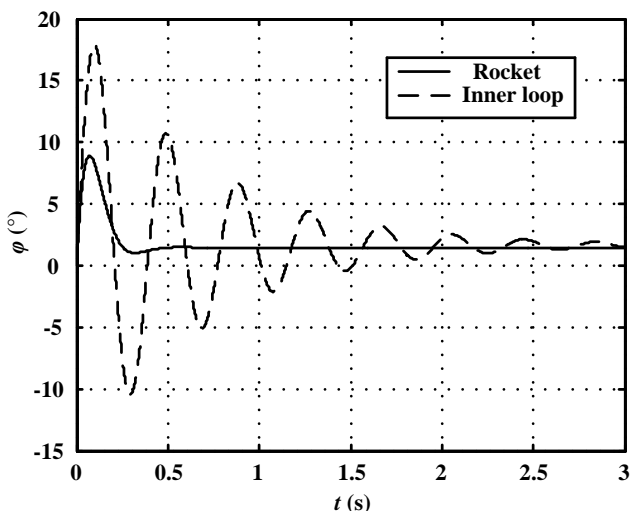


Fig. 4 Response curves of unit step input

The Bode curves of open loop transfer function are shown in Fig. 3. It can be seen that the phase margin of inner loop is 95.9deg, the cut-off frequency is 28.6287rad/s, and the inner loop is stabilized. The unit step response curves of inner loop and rocket are shown in Fig. 4. It can be seen that, the stable tendency of inner loop is better than it of rocket.

The Bode curves of open loop transfer function and the unit step response curves of inner loop are shown in Fig. 5 and Fig. 6 respectively, when the transfer functions of rudder and rate gyroscope are added in inner loop.

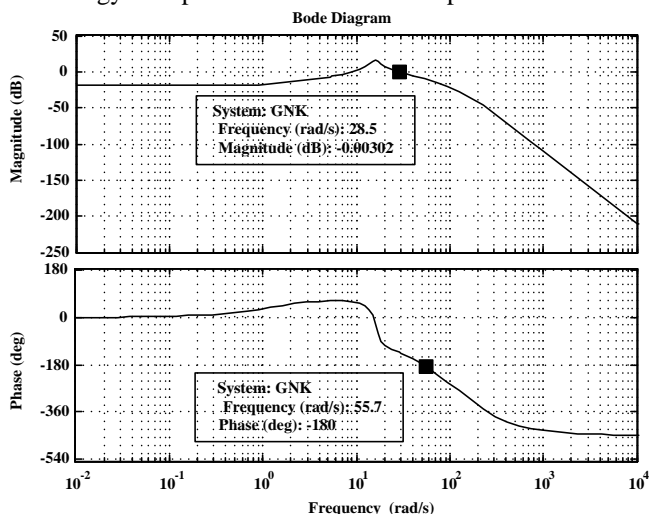


Fig. 5 Bode curves of open loop transfer function

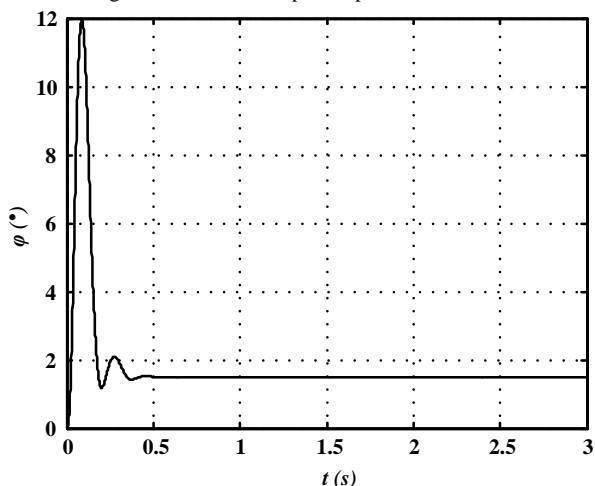


Fig. 6 Response curves of unit step input

It can be seen that the dynamic quality of inner loop becomes poor, there are vibrations in response and the adjustment time is about 0.5s. As shown in Fig.5, the magnitude margin of inner loop is 8.77dB, the phase margin of inner loop is 47deg, and the cut-off frequency is 28.5rad /s. In order to add the phase margin and improve the cut-off frequency, the corrective network should be designed and added in the feedback channel. The function of corrective network is compensate the inertia of rudder and rate gyroscope.

There are many kinds of corrective network, such as lead compensation, lag compensation, lead and lag compensation, lag and lead compensation. The lead and lag compensation is adopted in this inner loop. The form of lead and lag compensation is

$$W_1^{\phi}(s) = \frac{(T_1s+1)(T_3s+1)}{(T_2s+1)(T_4s+1)}$$

With repeated calculation, the parameter of corrective network, which will produce a good compensation effect, is  $T_1 = 30$ ,  $T_2 = 300$ ,  $T_3 = 3000$ ,  $T_4 = 3000$ .

The Bode curves of open loop transfer function and the unit step response curves of inner loop are shown in Fig. 7 and Fig. 8 respectively, when the corrective network are added in inner loop.

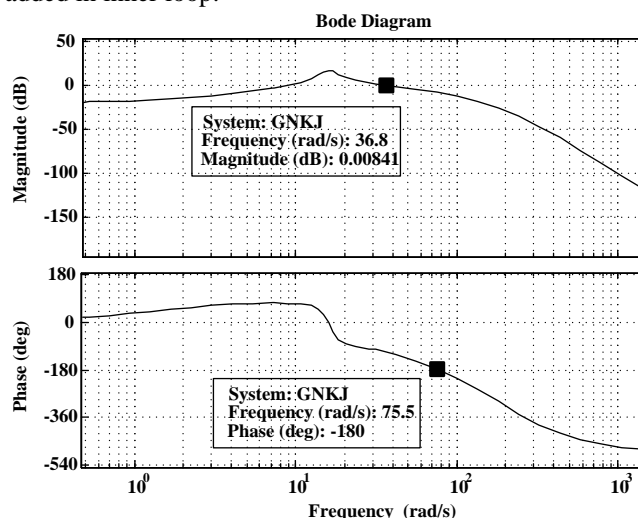


Fig. 7 Bode curves of open loop transfer function

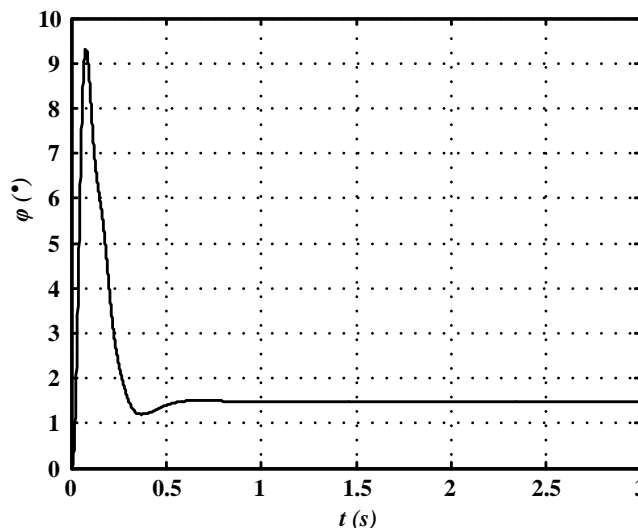


Fig. 8 Response curves of unit step input

It can be seen that the dynamic quality of inner loop becomes better, the number of vibrations is reduced, the adjustment time is 0.4s, and the phase margin of inner loop and the cut-off frequency are improved. The magnitude margin is 66deg, the cut-off frequency is 36.8rad/s.

D. Design of guided rocket autopilot outside loop

The static gain coefficient  $a_0^o$  and corrective network  $W_0^o(s)$  should be design.

The static gain coefficient  $a_0^o$  can obtained with (3). The estimate method has been study in reference [5].

$$a_0^o \approx 2.5a_2 / a_3 \quad (3)$$

With estimate and repeated simulation,  $a_0^o$  is designed as 2. Ignore the inertia of rudder and rate gyroscope, the Bode curves of open loop transfer function of outside loop is shown in Fig. 9, and the unit step response curve of outside loop is shown in Fig. 10.

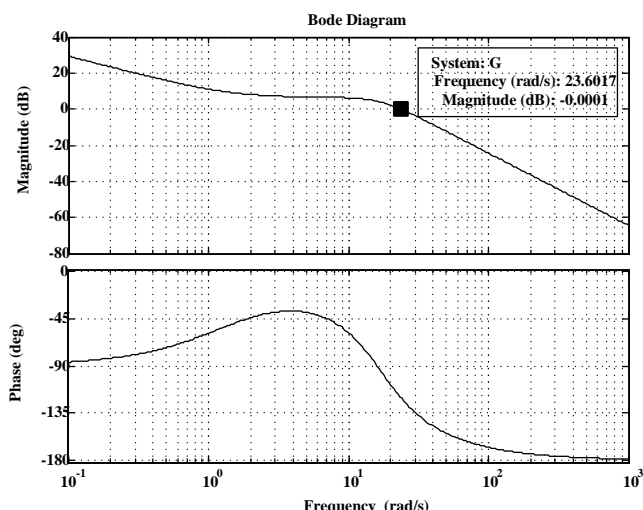


Fig. 9 Bode curves of open loop transfer function

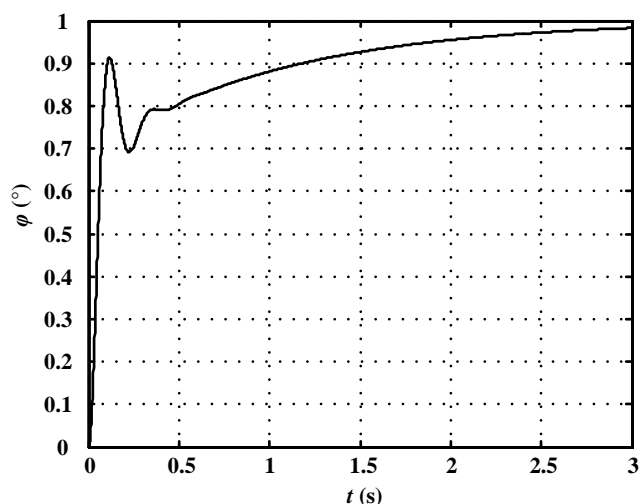


Fig. 10 Response curve of unit step input

As shown in Fig. 9, the phase margin of outside loop is 60.0919deg, the cut-off frequency is 23.6017rad/s, and the outside loop is stabilized. As shown in Fig. 10, the rise time

of unit step response is 0.2s and the adjustment time is 1.9s.

Consider the inertia of rudder and rate gyroscope, the Bode curves of open loop transfer function of outside loop is shown in Fig. 11, and the unit step response curve of outside loop is shown in Fig. 12.

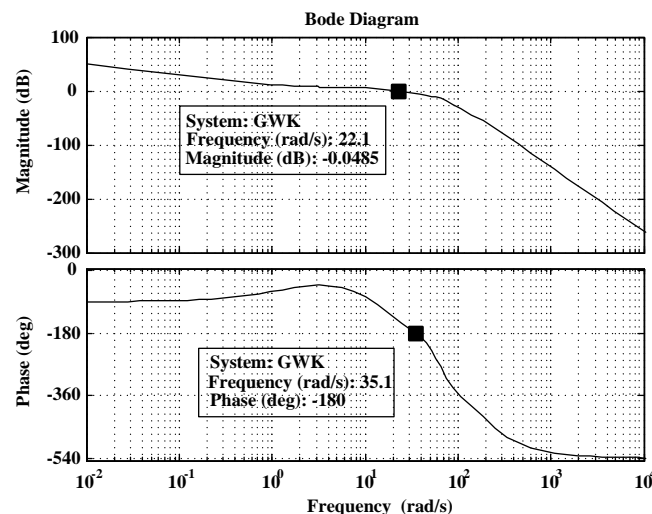


Fig. 11 Bode curves of open loop transfer function

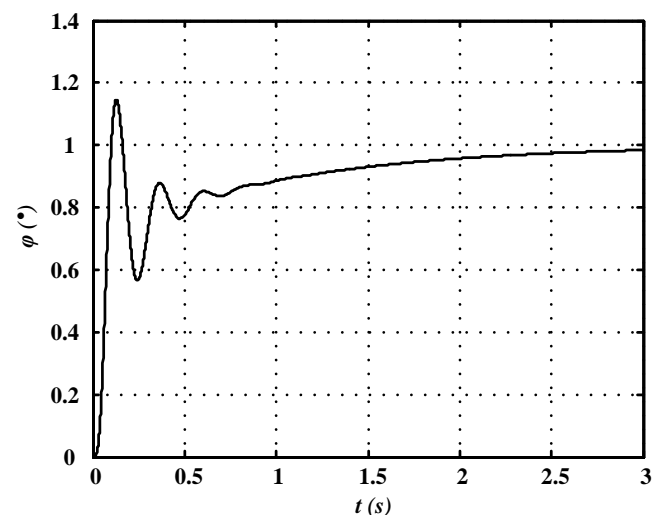


Fig. 12 Response curve of unit step input

It can be seen that the dynamic quality of outside loop becomes poor, and there are vibration in transitional processes. The adjustment time is 2.3s with the excitation of step signal. The magnitude margin of outside loop is 5.58dB, the phase margin of outside loop is 39deg, and the cut-off frequency is 23.6017rad/s. The stability margin is defective, and the corrective network should be designed to improve the stability of control system. The form of corrective network is:

$$W_0^o(s) = \frac{(T_1s + 1)(T_3s + 1)}{(T_2s + 1)(T_4s + 1)}$$

With repeated calculation, the parameter of corrective network, which will produce a good compensation effect, is  $T_1 = 35$ ,  $T_2 = 13$ ,  $T_3 = 35$ ,  $T_4 = 94$  ;

The Bode curves of open loop transfer function and the unit step response curves of inner loop are shown in Fig. 13 and Fig. 14 respectively, when the corrective network are added in outside loop.

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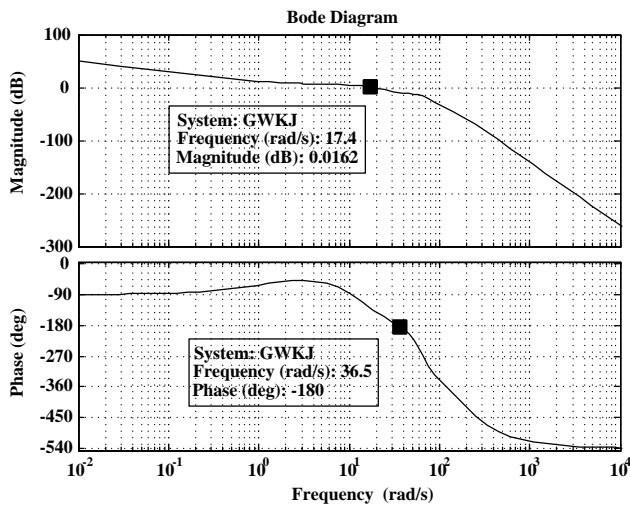


Fig. 13 Bode curves of open loop transfer function

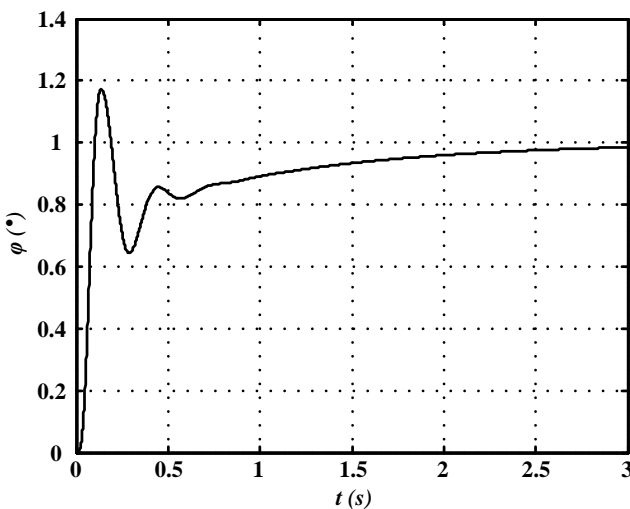


Fig. 14 Response curve of unit step input

It can be seen that the dynamic quality of outside loop becomes better, the response curve is smooth, and the adjustment time is 1.5s with the excitation of step signal. The magnitude margin of outside loop is 9.26dB, the phase margin of outside loop is 49deg, and the cut-off frequency is 17.4rad/s, and the stability of control system is improved.

IV. CONCLUSION

From the unit step response curves of inner loop and rocket, it can be seen that the damping coefficient is improved from actual value 0.093 to equivalence value 0.67203 with the function of velocity feedback loop, the overshoot is restrained, and the adjustment time is cut down. From the unit step response curves of outside loop, it can be seen that the overshoot of attitude angle is restrained with the function of angle feedback loop. From the Bode curves of open loop transfer function with corrective network and without corrective network, it can be seen that the dynamic quality of control system becomes poor with the effect of rudder and rate gyroscope, but it can be improved by the design of corrective network and the improved control system has a better stability.