# Attitude Dynamics of Spacecraft with Control by Relocatable Internal Position of Mass Center

Anton V. Doroshin, *Member, IAENG*

***Abstract*—In this paper the attitude dynamics of a spacecraft (SC) is considered under the control by the geometrical relocation of the internal position of the center of mass relatively the SC-frame. This relocation can, firstly, change the SC inertia tensor and, secondly, change the lever of force at the creation of the control torque from the jet thrust of the SC reaction-propulsion unit; so the SC attitude dynamics is complex and nonstationary. The internal relocation of the mass center is realized by moving the internal mass inside the SC-frame, and then the SC attitude control can be constructed basing on this mass internal motion.**

**In this work the displacements of the internal mass are formed by the control system proportionally to the components of the angular velocity of the SC. In this case the dynamics can have the regimes with non-trivial effects, including Shilnikov's attractors.**

***Index Terms*—spacecraft, attitude, control, internal movable mass, Shilnikov's attractor**

## I. Introduction

This work is dedicated to an investigation of nonlinear aspects of attitude dynamics of spacecraft (SC) with the geometrically relocatable internal position of the center of mass relatively the SC frame. The internal relocation of the mass center (fig.1) is realized by moving the internal mass **m** inside the SC-frame. This relocation can, firstly, change the SC inertia tensor and, secondly, change the lever of force at the creation of the control torque from the jet thrust (**P**) of the SC reaction-propulsion unit. Basing on this dynamical effects it is possible to build the control system for attitude control of SC relatively the inertial frame *OXYZ*, where the origin *O* corresponds to the point coincided with the actual center of mass of complete system (the main SC-frame and the internal mass).

Let us consider the motion of the SC with moving internal mass at its controlled displacements proportionally to the SC angular velocity components.

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A. V. Doroshin is with the Space Engineering Department (Division of Flight Dynamics and Control Systems), Samara National Research University (SSAU, 34, Moskovskoe Shosse str., Samara, 443086, Russia; e-mail: doran@inbox.ru; doroshin@ssau.ru; phone/fax: +7 846 335-18-63); IAENG Member No: 110131.

In is well known, the attitude dynamics of SC (rigid bodies with internal degrees of freedom) can be complex and nonstationary, and the phase space of the system can collect the complex irregular dynamical objects, e.g. strange chaotic attractors, chaotic homo/heteroclinic orbits. These nonlinear aspects are very interesting from the mathematical point of view [1-27], and, moreover, investigated properties of nonlinear phenomena can be used to design new effective control systems. Therefore, discovering and studying nonlinear irregular objects is the main goal of the work.

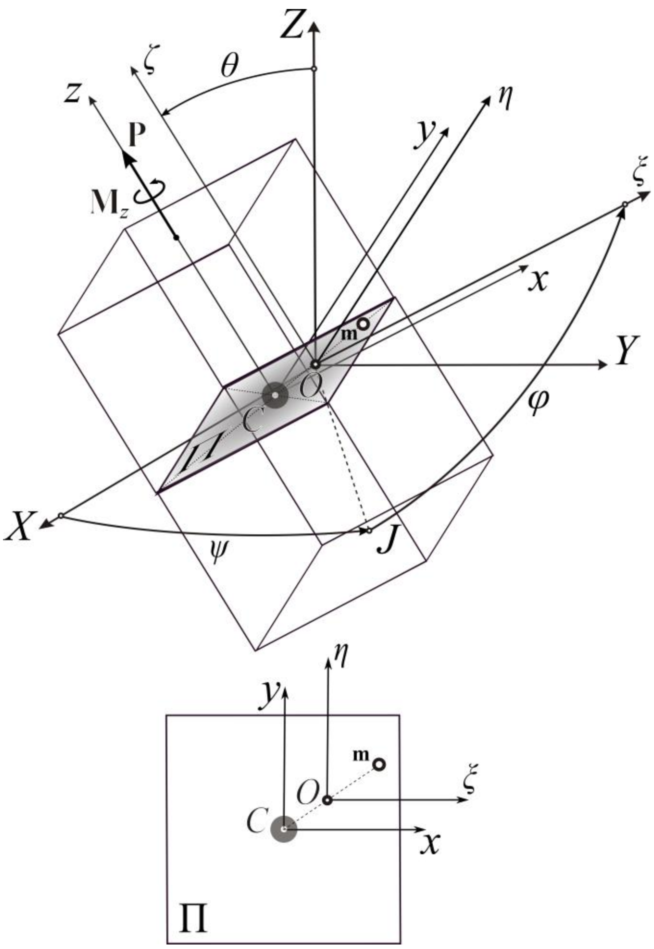


Fig. 1. The spacecraft with moving internal mass and corresponding coordinates systems

In the next sections is described the corresponding mechanical and mathematical models of the motion of SC with the relocatable internal mass. Also the numerical modeling results are presented, including the emergence of well-known Shilnikov's attractors [1-3] in the phase space.

## II. The Mechanical and Mathematical Models

Let us consider the attitude dynamics of SC which consists from the main body-frame (with its own mass center *C* and mass *M*) and internal moving mass **m**, which can relocate its position inside the main SC body only along plane Π (fig.1). The moving mass-point **m** in technical sense can correspond to the mass center of internal weights of a system of special regulators or multifunctional equipment which has technical/constructional opportunities to implement the controlled planar displacements relative the body-frame *Cxyz* depending on time ($x=x(t)$, $y=y(t)$). The mass center of complete system *O* has the following values of coordinates in the body-frame:

$$x_O(t) = mx(t)/(m+M) = \mu x(t); \quad y_O(t) = \mu y(t) \quad (1)$$

where $\mu = m/(m+M)$. Then it is possible to involve the main central connected coordinates frame $O\xi\eta\zeta$ of the complete mechanical system (fig.1) with the origin *O* in the system mass center and with axes parallel to axes of the body-frame *Cxyz*; and it is clear, that

$$\xi = x - x_O = (1-\mu)x; \quad \eta = y - y_O = (1-\mu)y; \quad \zeta = z \quad (2)$$

Assume that the inertia tensor of the main SC body in its own connected coordinates system *Cxyz* has the general diagonal form $\mathbf{I}_{xyz}=\mathrm{diag}[A_b, B_b, C_b]$. Taking into view the displacements of general axes, the inertia tensor of the SC body in the coordinates system $O\xi\eta\zeta$ will have the form which is depended on time (due to dependencies (1)):

$$\mathbf{I} = \mathbf{I}(t) = \begin{bmatrix} A_b + My_O^2 & -Mx_O y_O & 0 \\ -Mx_O y_O & B_b + Mx_O^2 & 0 \\ 0 & 0 & C_b + M\left(x_O^2 + y_O^2\right) \end{bmatrix} \quad (3)$$

Then the angular momentum of the SC-frame in connected coordinates system $O\xi\eta\zeta$ can be written

$$\mathbf{K}_{body} = \mathbf{I} \cdot \boldsymbol{\omega} \quad (4)$$

where $\boldsymbol{\omega} = [p,q,r]^T$ is the vector of the absolute angular velocity of the SC in projections on the connected coordinates system $O\xi\eta\zeta$.

The angular momentum of the moving mass relative the point *O* in projections on the axes $O\xi\eta\zeta$ can be written as follows:

$$\mathbf{K}_m = \mathbf{K}_m^e + \mathbf{K}_m^r;$$

$$\mathbf{K}_m^e = \begin{bmatrix} m\eta^2 & -m\xi\eta & 0 \\ -m\xi\eta & m\xi^2 & 0 \\ 0 & 0 & m\left(\eta^2+\xi^2\right) \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad (5)$$

$$\mathbf{K}_m^r = \left[0,\ 0,\ m\left(\dot{\eta}\xi - \dot{\xi}\eta\right)\right]^T$$

where $\mathbf{K}_m^e$ corresponds to the external part of the motion (the angular motion of the mass **m** around the point *O* as the part of the “frozen” SC-frame); and $\mathbf{K}_m^r$ is the relative angular momentum component of the “unfrozen” point (relatively the body).

Then the dynamical equations of the system angular motion follow from the law of the angular momentum changing, written with the help of the local derivation in the connected coordinates system $O\xi\eta\zeta$:

$$\frac{\tilde{d}}{dt}\left(\mathbf{K}_{body} + \mathbf{K}_m\right) + \boldsymbol{\omega} \times \left(\mathbf{K}_{body} + \mathbf{K}_m\right) = \mathbf{M}_O^e \quad (6)$$

where $\mathbf{M}_O^e$ is the vector of the external torques.

Let us consider the SC angular motion under the action only the jet thrust **P** and small spin-up torque $\mathbf{M}_z$ forming by the SC reaction-propulsion unit (fig.1). Assume that the modules of the force *P* and the torque $M_z$ are constant and quite small (the smallness of these modules allows to consider the SC as the system with constant mass, without considering the change of mass of the actuating medium in the propulsion unit). In this case the vector of external torques in projections on $O\xi\eta\zeta$ has the form:

$$\mathbf{M}_O^e = \left[-y_O P,\ x_O P,\ M_z\right]^T \quad (7)$$

Now for constructing the attitude control system it is needed to define the control laws for the *x*-*y*-displacements of the internal mass-point **m**. In this work the linear form of the feedback control system is selected, which provides for the internal mass-point the following displacements-dependencies (on *p*, *q*, *r* components):

$$\begin{cases} x = x(t) = c_p^x p(t) + c_q^x q(t) + c_r^x r(t) + c_0^x; \\ y = y(t) = c_p^y p(t) + c_q^y q(t) + c_r^y r(t) + c_0^y, \end{cases} \quad (8)$$

where $c_i^j$ are constants $\left(i = \{p,q,r,0\},\ j = \{x,y\}\right)$.

To describe the angular position of the SC in the inertial space *OXYZ* the well-known Euler’s kinematical equations can be added:

$$\begin{cases} \dot{\theta} = p\cos\varphi - q\sin\varphi; \\ \dot{\psi} = (p\sin\varphi + q\cos\varphi)/\sin\theta; \\ \dot{\varphi} = r - \mathrm{ctg}\,\theta(p\sin\varphi + q\cos\varphi) \end{cases} \quad (9)$$

So, the dynamical equations (6) together with the control laws (8) and the links (1), (2), and with kinematical equations (9) form the complete dynamical system for the modeling the attitude dynamics of the SC with the feedback control of the relocatable position of the center of mass.

The substantial simplification of the equations can be fulfilled at the assumption of the negligibly small relative angular momentum of the moving internal mass (in comparison with sum of the external angular momentum of the mass-point and the angular momentum of the main body), i.e. $\left|\mathbf{K}_m^r\right| << \left|\mathbf{K}_{body} + \mathbf{K}_m^e\right|$. And then it is possible to move the corresponding terms to the right part of dynamical equations and to consider it as a small perturbing torque. Moreover, in this work the following extreme simplification is taken:

$$\mathbf{K}_m^r \cong 0 \quad (10)$$

In spite of the simplification (10) the equations (6) in the scalar form remain cumbersome and implicit, therefore we do not present the reducing results.

In the next section the corresponding integration results of the equations (6) with control laws (8) at the simplification (10) are shown. As it will be numerically verified, this dynamical system has very interesting dynamical behavior, and, among other things, it contains Shilnikov’s attractors.

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## III. Modeling Results

Basing on the obtained in the previous section mathematical model we can realize numerical experiments with the integration dynamical equations (6) at different sets of the system parameters, initial conditions, and coefficients of the control laws (8).

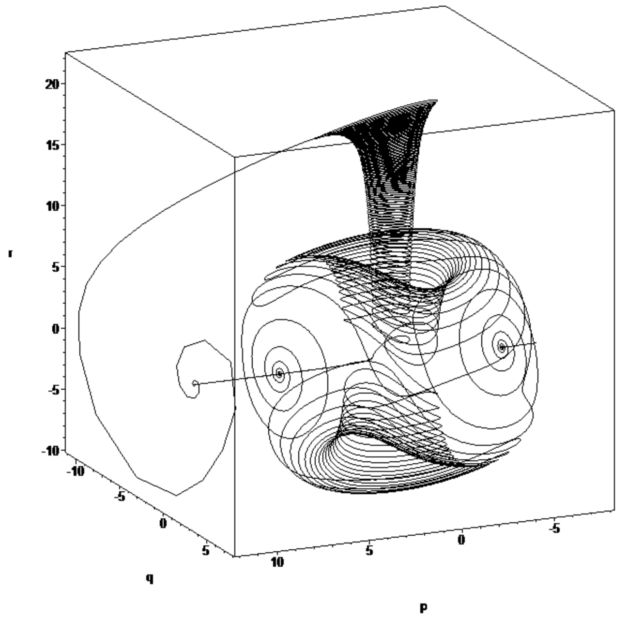
### A. *The Case A*

Let us to present the integration results (fig.2, fig.3) at the parameters and coefficients (tabl.A) which quite correspond to the class of small SC (micro-SC).

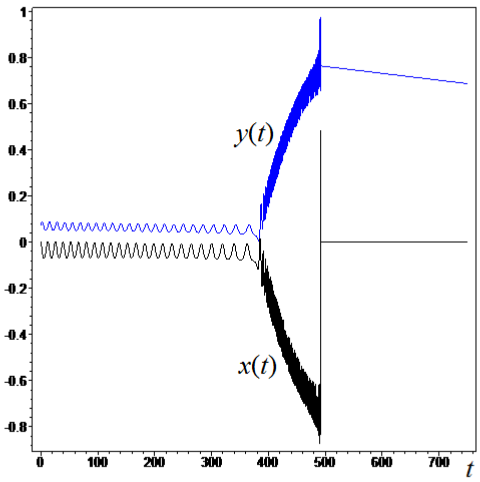
At the fig.2-a we can see the complex phase-trajectory (in the framework of the classical mechanic it is called as “polhode”), which evolves in the time and proceeds to the Shilnikov’s attractor depicted separately at the fig.3-a. To realize this phase trajectory the control system must generate (basing on the p-q-r-feedback) as the result the relative motion of the internal moving mass (fig.2-b), that also evolves to the repeated cycles (fig.3-b) along the Shilnikov’s attractor (fig.3-a). Also it is possible to indicate the interesting intermediate spiral section of the attractor.

TABLE A
The SC control parameters

| Inertia moments [kg·m²], Mass [kg], Force [N], Torque [N·m] | | Control coefficients $c_i^j$ [m s] | | Initial angular velocities [1/s] and angles [rad] | |
|---|---|---|---|---|---|
| $A_b$ | 8 | $c_p^x$ | 0.00025 | $p_0$ | 1.0 |
| $B_b$ | 6 | $c_q^x$ | 0.0625 | | |
| $C_b$ | 4 | $c_r^x$ | -0.0375 | | |
| $M$ | 60 | $c_0^x$ | 0 | $q_0$ | 0.0 |
| $m$ | 6 | $c_p^y$ | 0.075 | | |
| $P$ | 1 | $c_q^y$ | 0.00025 | | |
| $M_z$ | 1 | $c_r^y$ | 0.0375 | $r_0$ | 0.0 |
| $\mu$ | 0.09 | $c_0^y$ | 0 | | |

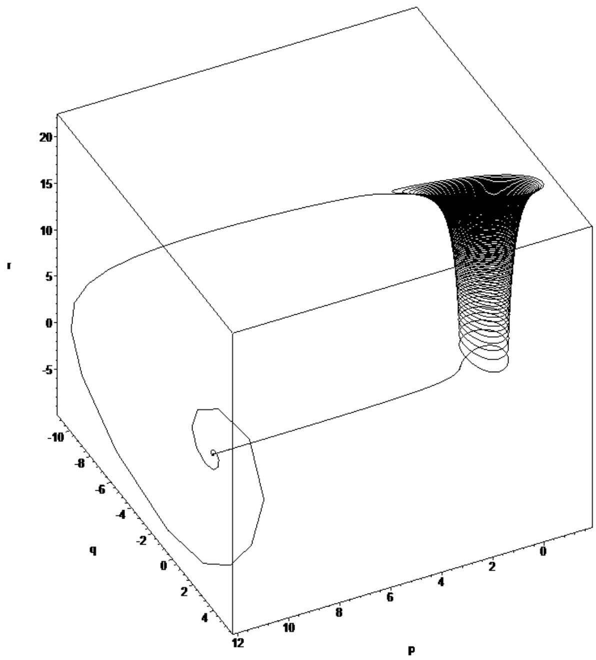



(a)




(b)

Fig. 2. The polhode/trajectory (a) in the phase space {p, q, r} and the time-history (b) for coordinates of the moving internal mass

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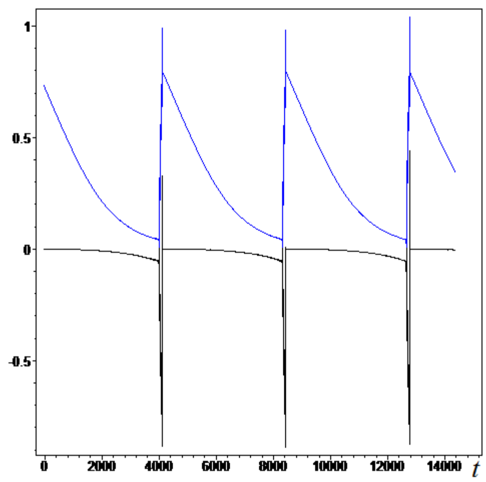


(b)

Fig. 3. The Shilnikov’s attractor (a) in the phase space {p, q, r} and the time-history (b) for coordinates of the moving internal mass

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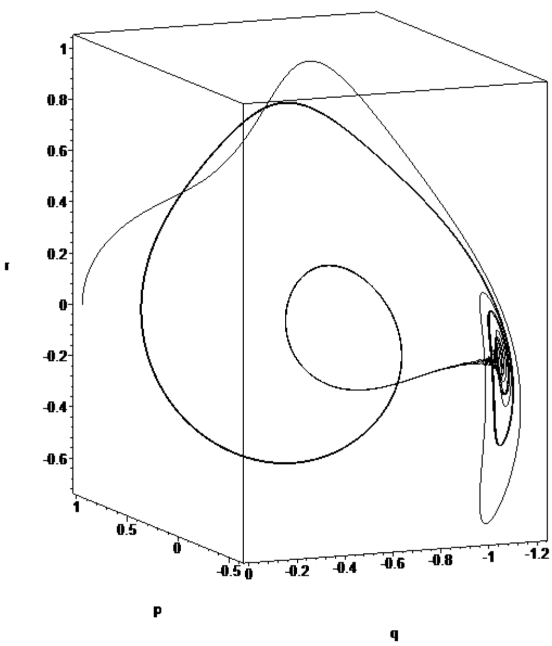
### B. *The Case B*

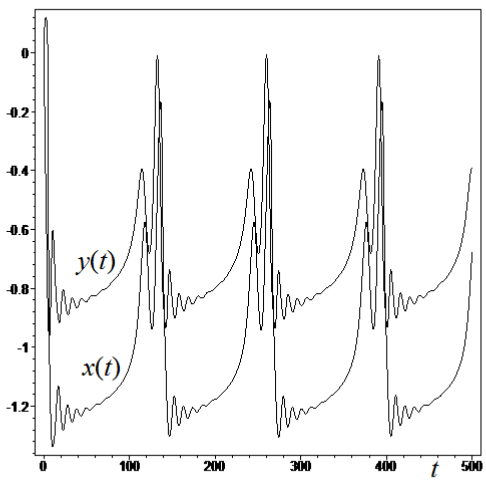
The second case of the Shilnikov's attractor (fig.4) can be initiated in the phase space at the parameters (tabl.B).

TABLE B
The SC control parameters

| INERTIA MOMENTS [kg·m²], MASS [kg], FORCE [N], TORQUE [N·m] | | CONTROL COEFFICIENTS $c_i^j$ [m s] | | INITIAL ANGULAR VELOCITIES [1/S] AND ANGLES [RAD] | |
|---|---|---|---|---|---|
| $A_b$ | 8 | $c_p^x$ | 0.0375 | $p_0$ | 1.0 |
| $B_b$ | 6 | $c_q^x$ | 0.75 | | |
| $C_b$ | 4 | $c_r^x$ | -0.375 | | |
| $M$ | 60 | $c_0^x$ | -0.0373 | $q_0$ | 0.0 |
| $m$ | 6 | $c_p^y$ | 0.75 | | |
| $P$ | 1 | $c_q^y$ | 0.0375 | | |
| $M_z$ | 1 | $c_r^y$ | 0.375 | $r_0$ | 0.0 |
| $\mu$ | 0.09 | $c_0^y$ | -0.75 | | |




(a)




(b)

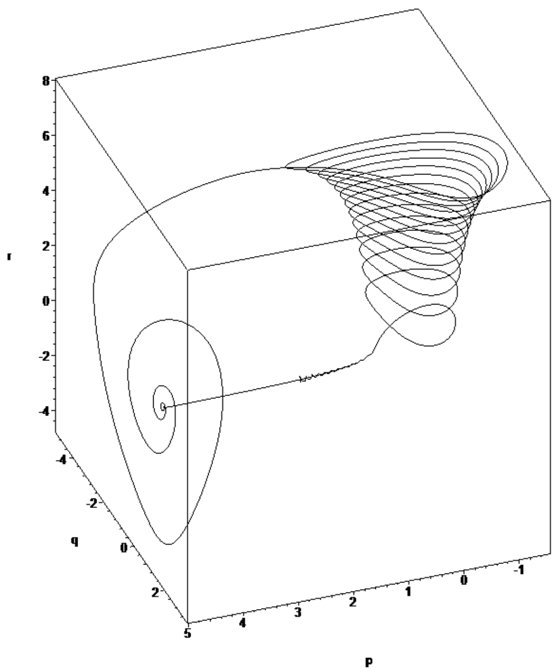
Fig. 4. The Shilnikov's attractor (a) in the phase space {p, q, r} and the time-history (b) for coordinates of the moving internal mass

### C. *The Case C*

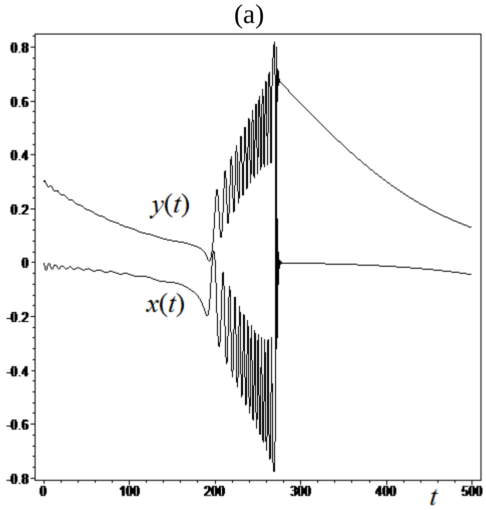
The additional case (tabl.C) of the dynamics with the Shilnikov's attractor (fig.5) is possible.

TABLE C
The SC control parameters

| INERTIA MOMENTS [kg·m²], MASS [kg], FORCE [N], TORQUE [N·m] | | CONTROL COEFFICIENTS $c_i^j$ [m s] | | INITIAL ANGULAR VELOCITIES [1/S] AND ANGLES [RAD] | |
|---|---|---|---|---|---|
| $A_b$ | 1.5 | $c_p^x$ | 0.0005 | $p_0$ | 0.2 |
| $B_b$ | 1.2 | $c_q^x$ | 0.125 | | |
| $C_b$ | 1 | $c_r^x$ | -0.075 | | |
| $M$ | 10 | $c_0^x$ | 0 | $q_0$ | 0.3 |
| $m$ | 1 | $c_p^y$ | 0.15 | | |
| $P$ | 1 | $c_q^y$ | 0.0005 | | |
| $M_z$ | 0.1 | $c_r^y$ | 0.075 | $r_0$ | 0.6 |
| $\mu$ | 0.09 | $c_0^y$ | 0 | | |




(a)




(b)

Fig. 5. The Shilnikov's attractor (a) in the phase space {p, q, r} and the time-history (b) for coordinates of the moving internal mass

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Also the strong dissipative regimes can realize (fig.6).

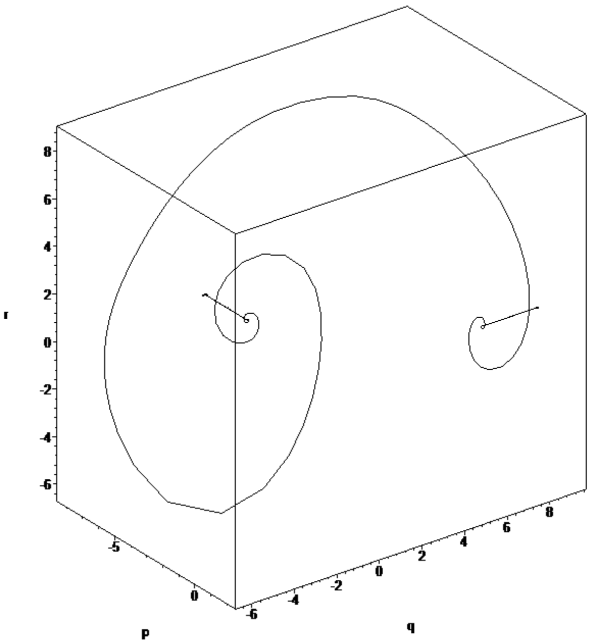


Fig. 6. The dissipative possible regime

So, the modeling results clearly show possibility of using the attitude control by the displacement of the mass center due to moving the internal mass, even in the cases of the intentional creating complex regimes with Shilnikov's attractors in SC dynamics.

## IV. CONCLUSION

In the paper the SC attitude dynamics was considered under the control by the geometrical relocation of the internal position of the center of mass. This relocation fulfilled with the help of moving internal mass.

Undoubtedly, the considered control method uses the well-known dynamical scheme, which based on the dynamics of rigid bodies with internal degrees of freedom (moving parts), but the suggested way of SC attitude control implementation can be indicated as original. Concretized constructional parameters, including mass-inertia and geometrical parameters, and real intervals of dynamical parameters, certainly, must be defined/predefined in the separate self-contained research.

The dynamics of the SC is complex, and the Shilnikov's attractors can realize. This nonlinear dynamical object in its turn can generate the Smale's horseshoes and produce the corresponding dynamical chaos. The extended study of the SC with attitude control by mowing internal mass and also chaotic aspects investigation in the SC motion along the Shilnikov's attractors are the next tasks for further research.

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