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The Problem of Synthesis the Control Laws with Uncertainties in External Disturbances

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***Abstract*—In the synthesis of control systems the dynamic properties of a closed-loop system are very important. These properties are determined by the choice of parameters of the basic control law. The article focuses on the development of the feedback, based on the implementation of the optimization approach to the problem of the best compensation effect of external disturbances.**

***Index Terms*—control law, synthesis, feedback.**

I. Introduction

MODERN systems of automatic motion control of marine vehicles operate in different dynamic modes defined by the specific assignment of command signals and external disturbances acting on the control object. A natural way to ensure all the required dynamic properties is to reach a compromise on the quality of control processes in different modes.

This compromise can be achieved with the use of multi-purpose control laws with special structure [1]–[6]. In this paper the approach to their formation is adopted. The main ideas of this approach were firstly presented in [7]–[11]. The mentioned structure includes some basic parts and several additional separate items to be adjusted for an actual, sailing environment.

This article primarily discusses questions related to the mathematical formalization of marine vessels motion as control objects. Also, the paper highlights the problem of uncertainties in the assignment of external disturbances mathematical models, which is one of the major difficulties in the analysis and synthesis of control systems.

In the paper an original method for minimizing the size of the set of reactions, taking into account the additional requirements to ensure the desired degree of stability for the closed-loop system is proposed. This approach is an omnibus technique with respect to the way of determining the reactions set size, that allows us to use it for arbitrary linear control laws with a fixed structure. The main attention in this method is paid to stability and quality of the dynamic processes.

II. The Problem of Synthesis of Stabilizing Control

Let us consider the linear system of differential equations, which presents a mathematical model of the marine vessel dynamics

$$\begin{aligned} \dot{x} &= Ax + B\delta + Dw(t), \\ \dot{\delta} &= u, \\ e &= Mx, \\ y &= Cx. \end{aligned} \quad (1)$$

Here $x \in E^n$ is a state vector (i.e. the deviation from the equilibrium position), $e \in E^{k_e}$ is a vector of controlled variables, $\delta \in E^m$ is a state vector of rudders deviations, $u \in E^m$ is a vector of controls, $y \in E^k$ is a vector of measured variables, A, B, C, D, M are constant matrices of corresponding dimensions. Further, we assume that the vessel moves under the influence of external disturbances, represented by a vector function $w(t), w \in E^l$.

The essence of an analytical synthesis problem is the formation of feedback (controller) on the measured output

$$u = W_y(s)y + W_\delta(s)\delta, \quad (2)$$

written in tf-form, where $W_y(s)$ and $W_\delta(s)\delta$ are transfer matrices with fractionally rational components. The search of these matrices is carried out in a solution of the corresponding mathematical problem. At the same time we need to provide the asymptotic stability of the zero equilibrium position for a closed-loop system without disturbances. Besides their choice is aimed at achieving the desired dynamic properties of the control system.

The main problem is the absence of any information about the function $w(t)$ during the motion process. In such situation the only fact of disturbances boundedness in a certain sense is postulated.

The presence of these uncertainties in the external disturbances definition significantly complicates the design process and requires the special formalization of the synthesis problem with a focus on achieving the desired result for any choice of the function $w(t)$ from the admissible set of functions.

We assume that the external disturbance $w(t)$ affecting the motion of the closed-loop system (1), (2) is an element of the normed space $\mathfrak{R}$ with the norm $\|w\|_r$.

Let us consider the admissible set $\mathfrak{R}_{wa} \subset \mathfrak{R}$ of disturbances, defining it by the relation

$$\mathfrak{R}_{wa} = \{w(t) \in \mathfrak{R} : \|w\|_r \leq w_0\},$$

where $w_0 > 0$ is the given real number.

We assume that the space $\mathfrak{R}$ is such that it includes the reaction $e(t)$ of the closed-loop system (1), (2) to the disturbance $w(t) \in \mathfrak{R}$, which is determined uniquely. Thus, the mathematical model of this system with zero initial conditions on the state vectors x and δ determines a linear operator $L : \mathfrak{R} \to \mathfrak{R}$. At the same time the set

$$\mathfrak{R}_{ea} = L(\mathfrak{R}_{wa}) = \{e(t) \in \mathfrak{R} : e = L(w)\}$$

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we will call the set of reactions to valid disturbances.

Note that due to the asymptotic stability of the zero equilibrium position of the closed-loop system (1), (2), the reactions set will be limited by the norm of the space $\mathfrak{R}$, i.e.

$$\mathfrak{R}_{ea} \subset \widetilde{\mathfrak{R}}_{ea} = \{e(t) \in \mathfrak{R} : \|e\|_r \le e_0\},$$

where $e_0 > 0$ is the real number.

The radius e_0 of the sphere $\widetilde{\mathfrak{R}}_{ea}$ will be called the set $\mathfrak{R}_{ea}$ of reactions size to valid disturbances, if this number is determined by the condition

$$e_0 = \sup_{w \in \mathfrak{R}_{wa}} \|L(w)\|.$$

It is obviously, that for a fixed mathematical model (1) of the control object and for a fixed number w_0 the size e_0 of the reactions set is uniquely determined by the choice of the transfer matrices W_y and W_δ in the feedback. Thus, in a set of pairs $\{W_y, W_\delta\}$ the following functional is determined

$$e_0 = J_{we}\left(\{W_y, W_\delta\}\right).$$

Its value plays defining role in the analysis and design of systems with uncertain external disturbances.

As mentioned above, the influence of external disturbances to the object is negative, i.e. a well-functioning control system should to suppress them in relation to the controlled output of the closed-loop system. If the set $\mathfrak{R}_{wa}$ of admissible disturbances $w(t)$ is given, then we need to take into account this uncertainty. It should be noted, that during designing the feedback it is unknown which specific disturbance will act to the system.

Mathematical formalization of such design problems can be performed in different ways, however, in this paper we assume that the defining value that characterizes the quality of the closed-loop system (1), (2) is the size e_0 of the reactions set. The smaller the value of the functional J_{we} is, the better the control system works with uncertainties.

We need to emphasize, that for a concrete formulation of the synthesis problem, it is necessary to specify the choice of the set $\mathfrak{R}$ to which the disturbances $w(t)$ belong, and to specify the definition of the controlled variables $e(t)$.

The problem of the synthesis of the stabilizing feedback (2) for the object with the mathematical model (1) in the presence of uncertainty in the definition of external disturbances will be called the optimization problem

$$e_0 = J_{we}\left(\{W_y, W_\delta\}\right) \to \inf_{\{W_y, W_\delta\} \in \Omega_a \subset \Omega} \tag{3}$$

of the analytical search the best pair $\{W_y, W_\delta\}$ providing the minimum size of the set of reactions and the implementation of the complex of structural and dynamic requirements to a closed-loop system. Here the set Ω_a is a narrowing of the set Ω of stabilizing controllers is defined by the introduction of these requirements.

We emphasize that for a particular formulation of the synthesis problem (3), first of all, we need to specify the set $\mathfrak{R}$, which owns the disturbance $w(t)$ and the controlled variables $e(t)$.

Depending on the environmental conditions and on the choice of the operating mode of the control system, the following three cases [20] are often considered.

1) $\mathfrak{R} = L_2$ is the space of bounded to the square vector functions $w(t)$ with finite 2-norms:

$$\|w\|_r^2 = \|w\|_2^2 = \int_0^\infty w'(t)w(t)dt.$$

Note, that the square of the norm here characterizes the energy of the disturbance.

2) $\mathfrak{R} = L_\infty$ is the space of essentially bounded vector functions $w(t)$ with finite ∞-norms:

$$\|w\|_r = \|w\|_\infty = \sup_{t \in [0,\infty)} w'(t)w(t).$$

These norms describe the intensity of external influences.

3) $\mathfrak{R} = L_1$ is the space of absolutely integrable vector functions $w(t)$ with finite 1-norms:

$$\|w\|_r = \|w\|_1 = \sup_{t \in [0,\infty)} \sqrt{w'(t)w(t)}.$$

Similar spaces can be entered for the variables $w = \{w[\nu]\}$ and $e = \{e[\nu]\}$ in discrete time $\nu \in N^1$. In this case they are spaces l_2, l_∞ and l_1 respectively.

In connection with considered particular cases of normed spaces $\mathfrak{R}$ of external disturbances, let introduce into consideration the norms of linear operators L induced by them of the closed-loop systems (1), (2). These operators will be specified by mathematical models of these systems in the tf-form

$$e = H(s)w, \tag{4}$$

where $H(s)$ is the transfer matrix from input w to the output e.

In connection with the stated above spaces of disturbances in the constructive methods of modern control theory the following matrix norms that characterize the quality of the LTI-system functioning [20] are used:

1) The norm of the space H_2 whose elements are matrices $H(s)$ with dimension $\dim e \times \dim w$ and strictly correct fractional-rational components:

$$\|H\|_2 = \sqrt{\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \mathrm{tr}[H'(-s)H(s)]ds}.$$

2) The norm of the space H_∞ whose elements are matrices $H(s)$ with the same dimension and strictly correct rational components:

$$\|H\|_\infty = \max_{\omega \in [0,\infty)} \overline{\sigma}(\omega).$$

3) The norm of the space H_1 for discrete-time LTI-systems whose elements are matrices $H(z)$ with correct rational components:

$$\|H\|_1 = \sum_{\nu=0}^{\infty} \sqrt{h'[\nu]h[\nu]},$$

where $h[\nu]$ are the samples of the impulse response of the closed-loop system.

To connect the norms of disturbances and the corresponding reactions the following relations are used in practice [20]:

- for the space $\mathfrak{R} = L_2$

$$\|e\|_2 \le \|H\|_\infty \|w\|_2; \tag{5}$$

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- for the space $\mathfrak{R} = l_\infty$

$$\|e\|_\infty \leq \|H\|_1 \|w\|_\infty. \tag{6}$$

Note that there are no input disturbances $w(t)$ in the set $\mathfrak{R}$, for which there is an equality in (5), (6). But then for the spaces $\mathfrak{R} = L_2$ and $\mathfrak{R} = l_\infty$ it is easy to determine the size e_0 of the set of reactions to disturbances $w(t) \in \mathfrak{R}_{wa}$:

$$e_0 = w_0 \|H\|_\infty \text{ or } e_0 = w_0 \|H\|_1.$$

It is obviously, that for a fixed mathematical model (1) of the control object the values of the matrix norms $\|H\|_\infty$ and $\|H\|_1$ are uniquely determined by the choice of transfer matrices W_y and W_δ in the feedback equation (2). Thus, the following functionals are defined in the set Ω of pairs $\{W_y, W_\delta\}$

$$J_\infty = J_\infty\left(\{W_y, W_\delta\}\right) = \|H\|_\infty,$$

$$J_1 = J_1\left(\{W_y, W_\delta\}\right) = \|H\|_1.$$

Then the question of searching the minimum of the functional on the admissible set $\Omega_a \subset \Omega$ is equivalent to the synthesis problem (3): i.e. for the spaces $\mathfrak{R} = L_2$ and $\mathfrak{R} = l_\infty$ we have

$$J_\infty = J_\infty\left(\{W_y, W_\delta\}\right) \rightarrow \min_{\{W_y, W_\delta\} \in \Omega_a \subset \Omega}, \tag{7}$$

$$J_1 = J_1\left(\{W_y, W_\delta\}\right) \rightarrow \min_{\{W_y, W_\delta\} \in \Omega_a \subset \Omega}. \tag{8}$$

Constructive methods for solving H_∞-optimization problem (7) under the condition $\Omega_a = \Omega$ are provided in numerous works on the control theory. In particular, the spectral approach for the SISO situation is proposed in [6] in two ways.

The problem (8) under the condition $\Omega_a = \Omega$ was first discussed and solved in [20], the further development of methods of its solutions is given in the works of Granichin.

The disadvantage of these approaches is the fact that for any narrowing $\Omega_a \subset \Omega$ of the set of stabilizing feedbacks the direct application of these methods is not possible. That requires a corresponding modification of the theory and construction of computing synthesis algorithms.

We note in particular that there is another method to estimate the size $J_w e$ of the set of reactions that is not directly associated with the matrix norms. This method is given significant attention in the paper. Its main advantage is the simplicity of calculation the functional values $J_w e$ and the relative simplicity of solution of the problem (3) at least for a set Ω of stabilizing controls.

Let us consider the question of solving the synthesis problem of the stabilizing feedback (2) for the object with the mathematical model (1) in the presence of uncertainty in assignment of disturbances $w(t)$ in a particular case with the introduction of the feasible set $\Omega_a \subset \Omega$. This set of pairs $\{W_y, W_\delta\}$ of transfer matrices we will define by two requirements:

a) the feedback structure (2) is initially fixed (for example, it can be included in the asymptotic observer) with the allocation of customizable vector $h \in E^p$ of numerical parameters, i.e., the controller has the form

$$u = W_y(s,h)y + W_\delta(s,h)\delta; \tag{9}$$

b) the selection of vector h must be within the admissible set

$$\Omega_h = \left\{h \in E^p : \delta_i(h) \in C_\Delta, i = \overline{1, n_d}\right\},$$

where $\delta_i(h)$ are roots of the n_d-degree characteristic polynomial $\Delta_3(s,h)$ of the closed-loop system (1), (9). In other words, for any given vector h from the given set the spectrum of roots must be located entirely in a predetermined region C_Δ of the complex plane. As this area we will take $C_\Delta = \{s = x \pm jy \in C^1 : x \leq -\alpha_d\}$, where $\alpha_d > 0$ is a given real number that determines the stability degree of the closed-loop system.

Now let us introduce into consideration the functional J_d characterizing the size of the set $\mathfrak{R}_{ea}$ of reactions for the system (1), (9) to the admissible disturbance $w \in \mathfrak{R}_{wa}$.

The problem of parametric minimizing the size of the reactions set for the closed-loop system (1), (9) in presence of an uncertainty in the assignment of disturbances will be called finite-dimensional problem of searching the extremum

$$J_d = J_d(h) \rightarrow \min_{h \in \Omega_h} \tag{10}$$

that is in this particular case obviously equivalent to the problem of the stabilizing feedback synthesis.

III. Problem Solution

To solve this problem an original method for minimizing the size of the reactions set, taking into account the additional requirements to ensure the desired degree of stability for the closed-loop system was developed. It consists of the following steps:

1. Take any point $\gamma \in E^{n_d}$ and form an auxiliary polynomial $\Delta^*(s,\gamma)$ according to the formulae

$$\Delta^*(s,\gamma) = \Delta^*(s,\gamma) \quad \text{if} \quad n_d \quad \text{is even, and}$$

$$\Delta^*(s,\gamma) = (s + a_{d+1}(\gamma, \alpha_d))\Delta^*(s,\gamma) \quad \text{if} \quad n_d \quad \text{is odd,}$$

where $\alpha_d > 0$ is the desired degree of stability of the polynomial. Here

$$\Delta^*(s,\gamma) = \Pi_{i=1}^{d}\left(s^2 + a_i^1(\gamma, \alpha_d)s + a_i^0(\gamma, \alpha_d)\right),$$

$$d = [n_d/2], a_i^1(\gamma, \alpha_d) = 2\alpha + \gamma_{i1}^2,$$

$$a_i^0(\gamma, \alpha_d) = \alpha_d^2 + \gamma_{i1}^2 \alpha_d + \gamma_{i2}^2, \quad i = \overline{1, d}$$

$$a_{d+1}(\gamma, \alpha_d) = \gamma_{d0}^2 + \alpha_d,$$

$$\gamma = \{\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, \ldots, \gamma_{d1}, \gamma_{d2}, \gamma_{d0}\}.$$

2. Construct the system of nonlinear equations

$$Q(h) = \chi(\gamma),$$

that provides the identity $\Delta_3(s,h) \equiv \Delta^*(s,\gamma)$, which is always consistent. If its solution is not the only one, it is necessary to implement an arbitrary choice of the vector $h_c \in E^{n_c}$ of free variables in relation to this system.

3. After substituting the received vector $\varepsilon = \{\gamma, h_c\} \in E^\lambda$ in the system from the second step we will find its solution $h = h^*(\varepsilon)$.

4. Substitute the found vector $h = h^*(\varepsilon)$ of configurable parameters in the feedback (9) and find for a closed-loop system (1), (9) the corresponding value of the size of the set of reactions $J_d = J_d\left(h^*(\varepsilon)\right) = J_d^*(\varepsilon)$.

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5. Using any valid numerical method for solving the problem

$$J_d^* = J_d^*(\varepsilon) \to \min_{\varepsilon \in E^\lambda}$$

to an unconditional extremum, set a new point ε and repeating steps 3 and 4 to minimize the function $J_d^*(\varepsilon)$.

6. The process will be completed after finding a point

$$\varepsilon_0 = \arg \min_{\varepsilon \in E^\lambda} J_d^*(\varepsilon)$$

defining the vector $h_0 = h^*(\varepsilon_0)$, which is accepted as a solution of the problem (4). Required stabilizing control as a solution of the synthesis problem (2) will be represented by the equation

$$u = W_y(s, h_0)y + W_\delta(s, h_0)\delta.$$

IV. Conclusion

An original method to minimize the size of the set of reactions with the additional requirements to ensure the desired level of stability for a closed system was offered. The proposed method does not rely on a specific way to determine the size and has a universal approach that allows it to be used for arbitrary linear control law with a fixed structure. Perspective direction of research is related to the multiprogram stabilization approach [4], [10–21].

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