A New Method for Forecasting via Feedback Control Theory

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Abstract—In this paper, we present a new method for forecasting time series data. Firstly, we give a brief review of five standard statistical techniques from the literature, namely the ARIMA, Holt's method, Holt-Winters method, Decomposition method and Regression analysis. Then, we apply feedback control theory to five well-known forecasting methods. By introducing the concept of mean absolute error, different results are obtained and compared via feedback control theory. In summary, the comparison results illustrate that the new method proposed in this paper provides improved forecasting accuracy and significantly decrease the forecasting error than the existing individual forecasting methods.

Index Terms—ARIMA, Holt's Method, Decomposition Method, Regression Analysis, Feedback Control Theory.

I. INTRODUCTION

Nowadays, people often use forecasting techniques to model and predict economic activities, population growth, stocks, insurance/re-insurance, industrial and science [1]. Over the past decades, several models have been developed for time series forecasting. ARIMA is the method first introduced by Box and Jenkins and until now become the most popular models for forecasting univariate time series data. This model has been originated from the autoregressive model (AR), the moving average model (MA) and the combination of the AR and MA, the ARMA models.

In the case where seasonal components are included in this model, then the model is called as the SARIMA model. Box and Jenkins procedure that contains three main stages to build an ARIMA model, i.e. model identification, model estimation and model checking is usually used for determining the best ARIMA model for certain time series data [2]. Moreover, the exponential smoothing methods of forecasting involve a mix of adaptive levels, growth rates and seasonal effects [3]. The exponential smoothing is an extension of ES designed for trend time series [4]. The Holt-Winters method (triple exponential smoothing) takes into account both seasonal changes and trends. They provide good forecasts with imply formulations, allowing the incorporation of error, trend and seasonal component in a comprehensive manner [5-9]. Furthermore, time series regression basically relates the dependent variable to functions of time describing trend and seasonal component. It is most profitably used when the components describing the time series to be forecast remain constant overtime [10] and it application by many researchers in many areas [11-14]. Additionally, decomposition models to forecast time series that exhibit trend and seasonal effects. These models have been found

useful when the parameters describing a time series are not changing over time [10]. There are a number of papers in the literature which deal with the extrapolation of time series through the extrapolation of individual components. However, in all applications of the classical decomposition technique, the residual component after the elimination of any trend, cyclical and seasonal variations is always assumed to be a random variable with a constant variance, and is therefore exclude from the forecasting process [15]. Then, we apply feedback control theory to five well-known forecasting methods. Moreover, we demonstrate an illustrative example of these methods with real data set and compare different results via feedback control theory by introducing the concept of mean absolute error (MAE).

In this paper, a new forecasting method is developed via feedback control theory with application to real data set. The five forecasting methods namely the ARIMA, Holt's method, Holt-Winters method, Decomposition method and Regression analysis are included in the analysis and can be implemented readily in a software package. The statistical software used in this paper are SPSS and Minitab.

In the following section, we review the five modeling approaches to time series forecasting and we prefer the time series via feedback control theory. Empirical results for comparing the forecasting techniques from five real data set are illustrated in Section 3. The final section provides the conclusion.

II. METHODOLOGY

In this paper, we focus on the basic principles and modeling process of the ARIMA, Exponential smoothing, Decomposition and Regression analysis.

2.1 The ARIMA Model [10]:

Classical Box-Jenkins models are describe the **stationary** time series. A time series is stationary if the statistical properties (for instance, the mean and the variance) of time series are essentially constant through time. If n-number of values seem to fluctuate with constant variation around a constant mean μ, then it is reasonable to believe that the time series is stationary. If the n values do not fluctuate around the constant mean or do not fluctuate with constant variation, then it is reasonable to believe that the time series is **nonstationary**. In this paper, we consider only nonstationary time series. In this case where the data have trend component, we call an autoregressive integrated moving average or ARIMA (p, d, q). While, the data have seasonal component. We call SARIMA (P, D, Q) and the data have both of trend and seasonal component, we call ARIMA $(p, d, q) \times (P, D, Q)_m$. Box-Jenkins procedure that contains three mail stages to build model, i.e. model identification, model estimation and model checking for determining the best model for certain

series data[16]. Hence an ARIMA (p, d, q) can be written as

$$\hat{y}_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p}$$

$$+ \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q},$$

Moreover, the SARIMA $(P, D, Q)_m$ can be written as

$$\hat{y}_t = \theta_0 + \varphi_1 y_{t-m} + \varphi_2 y_{t-2m} + \ldots$$

$$+ \varphi_P y_{t-Pm} + \varepsilon_t - \omega_1 \varepsilon_{t-m}$$

$$- \omega_2 \varepsilon_{t-2m} - \ldots - \omega_Q \varepsilon_{t-Qm}.$$

Therefore a more general seasonal ARIMA model orders $(p, d, q) \times (P, D, Q)_m$ with period m is

$$\Phi^*(B^s)\Phi(B)(1 - B)^d (1 - B^m)^D \hat{y}_t$$
$$= \delta + \Theta^*(B^m)\Theta(B)\varepsilon_t,$$
$$\hat{y}_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p}$$
$$+ \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q}$$
$$+ \varphi_1 y_{t-m} + \varphi_2 y_{t-2m} + \ldots + \varphi_P y_{t-Pm}$$
$$- \omega_1 \varepsilon_{t-m} - \omega_2 \varepsilon_{t-2m} - \ldots - \omega_Q \varepsilon_{t-Qm}.$$

where

y_t is the actual value at time period t.

p, d, q are the autoregressive, differencing and moving average term in the ARIMA model.

P, D, Q are the autoregressive, differencing and moving average term for the seasonal part of ARIMA model.

ϕ_i, θ_j are model parameters for ARIMA term order p and q respectively $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, q$.

φ_i, ω_j are model parameters for SARIMA term order P and Q respectively; $i = 1, 2, \ldots, P$ and $j = 1, 2, \ldots, Q$.

ε_t are random errors which assumed to be independently and identically distributed with a mean of zero and a constant variance of σ^2 [17].

m are the period of seasonal (for example weekly, monthly and quarterly).

2.2 Exponential Smoothing Technique [10]:

Exponential smoothing provides a forecasting method that is most effective when the components (trend and seasonal factors) of the time series may be changing over time. Simple exponential smoothing is suitable method for time series has no trend. While, Holt's method is appropriate when both the level (ℓ_t) and the growth rate (b_t) are changing. Furthermore, Holt-Winter methods are exponential smoothing procedures for seasonal data.

2.2.1 Holt's Method:

A point forecast made in time period t for $\hat{y}_{t+\tau}$ is $\hat{y}_{t+\tau} = \ell_t + \tau b_t$ $(\tau = 1, 2, \ldots)$. The smoothing equations are as follow $\ell_t = \alpha y_t + (1 - \alpha)[\ell_{t-1} + b_{t-1}], b_t = \gamma[\ell_t - \ell_{t-1}] + (1 - \gamma) b_{t-1}$.

2.2.2 Holt-Winters Method:

Holt-Winters method is designed for time series that exhibit linear trend at least locally. The additive Holt-Winters method is used for time series with constant (additive) seasonal variation, whereas the multiplicative Holt-Winters method is used for time series with increasing (multiplicative) seasonal variation.

2.2.3 Additive Holt-Winters Method

A point forecast made in time period t for $\hat{y}_{t+\tau}$ is

$$\hat{y}_{t+\tau}(t) = l_t + \tau b_t + sn_{t+\tau-m}, \quad (\tau = 1, 2, \ldots).$$

The smoothing equations are given by

$$\ell_t = \alpha(y_t - sn_{t-m}) + (1 - \alpha)[\ell_{t-1} + b_{t-1}],$$
$$b_t = \gamma[\ell_t - \ell_{t-1}] + (1 - \gamma) b_{t-1},$$
$$sn_t = \delta[y_t - \ell_t] + (1 - \delta) sn_{t-m}.$$

2.2.4 Multiplicative Holt-Winters Method

A point forecast made in time period t for $\hat{y}_{t+\tau}$ is

$$\hat{y}_{t+\tau}(t) = (l_t + \tau b_t) sn_{t+\tau-m}, \quad (\tau = 1, 2, \ldots).$$

The smoothing equations are given by

$$\ell_t = \alpha(y_t / sn_{t-m}) + (1 - \alpha)[\ell_{t-1} + b_{t-1}],$$
$$b_t = \gamma[\ell_t - \ell_{t-1}] + (1 - \gamma) b_{t-1},$$
$$sn_t = \delta[y_t / \ell_t] + (1 - \delta) sn_{t-m}.$$

where

ℓ_t is estimated for the level .

b_t is estimated for the growth rate.

sn_t is estimated for the seasonal factor of the time series in time series period t.

$sn_{t+\tau-m}$ is the "most recent" estimate of the seasonal factor for the reason corresponding to time period $t + \tau$.

α, γ, δ are the smoothing constants between 0 and 1.

ℓ_{t-1}, b_{t-1} are estimates in time period $t - 1$ for the level and growth rate.

sn_{t-m} is the estimate in time period $t - m$ for the seasonal factor.

m is the period of the seasonality.

τ is the τ-step-ahead forecast.

2.3 Decomposition Method [10]:

The basic idea behind these models is to decompose the time series into several factors: trend, seasonal, cyclical and irregular (error). Multiplicative decomposition appropriate for a time series that exhibits increasing or decreasing seasonal variation. When the parameters describing the series are not changing over time. Whereas, Additive decomposition found useful when a time series that exhibits constant seasonal variation. The multiplicative decomposition model is

$$Y_t = TR_t \times SN_t \times CL_t \times IR_t$$

The additive decomposition model is

$$Y_t = TR_t \times SN_t + CL_t + IR_t$$

where

Y_t are the observed value of the time series in time period t.

TR_t are trend component (or factor) in time period t.

SN_t are seasonal component (or factor) in time period t.

CL_t are cyclical component (or factor) in time period t.

IR_t are irregular component (or factor) in time period t.

The estimates tr_t, sn_t, cl_t and ir_t are generally used to describe the time series. We can also use these estimates to compute predictions. If there are no pattern in the irregular

and cycle component, we assume TR_t and CL_t to equal zero. The point forecast for the multiplicative decomposition model is

$$\hat{y}_t = tr_t \times sn_t.$$

The point forecast for the additive decomposition model is

$$\hat{y}_t = tr_t + sn_t.$$

2.4 Regression Analysis [18]:

Regression analysis is a statistical technique for modeling and investigating the relationships between an outcome or response variable and one or more predictor or regression variables. The end result of a regression analysis study is often to generate a model that can be used to forecast or predict future values of the response variable given specified values of the predictor variable.The simple linear regression model involves a single predictor variable,which can be written as

$$y = \beta_0 + \beta_1 x + \varepsilon,$$

where

y	is the response.
x	is the predictor variable.
β_0, β_1	are unknown parameters.
ε	is an error term.

The model parameters or regression coefficients β_0 and β_1 have a physical interpretation as the intercept and the slope of a straight line, respectively. The slope β_1 measures the change in the mean of the response variable y_t for a unit change in the predictor variable x. These parameters are typically unknown and must be estimated from a sample of data. The error term ε accounts for deviations of the actual data from the straight line specified by the model equation. We usually think of ε as a statistical error, so we define it as a random variable and will make some assumptions about its distribution. For example, we typically assume that ε is normally distributed with mean zero and variance σ^2, abbreviated as $N\left(0, \sigma^2\right)$.

Note that the variance is assumed to be constant; that is, it does not depend on the value of the predictor variable (or any other variable).

Additionally, [10] one useful from of the linear regression model is what we call the quadratic regression model. The quadratic regression model relating y to x is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x^2 + \varepsilon.$$

Then, we apply regression analysis to time series analysis. The simple linear regression is as follow

$$\hat{y}_t = b_0 + b_1 t.$$

The quadratic regression model is

$$\hat{y}_t = b_0 + b_1 t + b_2 t^2.$$

where b_0, b_1 are estimated using the method of least squares of the unknown parameter β_0 and β_1 respectively.

2.5 Time Series with Feedback Control Theory/

The Feedback Control Methodology

Striking developments have taken place since 1990 in feedback control theory [19]. The subject has become both more rigorous and more applicable. The rigorous is not for its own sake, but rather that even in an engineering discipline rigorous can lead to clarity and to methodical solutions to problems. The applicability is a consequence both of new problem formulations and new mathematical solutions to these problems [20-22]. Moreover, computers and software have changed the way engineering design is done. These developments suggest a fresh presentation of the subject, one that exploits these new developments while emphasizing their connection with classical control. Control systems are designed so that certain designated signals, such as tracking errors and actuator inputs, do not exceed pre-specified levels. Hindering the achievement of this goal are uncertainty about the plant to be controlled (the mathematical models that we use in representing real physical systems are idealizations) and errors in measuring signals (sensors can measure signals only to a certain accuracy). Despite the seemingly obvious requirement of bringing plant uncertainty explicitly into control problems, it was only in the early 1990s that control researchers re-established the link to the classical work of Bode and others by formulating a tractable mathematical notion of uncertainty in an input-output framework and developing rigorous mathematical techniques to cope with it.This formulates a precise problem, called the robust performance problem, with the goal of achieving specified signal levels in the face of plant uncertainty.

Without control systems there could be no manufacturing, no vehicles, no computers, no regulated environment-in short, no technology. Control systems are what make machines, in the broadest sense of the term, function as intended. Control systems are most often based on the principle of feedback, whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action. Consider the Time series with Feedback Control Theory/The Feedback control methodology:

$$\hat{y}_t = g(y_t) + u(t),$$

where

$\hat{y}_t$	is predicted value.
$u(t)$	is feedback control input.It is defined by $u(t) =$ median error.

Model Selection

Marina Theodosiou [15] conducted an expert remark about assessment error measures to select forecasting techniques. Scale-independent measures; mean absolute error (MAE), median absolute error (MdAE), mean square error (MSE) and root mean square error (RMSE) are proper for comparing the forecast performance between different data sets. These have been introduced by Tashman [23]. However, scale-dependent measures are based on the variability of the forecasting relative to the real observations, and there are appropriate when comparing various methods for the same data set. Therefore, in this paper, we focus only scale-dependent measures as we compared different methods for the same data series and only mean absolute

error results are reported here. The expressions is depicted below

$$MAE = \sum_{t=1}^{n} |y_t - \hat{y}_t| / n,$$

where

y_t is the actual value at time t.
$\hat{y}_t$ is the forecast value at time t.
n is the number of observations.

III. EMPIRICAL RESULTS

Five real data sets-the expert value of apple juice, the average gold prices (US dollar /oz), the number of tourist arrivals to Thailand by nationality (America), the foreign workers in Thailand, and the influenza patient numbers in Thailand-are considered in this paper to illustrate the effectiveness of the control theory. Each data set is divided into two samples of training and testing. The actual data sets are as follows

Example 1. The Expert Value of Apple Juice Cases

We considered all monthly series between January 1998 - September 2014, total 201 observations.We divided the data into two groups, the first group used a 16-year data set from January 1998 to December 2013, giving a total of 192 observations as the training data. The second group used 9 observations between January to September in 2014 to finding suitable methods. The time series plot are shown in **Fig.1**
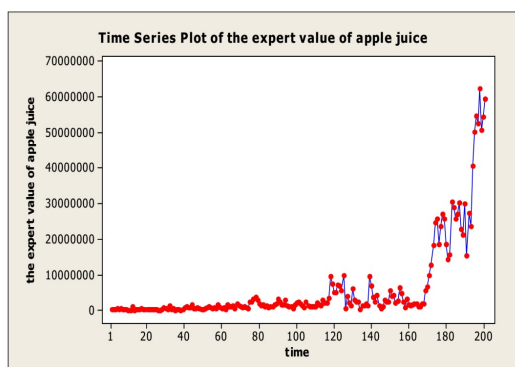

Fig. 1.   The expert value of apple juice series from January 1998 to September 2014.

It is noted that the expert value of apple juice series from Office of Agricultural Economics Ministry remained stable obviously being 1,005,632 baht. After that, the plot grew dramatically being from 9,544,905 to 59,666,747 baht.

Example 2. The Average Gold Prices (US Dollar/Oz)Cases

The last 10 observations is not be used to compute the forecast but to evaluate their accuracy. The plot is shown in **Fig.2**
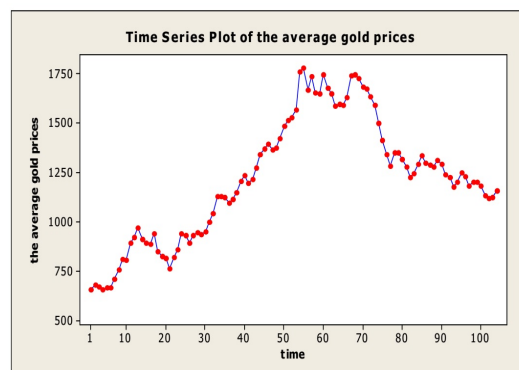

Fig. 2.   The average gold prices (US dollar/oz) between January 2007 and October 2015.

It is noted that the time plot of the average gold prices from gold traders association reached to the peak obviously 1,780 (US dollar/oz), then the graph declined slightly.

Example 3. The Number of Tourist Arrivals to Thailand by Nationality (America).

The last 9 observations is not be used to calculate the forecast but to evaluate their accuracy. The plot is shown in **Fig.3**
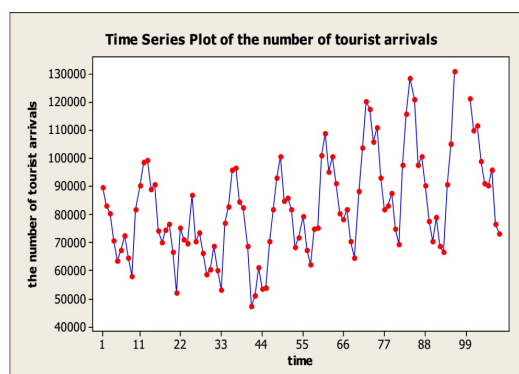

Fig. 3.   American tourist visitor in Thailand via Suvarnabhumi airport from January 2007 until September 2015.

The time series plot of the number of oversea visitor's arrivals to Thailand by nationality (America) from department of tourism reveals a periodicity of approximately 9 years.

Example 4. The Foreign Workers in Thailand

The in-sample and out-of-sample the data are the same as the paper [5, 24]. The time series plot isprovided in **Fig.4**
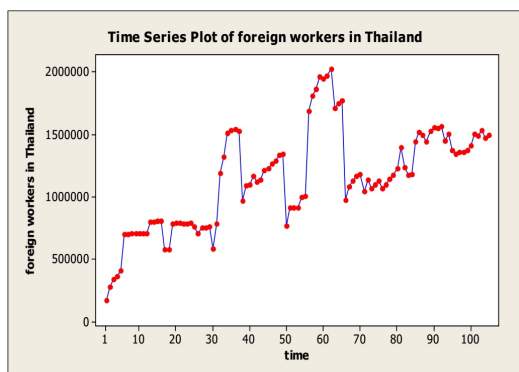
Fig. 4.   The foreign workers in Thailand from January 2007 until September 2015.

The plot of the foreign workers in Thailand from office of Foreign Workers Administration shows various changing turning points in the series.

Example 5. The Number of Influenza Patients in Thailand

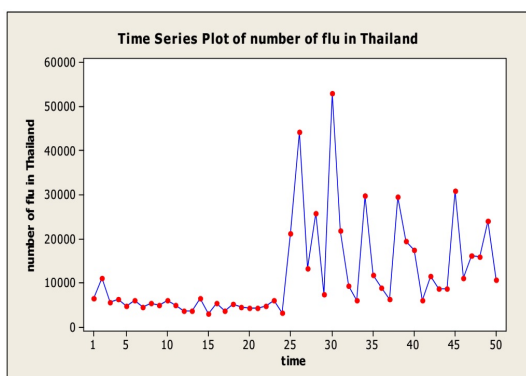Time plot of the number of influenza patients in Thailand are presented in **Fig.5**

Fig. 5.   Quarterly total influenza patients in Thailand, 2003-2015.

The number of influenza patients in Thailand from social and quality of life data base system reveals remained stable for 6 years and then the plot provided a periodicity of approximately 7 years.

Table I. The results of mean absolute error (MAE) obtained using well known time series models with application to feedback control theory

TABLE I
COMPARISONS MAE VALUES OF VARIOUS METHODS WITH APPLICATION TO FEEDBACK CONTROL THEORY OBTAINED FROM DIFFERENT DATA SETS.

Data	Method		MAE	Reduce rate
Apple Juice	ARIMA(1,1,2)	Control	167,036.56	72.08%
		Without Control	598,236.32	
	Holt	Control	131,939.20	67.84%
		Without Control	410,242.20	
Gold prices	ARIMA(0,2,1)	Control	17.01	73.87%
		Without Control	65.09	
	Regression	Control	102.93	79.49%
		Without Control	501.75	
Tourism	$ARIMA(0,1,0)(2,1,0)_{12}$	Control	2,784.86	70.84%
		Without Control	9,549.98	
	Holt-Winters'	Control	2,979.03	67.75%
		Without Control	9,238.38	
Foreign Workers	$ARIMA(0,1,0)(0,1,1)_{12}$	Control	83,670.18	37.60%
		Without Control	134,088.9	
	Holt-Winters'	Control	119,036.20	26.59%
		Without Control	162,154.30	
Influenza patients	$SARIMA(0,1,1)_4$	Control	3,997.01	41.16%
		Without Control	6,792.74	
	Decomposition	Control	6,448.18	29.55%
		Without Control	9,152.36	

IV. CONCLUSION

Time series analysis is one of the very demanding subjects over the last few decades, since can be applied for financial, economic, engineering and scientific modeling. The five other standard statistical techniques from the literature, namely the ARIMA,Holt's method, Holt-Winters method, Decomposition method and Regression analysis are well known for many researchers. The aim of this research is prefer a new method for forecasting via feedback control theory with application to several non-stationary real data sets. The performance evaluation results indicated that this method can perform well. As it is clear that the errors measures are decline all of cases. Future work can include stationary data and apply adaptive control theory to time series analysis.

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