Chance Constrained Multi-objective Programming for Supplier Selection and Order Allocation under Uncertainty

Xi Li[1], Tomohiro Murata[2]

***Abstract*—**This paper proposes a chance-constrained multi-objective goal programming model for supplier selection problem with uncertain factors. Considering uncertain factors of demand, capacity and lead time and several objectives, the proposed approach provides chance constraints leading to a order allocation decision-making result. The decision keeps the confidence and risk of constraints to a certain level. Therefore, the optimization model makes sure of the procurement cost, including purchasing cost and penalty cost, being restricted within an acceptable level according to the confidence and risk value initially set. The paper also provides a comparison between the conventional model (deterministic constraints) and proposed model (chance constraints) and shows the superiority of the latter.

***Keywords*—**supplier selection, chance constraint, goal programming, uncertainty

I. Introduction

SUPPLIER selection problem is a multi-objective multi-criteria problem in which both conflicting objectives and various factors should be considered to select a proper supplier portfolio. The contemporary supply management is to maintain long term partnership with suppliers, and use fewer but reliable suppliers [1]. Furthermore, by using optimization programming method, the demand allocation of the portfolio can be achieved.

Now due to global sourcing, it's obvious that supplier selection must deal with multi-product situation which means company has to make a decision of how much of every kind of product should be purchased from which supplier. Management should split order quantities among the available suppliers for a variety of reasons including creating a constant environment of competitiveness (multiple sourcing) [2]. The company should consider both the qualified suppliers and its related order allocation.

Vendor selection decisions are complicated by the fact that various criteria must be considered in the decision making process [3]. Choosing the right suppliers involves much more than scanning a series of price list, and choices will depend on a wide range of factors. In previous researches, the criteria considered in a supplier selection model covers a large range, and cost, quality and lead time are the most significant ones. Many factors influencing international supplier selection decisions are in conflict with one another. For instance, the low price of purchased materials from a certain foreign supplier can be offset by the firm's loose quality standards or chronic financial instability [4]. And 3 major objectives including cost minimization, quality maximization and lead time minimization are also centered in previous researches and usually decision-makers have aspired goals for each objectives.

So for solving the problem, there is variety of approaches. DM techniques are identified from three perspectives: (1) Multi-criteria decision making (MCDM) techniques, (2) Mathematical programming (MP) techniques, and (3) Artificial intelligence (AI) techniques [5]. However, not all methods are equally useful in every possible purchasing situation [6]. From these technologies, optimization programming (especially goal programming) is mostly used. Because goal programming is a multi-objective optimization method in which each of measures is given a goal or target to be achieved. What's more, objectives could be achieved by a certain priority order. Goal programming is used as a multi-criteria decision analysis tool [7].

Though optimization programming method can be effectively and frequently used for such kind of problem, another trend in current supplier selection environment which includes fluctuating demand, unreliable capacity and lead time adds risk in supply chain management. Risk is an inherent part of supply chain operations and the presence of outside suppliers is a major driver of supply chain risks. The study of uncertainty is centered on risks related to capacity, demand and variable cost uncertainty [8]. Few researches have considered the uncertainty of lead time. In real scenario, it is a problem with uncertain capacity, lead time and customer demand. The company will face a loss in case the demand can't be satisfied or supplier fails in delivering adequate products on time. Therefore, effective and cost efficient supplier selection in a stochastic or uncertain scenario helps an organization in achieving its goal [9].

The problem of uncertainty in supplier selection process has been studied and method to solve it has been suggested for long. Apart from fuzzy technology, one of the most popular

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methodologies applied in a supplier selection problem is chance constrained programming. The problem of stochastic (chance-constrained) programming is defined as follows: select certain random variable as function of random variables with known distributions in such a manner as constraints on these variables which must be maintained at prescribed levels of probability [10]. However, it is still not recommended to combine chance constrained programming in a multi-objective goal programming to solve the supplier selection and order allocation problem under uncertainty.

Therefore, the paper proposes a chance constrained goal programming method to help solve the multi-objective supplier selection problem with uncertain factors. There are 3 major purposes in the research. First is to complete multi-objective supplier selection programming with cost, product quality and lead time. Second is to propose robust supplier selection and order allocation method with chance constrained goal programming under uncertainties. Third is to evaluate the proposed method by experiment and comparison with conventional supplier selection goal programming.

II. Model Construction

A. Objective

There are 3 objectives making up the multi-objective part in supplier selection programming model: cost minimization, quality maximization and lead time minimization.

Minimize $\sum_{j=1}^{J}\sum_{k=1}^{K}\mu_{c_{jk}}h_{jk}x_{jk}+\sum_{j=1}^{J}\sum_{k=1}^{K}F_j x_{jk}$
$=\sum_{j=1}^{J}\sum_{k=1}^{K}(\mu_{t_{jk}}+\mu_{w_{jk}})h_{jk}x_{jk}+\sum_{j=1}^{J}\sum_{k=1}^{K}F_j x_{jk}$ (1)

Maximize $\sum_{j=1}^{J}\sum_{k=1}^{K}Q_{jk}\mathrm{h}_{jk}x_{jk}$ (2)

Minimize $\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}\mathrm{h}_{jk}x_{jk}$ (3)

Eq. (1) to Eq. (3) represent objectives of cost minimization, quality maximization and lead time minimization respectively where j means j^{th} supplier, *k* means k^{th} product, h_{jk} means the amount of product *k* delivered from supplier *j* and x_{jk} is a binary variable equal to 1 if supplier *j* is assigned as supplier of product *k*, 0 otherwise.

These are objectives that ought to be achieved simultaneously restricted by the constraints.

B. Conventional constraint (deterministic)

$\sum_{j=1}^{J}h_{jk}\geq D_k \quad \forall k\in K$ (4)

$h_{jk}\leq cap_{jk}x_{jk} \quad \forall j\in J,\ k\in K$ (5)

$\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}h_{jk}x_{jk}\leq A_k \quad \forall j\in J,\ k\in K$ (6)

$x_{jk}\in[0,1] \quad \forall j\in J,\ k\in K$ (7)

$h_{jk}\geq 0 \quad \forall j\in J,\ k\in K$ (8)

Eq. (4) represents demand constraint meaning that the amount of product *k* ordered from supplier *j* must exceed the customer's total demand. Eq. (5) represents capacity constraint meaning that the amount of product *k* ordered from supplier *j* should not exceed the real capacity of product *k* of supplier *j*. Eq. (6) represents lead time constraint meaning that the total lead time of one product required by the customer or the market.

C. Chance constraint (stochastic)

$P\left(\sum_{j=1}^{J}h_{jk}\geq D_k\right)\geq\alpha_k \quad \forall k\in K$ (9)

$P\left(h_{jk}\leq cap_{jk}x_{jk}\right)\geq\alpha_{jk} \quad \forall j\in J,\ k\in K$ (10)

$P\left(\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}h_{jk}x_{jk}\geq A_k\right)\leq\alpha_l \quad \forall j\in J,\ k\in K$ (11)

There is uncertainty or risk when the fluctuating demand, supplier capacity and lead time required by customer is unknown or uncertain. Then to respond to this uncertainty, chance constraint that keeps the condition to be achieved at a certain level is proposed.

Eq. (9) means that the probability that total supply of each product satisfies the aspired demand of each product must exceed α_k of confidence level. Eq. (10) means that the probability that supply of each product satisfies the real capacity of each product must exceed α_{jk} of confidence level. Eq. (11) means that the probability that total lead time exceeds the aspired lead time must be less than α_l.

Stochastic form could not be solved in a optimization model and according to chance constrained methodology, it can be transformed to its equivalent deterministic form.

For demand chance constraint:

$P\left(\sum_{j=1}^{J}h_{jk}\geq D_k\right)\geq\alpha_k \quad \forall k\in K$
$\leftrightarrow \sum_{j=1}^{J}h_{jk}\geq F_{D_k}^{-1}(\alpha_k) \quad \forall k\in K$ (12)

For capacity chance constraint:

$P\left(h_{jk}\leq cap_{jk}x_{jk}\right)\geq\alpha_{jk} \quad \forall j\in J,\ k\in K$
$\leftrightarrow h_{jk}\leq F_{cap_{jk}x_{jk}}^{-1}\left(1-\alpha_{jk}\right) \quad \forall j\in J,\ k\in K$ (13)

For lead time chance constraint:

$P\left(\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}h_{jk}x_{jk}\geq A_1\right)\leq\alpha_l$
$\leftrightarrow \sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}h_{jk}x_{jk}\leq F_{A_k}^{-1}(\alpha_l)$ (14)

Assuming D_k follows probability distribution with cumulative distribution function F_{D_k} (e.g.: normal distribution). Then (9) can be transformed to the equivalent condition (12) by using inverse cumulative probability distribution function. Eq. (13) is transformed assuming $cap_{jk}x_{jk}$ follows cumulative distribution function $F_{cap_{jk}x_{jk}}$ with known mean and standard deviation. It is same with (14) and the data is based on historical evidence.

D. Chance constrained goal programming

The chance constrained goal programming model is depicted as follows where d_i represents deviation variable of each goal and λ_1 represents weights of each goal. Z_1, Z_2 and Z_3 are goal values of each goal condition from (16) to (18).

Minimize $\lambda_1 d_1^+ + \lambda_2 d_2^- + \lambda_3 d_3^+$ (15)

Subject to

$\sum_{j=1}^{J}\sum_{k=1}^{K}\mu_{c_{jk}}\mathrm{h}_{jk}x_{jk}+\sum_{j=1}^{J}\sum_{k=1}^{K}F_j x_{jk}+d_1^- - d_1^+ = Z_1$ (16)

$\sum_{j=1}^{J}\sum_{k=1}^{K}Q_{jk}\mathrm{h}_{jk}x_{jk}+d_2^- - d_2^+ = Z_2$ (17)

$\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}\mathrm{h}_{jk}\mathrm{x}_{jk}+d_3^- - d_3^+ = Z_3$ (18)

$\sum_{j=1}^{J}h_{jk}\geq F_{D_k}^{-1}(\alpha_k) \quad \forall k\in K$ (12)

$h_{jk}\leq F_{cap_{jk}x_{jk}}^{-1}\left(1-\alpha_{jk}\right) \quad \forall j\in J,\ k\in K$ (13)

$\sum_{j=1}^{J}\sum_{k=1}^{K}L_{jk}h_{jk}x_{jk}\leq F_{A_k}^{-1}(\alpha_l)$ (14)

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$x_{jk} \in [0,1] \quad \forall j \in J,\ k \in K$ (7)

$h_{jk} \geq 0 \quad \forall j \in J,\ k \in K$ (8)

The achievement function (15) is a minimization function because we require the total cost and lead time should be less than or equal to the goal value and quality level should be greater than or equal to the goal value.

III. Experiment

A. Case description, source of data and optimization tool

The paper chose a single-buyer 5-supplier and 3-product situation. All of data used in the experiment is randomly generated under normality assumption. Specific information is shown in table I. Confidence level is initially set at α_k=80%, α_{jk}=80% and risk level is set at α_l=0.2.

LINGO 11 is used to solve both conventional and proposed model.

TABLE I
DATA PREPARED FOR EXPERIMENT

Data	Data type	Data size
Demand	mean and variable	3 (each product)
Capacity	mean and variable	15 (each product of each supplier)
Cost	mean	15
Quality	percentage	15
Lead time	mean and variable	15

B. Experiment purpose

The objective of the experiment is to evaluate the effectiveness of the proposed model by comparing the cost both conventional (deterministic) and (stochastic) model would result in.

C. Experiment process

The experiment is carried out in the following steps:

1) First of all, goal values should be obtained by solving the conventional model. The paper set the optimal value of an objective from solution ignoring other objectives as its goal value. The following model show2 the process of getting goal value of objective (1).

Single-objective optimization:

$$\text{Minimize } \sum_{j=1}^{J} \sum_{k=1}^{K} \mu_{c_{jk}} h_{jk} x_{jk} + \sum_{j=1}^{J} \sum_{k=1}^{K} F_j x_{jk} \quad (1)$$

Subject to

$P\left(\sum_{j=1}^{J} h_{jk} \geq D_k\right) \geq \alpha_k \quad \forall k \in K$ (9)

$P\left(h_{jk} \leq cap_{jk} x_{jk}\right) \geq \alpha_{jk} \quad \forall j \in J,\ k \in K$ (10)

$P\left(\sum_{j=1}^{J} \sum_{k=1}^{K} L_{jk} h_{jk} x_{jk} \geq A_k\right) \leq \alpha_l \quad \forall j \in J,\ k \in K$ (11)

$x_{jk} \in [0,1] \quad \forall j \in J,\ k \in K$ (7)

$h_{jk} \geq 0 \quad \forall j \in J,\ k \in K$ (8)

The other 2 goal values can be obtained by the same way.

2) Solve the conventional model (deterministic) in LINGO 11 and get the solution. Schedule with certain data which means there is no uncertainty on demand, capacity and lead time. Use average demand, full (100%) capacity and average lead time.
3) Solve the proposed model (stochastic) in LINGO 11and get the solution. Schedule with uncertain constraints. Use 80%probable demand, 80% probable capacity and 20% delay risk.
4) Randomly select 10 sets of data as real situations and use the data in the deterministic model and get related solutions. The results are regarded as comparative items.

D. Result

Table II and table III show the results of order allocation solution using conventional model and proposed model as follows:

TABLE II
SOLUTION OF CONVENTIONAL MODEL

	h_{jk}			purchasing cost
	P1	P2	P3	
S1	177	0	0	10532
S2	73	57	0	
S3	0	0	93	
S4	27	189	219	
S5	0	1	0	

TABLE III
SOLUTION OF PROPOSED MODEL

	h_{jk}			purchasing cost
	P1	P2	P3	
S1	143	0	0	10525
S2	62	0	0	
S3	0	0	41	
S4	52	152	189	
S5	0	78	57	

Table IV shows the results of conventional model under 10 randomly generated situations:

TABLE IV
SOLUTION OF 10 SITUATIONS

situation	procurement cost of conventional model for 10 situations
1	10973
2	10527
3	10026
4	9720
5	10532
6	10529
7	8709
8	10531
9	9583
10	9836
Average	10096.6

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IV. EVALUATION

A. Evaluation measurement

In this paper, a measurement function is designed to evaluate the effectiveness and superiority of the proposed model:

Procurement cost C = purchasing cost + penalty cost.

In this function, purchasing cost represents the cost directly calculated using order allocation solution and related unit cost and fixed cost. And penalty cost is calculated when the supply of each product is not satisfied according to the real demand, the order allocation exceeds the real capacity or total lead time of each product exceeds the optimized value compared with the result from real situations. The average of cost C of 10 situations is calculated to make the comparison between conventional model and proposed model.

B. Evaluation result

TABLE V
COST C OF CONVENTIONAL MODEL UNDER 10 SITUATIONS

situation	penalty cost	cost C
1	3160	13692
2	3140	13672
3	4104	14636
4	1176	11708
5	9076	19608
6	6144	16676
7	2432	12964
8	4504	15036
9	5996	16528
10	3072	13604
Average	5752.4	16284.4

Table V shows results of cost C of conventional model under 10 situations.

TABLE VI
COST C OF PROPOSED MODEL UNDER 10 SITUATIONS

situation	penalty cost	cost C
1	1344	11869
2	4744	15269
3	6120	16645
4	3472	13997
5	3728	14253
6	4148	14673
7	1824	12349
8	5136	15661
9	6400	16925
10	4604	15129
Average	4152	14677

Table VI shows results of cost C of proposed model under 10 situations.

Procuration cost	
Deterministic model solution	16277.4
Proposed model solution	14684.0

Fig. 1 Comparison between procurement cost of conventional model and proposed model under uncertain environment

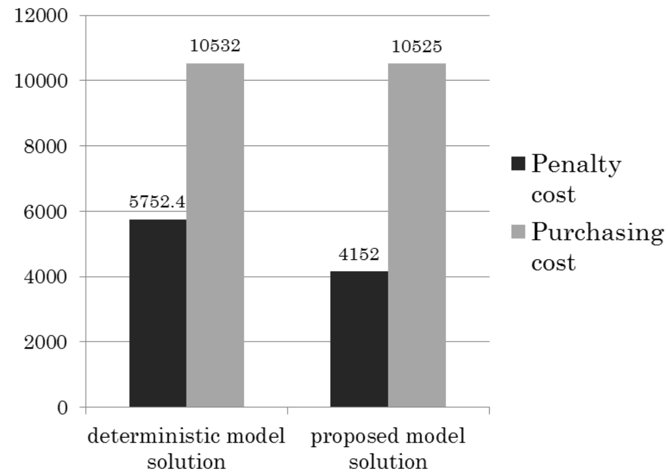

Fig. 2 Comparison between purchasing cost of conventional model and proposed model under uncertain environment

Fig. 1 and 2 show the comparison between 2 models under uncertain environment.

C. Analysis

It is obvious that the result of conventional model and proposed model is different and the former leads to a lower purchasing cost. With known data of demand, capacity and lead time, the constraints in the multi-objective problem could be fixed and constructed with uncertain value. Then the solution would be most compromised without penalty cost. However with chance constraints (or confidence level assigned to constraints), there must be deviation of demand, capacity and lead time from the real situation so that not only the purchasing cost is much larger than that of deterministic model, there appears an evident penalty cost as well. Furthermore, with uncertain data, the deterministic model no longer kept its advantages of low procurement cost because of significant penalty cost.

Specifically, On the one hand, the procurement cost decreased by 9.8% using proposed model under uncertain environment.

On the other hand, the purchasing cost of deterministic model (without chance constraints) is similar with that of proposed model. However, the penalty cost decreased by 27.8% using proposed model.

From the analysis above, we can draw a conclusion: The proposed model is proved robust under uncertain environment. And it relaxes the value range of uncertain factors to diminish the penalty cost and maintain the purchasing cost at the same time. Therefore the proposed model is superior to deterministic model under uncertain environment.

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V. CONCLUSION

Supplier selection is one of the most important stages of supply chain management. Due to global sourcing, firms have to make a decision of optimal order allocation under uncertainties in demand, capacity and lead time which increase in global manufacturing industry.

The paper provides a chance constrained multi-objective optimization model to handle the uncertainty factors including demand, capacity and lead time associated with supplier which can minimize total cost and lead time and maximize quality level of supply goods.

Deterministic equivalents of capacity, demand, cost and lead time chance constraints are derived under the normality assumption.

Goal programming methodology is used to solve the multi-objective optimization. The goal values are generated from single-objective model with each objective at one time.

The research also proposes an evaluating method using a procurement cost function (purchasing cost plus penalty cost) to compare the cost of conventional method and that of proposed method to test its superiority in multi-objective and robust supplier selection from a cost perspective. The experiment proves that the proposed model is superior to the conventional model under uncertain environment.

The research achievements can be applied to other similar multi-objective optimization problem under uncertainty. It can be improved by incorporating other qualitative issues including objective and constraint. However, it also has some drawbacks. The proposed method can only solve small-scale multi-objective optimization problem. Large-scale problem requires suitable optimization method such as genetic algorithm. This is one of the points to be considered in the future research.

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