A Hyperexponential Approximation to Finite- and Infinite-time Ruin Probabilities of Compound Poisson Processes

Amir T. Payandeh Najafabadi, Member IAENG *

Abstract—This article considers the problem of evaluating infinite-time (or finite-time) ruin probability under a given compound Poisson surplus process. By approximating the claim size distribution by a finite mixture exponential, say Hyperexponential, distribution. It restates the infinite-time (or finite-time) ruin probability as a solvable ordinary differential equation (or a partial differential equation). Application of our findings has been given though a simulation study.

Keywords: Ruin probability, Compound Poisson Processes, mixture exponential (Hyperexponential) distribution, Heavy-tailed distributions

1 Introduction

Consider the following compound Poisson process

\[ U_t = u + ct - \sum_{j=1}^{N(t)} X_j, \]  

(1)

where \( X_1, X_2, \cdots \) are a sequence of i.i.d. random variables with common density function \( f_X(\cdot) \), \( N(t) \) is a Poisson process with intensity rate \( \lambda \), \( u \) and \( c \) stand for initial wealth/reserve and premium of the process, respectively.

The finite-time and infinite-time ruin probabilities for the above compound Poisson process are, respectively, denoted by \( \psi(u; T) \) and \( \psi(u) \) and defined by

\[ \psi(u; T) = P(\tau_u \leq T); \quad & \quad \psi(u) = P(\tau_u < \infty), \]  

(2)

where \( \tau_u = \inf \{ t : U_t \leq 0 | U_0 = u \} \) is the hitting time.

Ref. [15] among others, established that an infinite-time ruin probability \( \psi(u; T) \) can be restated as the following integro-differential equation

\[ c\tilde{\psi}^{(1)}(u; T) = \lambda \psi(u; T) - \int_0^u \psi(u - x)f_X(x)dx = 0, \]  

(3)

A finite-time ruin probability \( \psi(u; T) \) under a compound Poisson process can be restated as the following partial integro-differential equation

\[ \frac{\partial \psi(u; T)}{\partial u} - \frac{\partial \tilde{\psi}(u; T)}{\partial T} - \frac{\lambda}{c} \tilde{\psi}(u; T) \]

\[ + \frac{\lambda}{c} \int_0^u \psi(u - x)f_X(x)dx = 0, \]  

(4)

where \( \tilde{\psi}(u; T) = 1 - \psi(u; T) \), \( \lim_{u \to \infty} \psi(u; T) = 0 \) for all \( T > 0 \) and \( \psi(u; T = 0) = 0 \) for all \( u \geq 0 \), see Ref [23].

Since the compound Poisson surplus process plays a vital role in many actuarial models, several authors studies ruin probability under surplus process (1). An excellent review for infinite-time ruin probability can be found in [3]. For finite-time ruin probability: [23] showed for exponential claim size distribution partial integro-differential equation (4) can be transformed into a second-order partial differential equation. Ref. [1] considered a compound Poisson surplus process with constant force of real interest. Then, they restated finite-time ruin probability \( \psi_T(u) \) as a gamma series expansion. Ref. [14] provided a global Lagrange type approximation in the z-space for \( \psi_T(u) \) under surplus process (1). Ref. [2] and [29] employed the Padé approximant method to approximate \( \psi_T(u) \) under surplus process (1).

This article in the first step approximates claim size density function \( f_X(\cdot) \) with a finite mixture exponential, say Hyperexponential, density function \( f_X^\star(\cdot) \). Then, it transforms two integro-differential equations (3) and (4), respectively, into an ordinary differential equation (ODE) and a partial differential equation (PDE). A simulation study has been conducted to show practical application of our findings.

The rest of this article is organized as follows. Some mathematical background for the problem has been collected in Section 2. Section 3 provides the main contribution of this article. Applications of the results have been given in Section 4.

2 Preliminaries

From hereafter now, we set \( \sum_{j=1}^{b} A_j = 0 \), for \( b < a \).
The following recalls the exponential type functions which plays a vital role in the rest of this article.

**Definition 1.** An $L_1(\mathbb{R}) \cap L_2(\mathbb{R})$ function $f$ is said to be of exponential type $T$ on $\mathbb{C}$ if there are positive constants $M$ and $T$ such that $|f(\omega)| \leq M \exp(T|\omega|)$, for $\omega \in \mathbb{C}$.

The Fourier transforms of exponential type functions are continuous functions which are infinitely differentiable everywhere and are given by a Taylor series expansion over every compact interval, see [6], [18], [19], and [30] for more details.

From the Hausdorff-Young Theorem, one can observe that if $(s_n)$ is a sequence of functions converging, in $L^2(\mathbb{R})$ sense, to $s$. Then, the Fourier transforms of $s_n$ converge, in $L^2(\mathbb{R})$ sense, to the Fourier transform of $s$, see [17] for more details. Using [13]'s method, [12] and [20] showed that most of the common distributions do have characteristic functions that can be extend to meromorphic functions.

The following from [16] and [26] recalls Hausdorff-Young inequality for the Laplace transform.

**Lemma 1.** Suppose $h(\cdot)$ is a given and nonnegative function that $f \in L^1(\mathbb{R}^+) \cap L^2(\mathbb{R}^+)$. Then, $||h|| = \frac{1}{\pi}||L(h)||_2$, where $L(h)$ stands for the Laplace transform.

The Schwarz integrability condition states that in situation that all partial derivatives of a bivariate function exist and are continuous, one may change order of partial derivatives, see [4] for more details.

The following lemma provides useful results for the next section.

**Lemma 2.** Suppose $k(\cdot)$ is a given and differentiable function and $y(\cdot)$ is an unknown function that satisfy

$$
\int_0^x y(t) \left( \sum_{i=1}^n \omega_i \mu_i e^{-\mu_i(x-t)} \right) dt = k(x), \ x \geq 0, \quad (5)
$$

where $\omega_i, \mu_i$ and $\mu_i$ are given and nonnegative constants. Then, the above integral equation can be transformed into differential equation

$$
0 = \sum_{i=1}^n \omega_i \mu_i y^{(n-1)}(x) + \sum_{i=1}^n \sum_{j \neq i} \omega_i \mu_i \mu_j y^{(n-2)}(x) - \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} \omega_i \mu_i \mu_j \mu_k y^{(n-3)}(x) + \cdots + (-1)^n \sum_{i=1}^n \prod_{j \neq i} \mu_j y^{(0)}(x) - k^{(n)}(x) - \sum_{i=1}^n \mu_i k^{(n-1)}(x) - \sum_{i=1}^n \sum_{j \neq i} \mu_i \mu_j k^{(n-2)}(x) - \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} \mu_i \mu_j \mu_k k^{(n-3)}(x) - \cdots - \sum_{i=1}^n \mu_i k^{(0)}(x).
$$

**Proof.** For $n = 1$ see [23]. For $n > 1$, set $A_i = \omega_i \mu_i$ and $h_i(x) = \int_0^x y(t) e^{-\mu_i(x-t)} dt$. Using the fact that the $n^{th}$ derivatives $h_i(x)$ with respect to $x$ is

$$
h_i^{(n)}(x) = (-\mu_i)^n h_i(x) + \sum_{j=0}^{n-1} (-\mu_i)^{n-j} y^{(j)}(x),
$$

one may restate all first $n$ derivatives of (5) as the following system of equation.

$$
\begin{align*}
&k^{(n)}(x) = \sum_{i=1}^n A_i \left[ y^{(n-1)}(x) - \mu_i y^{(n-2)}(x) + \cdots + (-\mu_i)^{n-1} y^{(0)}(x) - (-\mu_i)^n h_i(x) \right], \\
&\vdots \\
&k^{(0)}(x) = \sum_{i=1}^n A_i h_i(x).
\end{align*}
$$

Multiplying both sides of: the first equation by $1$, the second equation by $\sum_{i=1}^n \mu_i$; the third equation by $\sum_{i=1}^n \sum_{j=1}^n \mu_j \mu_k$; the forth equation by $\sum_{i=1}^n \sum_{j=1}^n \sum_{k>j} \mu_j \mu_k \mu_i$; and so on until the last equation which multiplying its both sides by $\prod_{i=1}^n \mu_i$, then adding together all equations leads to the desired results. \hfill \Box

**Hyperexponential distributions**

The Hyperexponential (or mixture exponential) distribution is characterized by the number of $n$ exponential distributions with means $1/\mu_i$ and associated weight $\omega_i \in \mathbb{R}$ (i.e. $\sum_{i=1}^n \omega_i = 1$). The density function for an $n$-component Hyperexponential distribution is given by

$$
f_X(x) = \sum_{i=1}^n \omega_i e^{-\mu_i x}, \ x \geq 0. \quad (6)
$$

Ref. [7] showed that, one may approximate a large class of distributions, including several heavy tail distributions such as Pareto and Weibull distributions, arbitrarily closely, by Hyperexponential distributions. Ref. [8] established that a survival function at $x^*$, for all $x > 0$, is a completely monotone function if and only if its corresponding density function is a mixture of Weibull distributions with fixed shape parameter $1/\gamma$. Ref. [9] showed that any Weibull distribution with shape parameter less than 1 can be restated as a Hyperexponential distributions.

Using the Hausdorff-Young-Theorem, the following provides error bound for approximating the claim size density function $f_X(\cdot)$ by Hyperexponential density function $f^H(\cdot)$, given by (6).

**Lemma 3.** Suppose random claim size $X$ is surplus process (1) has density function $f_X(\cdot)$ and characteristic function $\theta_X(\cdot)$. Moreover, suppose that characteristic function $\theta_Y(\cdot)$ is (or can be extend to) a meromorphic function. Then, (1) density function of compound sum $S(t) = \sum_{i=1}^N X_i$, say $f_{S(t)}(\cdot)$, can be approximated by density function $f_{S^H(t)}(\cdot)$, where $S(t) = \sum_{i=1}^{N(t)} Y_i$ and $Y_i$ is a $n$-component Hyperexponential dis-
3 Ruin Probability

This section utilizes integro-differential Equations (3) and (4) to derive an approximate formula for the infinite (and finite) time ruin probability of a compound Poisson process (1). We seek an analytical solution \( \hat{\psi}(\cdot) \) which is an exponential type function. In the other word, we assume:

\[
\hat{\psi}(\omega) \leq Me^{T|\omega|}, \quad \omega \in \mathbb{C}, \text{ for some real numbers } M \text{ and } T \text{ in } \mathbb{R}.
\]

If this assumption is not met, as might be the case if, for example, there are point masses in \( \hat{\psi}(\cdot) \), our method works, but our error bounds may not be valid anymore.

The following theorem provides an \((n+1)\)-order ODE for infinite-time ruin probability \( \psi(\cdot) \) in the situation that claim size distribution \( X \) has been approximated by an \( n \)-component Hyperexponential density function \( f_X(\cdot) \).

**Theorem 1.** Suppose claim size density function \( f_X(\cdot) \) has been approximated by an \( n \)-component Hyperexponential density function \( f_X(\cdot) \). Then, infinite-time survivals probability \( \psi(\cdot) \) of a compound Poisson process (1) can be approximated by infinite-time survivals probability \( \hat{\psi}(\cdot) \) which can be evaluated using the following \((n+1)\)-order ODE:

\[
\begin{align*}
\frac{d^n}{dt^n} \sum_{i=1}^{n} \lambda_i \psi_i(\cdot) &+ \sum_{1 \leq j \neq i}^{n} \sum_{k} \lambda_{jk} \psi_j(\cdot) \psi_k(\cdot) = 0, \\
\end{align*}
\]

where \( \lambda_i \) are the arrival rates and \( \psi_i(\cdot) \) are the exponential decay factors.

Theorem 2. Suppose claim size density function \( f_X(\cdot) \) has been approximated by an \( n \)-component Hyperexponential density function \( f_X(\cdot) \). Then, the infinite-time survival probability \( \psi(u) \) of compound Poisson process (1) can be approximated by \( \hat{\psi}(u) \), given by Theorem (1), and its error satisfies

\[
|\psi(u) - \hat{\psi}(u)| \leq \frac{a_1}{\sqrt{\pi}} ||\phi(u) - \hat{\phi}(u)||_2, \quad a_1 = \sup \{ \phi(s), \sum_{i=1}^{n} \psi_i(s) \} \text{ and } \phi(s) \text{ stands for the characteristic function of random claim } X.
\]

The following provides error bound for approximating infinite-time survivals probability \( \psi(\cdot) \) by \( \hat{\psi}(\cdot) \).

**Theorem 3.** Suppose claim size density function \( f_X(\cdot) \) has been approximated by an \( n \)-component Hyperexponential density function \( f_X(\cdot) \). Then, finite-time survivals probability \( \psi(u; T) \) of a compound Poisson process (1) can be approximated by finite-time survivals probability \( \hat{\psi}(u; T) \) which can be evaluated using the following
$(n + 1)$-order PDE.

\[
\sum_{i=1}^{n} \lambda_i \partial_{u_i} \psi_0^{(n)}(u; T)\right) + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} \partial_{u_i} \partial_{u_j} \psi_0^{(n-2)}(u; T) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n+1} \sum_{k=1}^{n+1} \lambda_{ijk} \partial_{u_i} \partial_{u_j} \partial_{u_k} \psi_0^{(n-3)}(u; T) \\
+ \sum_{i=1}^{n} \sum_{j=1}^{n+1} \sum_{k=1}^{n+1} \sum_{l=1}^{n+1} \lambda_{ijkl} \partial_{u_i} \partial_{u_j} \partial_{u_k} \partial_{u_l} \psi_0^{(n-4)}(u; T) \\
- \cdots + (-1)^n \prod_{i=1}^{n} \lambda_i \partial_{u_i} \psi_0^{(n)}(u; T)
\]

where \( \psi_0^{(n)}(u; T) = \lim_{u \to 0} \frac{\partial^n}{\partial u^n} \psi_0(u; T) \) and boundary conditions that satisfy \( c_0^{(n)}(0; T) = -c_0 \psi_0^{(n-1)}(0; T) \). The above finding along with an application of the central limit theorem, see [25] and [5], among others, for more details.

The following provides error bound for approximating finite-time survivals probability \( \tilde{\psi}(u; T) \) by \( \psi_0(u; T) \).

**Theorem 4.** Suppose claim size density function \( f_X(\cdot) \) has been approximated by an \( n \)-component Hyperexponential distribution function \( f_X^{(n)}(\cdot) \). Then, the infinite-time survival probability \( \psi(u; T) \) of compound Poisson process (1) can be approximated by \( \psi_0(u; T) \), given by Theorem (3), and its error satisfies

\[
\|\text{Error}\|_2 \leq \frac{\lambda}{\sqrt{n}} \left( \frac{c_0 \psi_0(0; T)}{a_0^2} + a_1 T + \psi_0(0; T) a_2^2 a_3 \right),
\]

where \( \text{Error} = \psi(u; T) - \psi_0(u; T) \), and \( a_1 = \sup \{ \psi_0(s; T) \psi(s; B) \psi(s; B) \psi_0(s; T) \} \), for all \( s \) and \( T \). Using the central limit theorem for compound sum

\[
\sum_{i=1}^{N(t)} X_i \text{ (see [10], §2.5, or [27], §1.9), one may provide the following approximation for expression } \psi_0(u; T) = \int_0^T F_S(x)dx/c(T),
\]

where \( \psi_0^{(n)}(u; T) = \lim_{u \to 0} \frac{\partial}{\partial u} \psi_0(u; T) \).

Using the central limit theorem for compound sum

\[
\sum_{i=1}^{N(t)} X_i \text{ (see [10], §2.5, or [27], §1.9), one may provide the following approximation for expression } \psi_0(u; T) = \int_0^T F_S(x)dx/c(T),
\]

where \( m_i = E(X^i) \), for \( i = 1, 2, \) and \( \Phi(\cdot) \) stands for cumulative distribution function for standard normal distribution, see [10], §2.5 or [27], §1.9, for other parametric approximation approaches and [24] for a nonparametric approximation approach. For heavy tailed random claim size \( X \) that the ordinary central limit theorem does not work properly. One has to employ an appropriated version of the central limit theorem, see [25] and [5], among others, for more details.
\(a^{-2}\|h_1 - h_2\|_2\), and \(a = \sup\{h_1, h_2\}\), from [11], triangle inequality, and the Mean value theorem (i.e., 

\[
\left(\exp\left(\frac{A(s)}{cT} + \ln\left(\frac{1}{a} - c\omega(0, T)\right)\right) - \exp\left(\frac{A(s)}{cT} + \ln\left(\frac{1}{a} - c\omega(0, T)\right)\right)\right)/\left(\exp\left(\frac{A(s)}{cT} + \ln\left(\frac{1}{a} - c\omega(0, T)\right)\right)\right) 
\]

\[
\leq \left(\frac{1}{a} - c\omega(0, T)\right) - A(s)/cT - \ln\left(\frac{1}{a} - c\omega(0, T)\right)\right) \leq b \text{ where } b = \sup\{A(s)/cT + \ln\left(\frac{1}{a} - c\omega(0, T)\right)\}.
\]

Application of inequality \(\|\ln h_1 - \ln h_2\|_2 \leq \|h_1 - h_2\|_2/a\), where \(a = \sup\{h_1, h_2\}\), from [11] completes the desired proof. \(\square\)

4 Simulation Study

Consider compound Poisson process (1) with intensity rate \(\lambda = 1\) and premium \(c = 1\). This section conducts two simulation studies to show practical application the about findings.

**Example 1.** Suppose random claim \(X\) in compound Poisson process (1) has been distributed according to Weibull(0.3, 0.26053). Ref. [7] using a three-moment matching algorithm showed that density function of random claim \(X\) can be approximated by the following 2-component Hyperexponential density function

\[
f_X(x) = 0.000095e^{−0.019x} + 1.348225e^{−1.355x}. \tag{7}
\]

**For infinite-time ruin probability:** Application of Theorem (1) leads to the following second order ODE

\[
1.1\psi^{(3)}(u) + 0.5114\psi^{(2)}(u) + 0.0026395\psi^{(1)}(u) = 0
\]

with initial conditions \(\psi(0) = 0.0999009009\), \(\lim_{u→0}\psi^{(1)}(u) = 0.08264462809\), and \(\lim_{u→0}\psi^{(2)}(u) = −0.03629929491\).

Solving the above ODE, one may approximate finite-time survival probability \(\psi(u)\) of compound Poisson process (1) by \(\psi(u) = 0.9974815963 - 0.2101123039e^{−0.01502369720u} - 0.2873692025e^{−0.8589763028u}\).

Figure 1 illustrates behavior for such approximated infinite-time ruin probability.

**For finite-time ruin probability:** Application of Theorem (3) leads to the following PDE \(1.1\frac{\partial^2}{\partial u^2}\psi(u; T) + 0.5114\frac{\partial}{\partial u}\psi(u; T) + 0.0539995\frac{\partial^2}{\partial u^2}\psi(u; T) - 1.1\frac{\partial}{\partial u}\psi(u; T) - 0.0283195\frac{\partial}{\partial u}\psi(u; T) = 0\) with initial conditions \(\psi(0; T) = \beta(T)\), \(\lim_{u→0}\psi(u; T) = 1\), \(\lim_{u→∞}\frac{\partial}{\partial u}\psi(u; T) = 0.9091\beta(T) + \frac{\partial}{\partial u}\beta(T)\), where \(\beta(T) = \frac{1}{1+T}\Phi\left(\frac{−T}{\sqrt{20}}\right)\)

Solving the above PDE, one may approximate finite-time survival probability \(\psi(u; T)\) of compound Poisson process (1) by \(\psi(u; T)\), that its behavior (for \(T = 50, 100, 200\)) has been illustrated in Figure 2.

**Example 2.** Suppose random claim \(X\) in compound Poisson process (1) has been distributed according to Gamma(0.7310, 1). Ref. [29] using the Padé approximant method showed that density function of random claim \(X\) can be approximated by the following 3-component Hyperexponential density function

\[
f_X(x) = 0.8099e^{−3.2198x} + 0.3616e^{−1.4465x} + 0.5198e^{−1.0396x}. \tag{8}
\]

**For infinite-time ruin probability:** Application of Theorem (1) leads to the following second order ODE \(1.1\psi^{(3)}(u) + 5.29849\psi^{(2)}(u) + 6.47950\psi^{(1)}(u) + 1.79791\psi^{(0)}(u) = 0\) with initial conditions \(\psi(0) = 0.3354861821\), \(\lim_{u→0}\psi^{(1)}(u) = 0.3049874383\), \(\lim_{u→0}\psi^{(2)}(u) = 0.2385744292\), and \(\lim_{u→∞}\psi(u) = 1\).

Solving the above ODE, one may approximate infinite-time survival probability \(\psi(u)\) of compound Poisson process (1) by \(\psi(u) = 1 - 0.013037e^{−3.0739978u} - 0.008568e^{−1.3496303u} - 0.642998e^{−0.3940982u}\). Figure 3 illustrates behavior for such approximated infinite-time ruin probability.

**For finite-time ruin probability:** Application of Theorem (3) leads to the following PDE \(1.1\frac{\partial^2}{\partial u^2}\psi(u; T) + 5.29849\frac{\partial}{\partial u}\psi(u; T) + 6.47950\frac{\partial^2}{\partial u^2}\psi(u; T) + 1.79791\frac{\partial}{\partial u}\psi(u; T) - 5.35914\frac{\partial}{\partial u}\psi(u; T) = \)
10.5141 \frac{\partial^2}{\partial u^2} \tilde{\psi}(u; T) - 6.2985 \frac{\partial^3}{\partial u^3} \tilde{\psi}(u; T) = 0 \text{ with initial conditions}
\tilde{\psi}(u; 0) = 0, \tilde{\psi}(0; T) = \beta(T), \lim_{u \to \infty} \tilde{\psi}(u; T) = 1, 
\lim_{u \to 0} \frac{\partial}{\partial u} \tilde{\psi}(u; T) = 0.9091 \beta(T) + \frac{\partial}{\partial T} \beta(T), \lim_{u \to 0} \frac{\partial}{\partial u} \tilde{\psi}(u; T) = -0.71113 \beta(T) + 0.9091 \frac{\partial}{\partial T} \beta(T) + \frac{\partial^2}{\partial u^2} \beta(T), \text{where}

\beta(T) = \frac{1}{1 + T} \int_0^{1.1T} \Phi \left( \frac{x - 0.7909651999T}{\sqrt{0.7909651999T}} \right) dx.

Solving the above PDE, one may approximate finite-time survival probability \( \tilde{\psi}(u; T) \) of compound Poisson process (1) by \( \tilde{\psi}(u; T) \), that its behavior (for \( T = 50, 100, 200 \)) has been illustrated in Figure 4.

Figure 3: Behavior for approximated infinite-time ruin probability \( \psi_{\infty}(u) \).

Figure 4: Behavior for approximated finite-time ruin probability \( \psi_{\infty}(u; T) \), for \( T = 50, 100, 200 \).

It worthwhile to mention that: A given density function (or a density function corresponding to a given data set) can be approximated by a Hyperexponential distribution using a Matlab package called “bayesf”, see [28] for more details.

5 Conclusion and Suggestions

This article approximates claim size density function \( f_X(x) \) by a \( n \)-component Hyperexponential density function \( f_{X_{\mathbf{GHE}}}^{(n)}(\cdot) \). Then, it restates the problem of finding an infinite-time (or finite-time) ruin probability as a \((n + 1)\)-order ordinary differential equation (or a partial differential equation for finite-time ruin probability).

Application of our findings has been given though a simulation study.

Certainly the following generalized Hyperexponential distribution can be provided a more accurate approximation in the situation that the true density function (or recorded data) has more than one mode.

\[
g_{X}^{GHE}(x) = \sum_{i=1}^{20} \omega_i e^{-\mu_i (x-b_i)} I_{[b_i, \infty]}(x).
\]

In such situation the finite and infinite ruin probabilities can be evaluated using the following lemma.

**Lemma 4.** Suppose claim size density function \( f_X(x) \) has been approximated by generalized Hyperexponential distribution \( g_{X}^{GHE}(\cdot) \). The survival probability can be found by the following two inverse Laplace transforms.

(i) The Laplace transform of the infinite-time survival probability can be found by the following equation

\[
L(\psi(u); u; s) = \frac{c e^{\psi(0)}}{cs - \lambda + \lambda \sum_{i=1}^{20} \frac{e^{-\mu_i s}}{\mu_i + s} e^{-sb_i}}.
\]

(ii) The Laplace transform of the finite-time survival probability can be found by the following equation

\[
L(\psi(u); T; u; s) = \frac{c e^{\psi(0); T; B(s)}}{B(s)} + \left( \frac{1}{s} - \frac{c e^{\psi(0); T; B(s)}}{B(s)} \right) e^{\frac{\mu_i s}{\mu_i + s}},
\]

where \( B(s) = cs - \lambda + \lambda \sum_{i=1}^{20} \frac{e^{-\mu_i s}}{\mu_i + s} e^{-sb_i} \).

**Proof.** The desired result arrives by taking a Laplace transform from both sides of equations (3) and (4) and solving corresponding first-order PDE with boundary condition \( \psi(u; 0) = 1 \) or \( L(\psi(u); 0; u; s) = 1/s \). Another possibility can be using a Lévy process to evaluate the ruin probability, see [21] and [22] for more details.

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References


