Generally Weighted Moving Average Sign Control Chart using Repetitive Sampling

S. Phanyaem

Abstract—**The Double Generally Weighted Moving Average Sign (DGWMA Sign) and Double Exponentially Weighted Moving Average Sign (DEWMA Sign) control charts are used to control and monitor the mean of process. The objective of this paper is to compare the efficiency of DGWMA Sign and DEWMA Sign control charts for detecting the mean shift in the process. The data is generated by Monte Carlo Simulation technique. The criteria for evaluating the performance of control charts are referred to as, the Average Run Length. The DGWMA Sign control chart is more sensitive than DEWMA Sign control chart in the case of process has small shifts in mean.**

Index Terms—**Double Generally Weighted Moving Average Sign Control Chart, Double Exponentially Weighted Moving Average Sign Control Chart, Average Run Length**

I. Introduction

Statistical Process Control (SPC) techniques are widely used in industry for process monitoring and quality improvement. Control charts are applied in engineering, public health, economics, finance, medicine and in other areas of applications. Traditional charts are based on an assumption that process data is statistically independent and normally distributed when the process is in control. Data in real production process arises from a process with a non-normal or unknown distribution. Hence, the commonly used Shewhart control chart, which requires normality of the monitoring statistic is not suitable. Thus, Nonparametric control chart seems to be a reasonable alternative.

The Shewhart control chart detects relatively large shifts in the process mean ($\geq 1.5\sigma$) and was first proposed by Walter A. Shewhart [1]. An alternative control chart is primarily used to detect smaller shifts, namely Exponentially Weighted Moving Average (EWMA) control chart. Roberts, S.W. [2] originally developed the EWMA control chart. It has used in various industries especially the chemical industry. The EWMA control chart is used in statistical process control to monitor the output of manufacturing process by tracking the moving average of performance over lifetime of the process.

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S. Phanyaem is with the Applied Statistics Department, King Mongkut's University of Technology North Bangkok, Bangkok, 10800, THAILAND (phone : 662-555-2000 e-mail: suvimol.p@sci. kmutnb.ac.th).

A Generally Weighted Moving Average (GWMA) control chart established by Sheu S. and Lin TC. [3]. The GWMA control chart with time-varying control limits to detect shift more sensitively performs better in detecting small shifts of the process mean. Yang S., Lin L. and Cheng SW. [4] presented the Nonparametric EWMA Sign control chart for monitoring and detecting possible deviation from the process target. Muhammad A., Muhammad Azam and Chi-Hyuck J. [5] proposed the Nonparametric control chart based on EWMA sign statistics by using repetitive sampling. Shin L. [6] proposed the Nonparametric Generally Weighted Moving Average Sign (GWMA Sign) control chart for improving the detection capability in small process shift.

A common characteristic used for comparing performance of control charts is Average Run Length (ARL), the expected number of observations taken from an in-control process until the control chart falsely signals out-of-control is denoted by ARL_0. An ARL_0 will be regarded as acceptable if it is large enough to keep the level of false alarms at an acceptable level. A second common characteristic is the expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control is denoted by ARL_1.

The aim of this paper is concerned with the use control charts for detecting the shifts in mean. The main objective is to provide a comparative study of the competitor control charts: Double Generally Weighted Moving Average Sign (DGWMA Sign) and Double Exponentially Weighted Moving Average Sign (DEWMA Sign) control charts with normal observations.

This paper is divided into five sections: in Section 1, we introduce the statistical process control charts. In Section 2 presents the characteristics of DGWMA Sign control chart. In Section 3 proposes characteristics of DEWMA Sign chart. In Section 4, we present the results of comparison. Finally, we provide a conclusion.

II. Characteristics of DGWMA Sign Control Charts

The Double Generally Weighted Moving Average Sign (DGWMA Sign) control chart was introduced by Shin L. [6]. Let X be the characteristic following normal distribution with mean μ and variance σ^2, where μ is the target value (T) of the mean. Consequently, the deviation of the process from the target value should be $Y = X - T$. Thus, the process proportion is denoted by p. The process is regarded as in-

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control state when $p = P(Y > 0) = 0.5$ and the process is considered to be out-of-control state when $p \neq 0.5$.

For the monitoring purpose, a random sample of size (n) is selected at each subgroup from the process. Define

$$I_{jt} = \begin{cases} 1, & Y_{jt} = X_{jt} - T > 0 \; ; \; j = 1,2,...,n \; ; \; t = 1,2,... \\ 0, & otherwise \end{cases}$$

Let N_t represent the total number of value for $N_t = \sum_{j=1}^{n} I_{jt}$

Thus, N_t is binomial distribution with parameter n and $p = 0.5$ for the in-control process.

Let G_t denote the generally weighted moving average in the plotted test statistics at time t. The DGWMA Sign statistic is defined as follows:

$$G_t = \sum_{j=1}^{t} (q^{(j-1)^{\alpha}} - q^{j^{\alpha}}) N_{t-j+1} + q^{t^{\alpha}} T \quad (1)$$

where q is a constant parameter, $0 \leq q < 1$.

α is an exponential smoothing parameter, $0 < \alpha \leq 1$.

When the process is in control, the mean and the variance of G_t are given as:

$$E(G_t) = E\left(\sum_{j=1}^{t} \left(q^{(j-1)^{\alpha}} - q^{j^{\alpha}} \right) N_{t-j+1} + q^{t^{\alpha}} T \right) = \frac{n}{2}$$

$$Var(G_t) = Var\left(\sum_{j=1}^{t} \left(q^{(j-1)^{\alpha}} - q^{j^{\alpha}} \right) N_{t-j+1} + q^{t^{\alpha}} T \right)$$

$$= \sum_{j=1}^{t} \left(q^{(j-1)^{\alpha}} - q^{j^{\alpha}} \right)^2 Var(N_j)$$

$$= Q.\frac{n}{4}$$

where $Q = \lim_{t \to \infty} \left(\sum_{j=1}^{t} \left(q^{(j-1)^{\alpha}} - q^{j^{\alpha}} \right)^2 \right)$.

The two upper control limits of Double Generally Weighted Moving Average Sign (DGWMA Sign) control chart are:

$$UCL_{DGWMA1} = \frac{n}{2} + L_1 \sqrt{Q.\left(\frac{n}{4}\right)} \quad (2)$$

and

$$UCL_{DGWMA2} = \frac{n}{2} + L_2 \sqrt{Q.\left(\frac{n}{4}\right)}. \quad (3)$$

The two lower control limits of Double Generally Weighted Moving Average Sign (DGWMA Sign) control chart are:

$$LCL_{DGWMA1} = \frac{n}{2} - L_1 \sqrt{Q.\left(\frac{n}{4}\right)} \quad (4)$$

and

$$LCL_{DGWMA2} = \frac{n}{2} - L_2 \sqrt{Q.\left(\frac{n}{4}\right)} \quad (5)$$

where L_1 and L_2 are the control limit coefficients, $L_1 \geq L_2$ to be determined for DGWMA Sign control chart.

III. Characteristics of DEWMA Sign Control Charts

The Double Exponentially Weighted Moving Average Sign control chart was introduced by Muhammad A. [5], which used for monitoring the process.

Assume that the quality characteristic X has a target value T. The deviation from the process target at any time denote by $Y = X - T$. Let $p = P(Y > 0)$ is the process proportion. If the process is in-control $p = 0.5$, when the process is out-of-control $p \neq 0.5$. To determine

$$I_{jt} = \begin{cases} 1, & Y_{jt} = X_{jt} - T > 0 \; ; \; j = 1,2,...,n \; ; \; t = 1,2,... \\ 0, & otherwise \end{cases}$$

Let M_t represents the t^{th} sequentially recorded number of $Y_{jt} = X_{jt} - T > 0$ from the process, which calculated by:

$$M_t = \sum_{j=1}^{n} I_j.$$

The DEWMA Sign statistic is given as follows:

$$D_t = \lambda M_t + (1-\lambda) D_{t-1} \quad (6)$$

where λ is an exponential smoothing parameter, $0 < \lambda \leq 1$.

The mean of the DEWMA Sign statistic is given by:

$$E(D_t) = \frac{n}{2}.$$

And the variance of the DEWMA Sign statistic is given by:

$$Var(D_t) = \frac{\lambda}{2-\lambda}\left(\frac{n}{4}\right).$$

Muhammad A. [2] proposed the double control limits for DEWMA control chart based on repetitive sampling. Thus two upper control limits for DEWMA Sign control chart denoted by UCL_{DEWMA1} and UCL_{DEWMA2} are given as follows:

$$UCL_{DEWMA1} = \frac{n}{2} + k_1 \sqrt{\frac{\lambda}{2-\lambda}\left(\frac{n}{4}\right)} \quad (7)$$

$$UCL_{DEWMA2} = \frac{n}{2} + k_2 \sqrt{\frac{\lambda}{2-\lambda}\left(\frac{n}{4}\right)} \quad (8)$$

And two lower control limits denoted by LCL_{DEWMA1} and LCL_{DEWMA2} are given as follows:

$$LCL_{DEWMA1} = \frac{n}{2} - k_1 \sqrt{\frac{\lambda}{2-\lambda}\left(\frac{n}{4}\right)} \quad (9)$$

$$LCL_{DEWMA2} = \frac{n}{2} - k_2 \sqrt{\frac{\lambda}{2-\lambda}\left(\frac{n}{4}\right)} \quad (10)$$

where k_1 and k_2 are the control limit coefficients, $k_1 \geq k_2$ to be determined for DEWMA Sign control chart.

IV. Comparison of Performance of Control Charts

Generally, the performance of control charts evaluated by the ARL. The ARL should be sufficiently large to avoid false alarms when the process is in-control, but it should be sufficiently small to rapidly detect shifts when the process is out-of-control.

We calculate the ARL of Double Generally Weighted

Moving Average Sign (DGWMA Sign) and Double Exponentially Weighted Moving Average Sign (DEWMA Sign) control charts by using Monte Carlo simulations technique.

In this section, we compare the efficiency of control charts between DGWMA Sign and DEWMA Sign control charts. We consider the case in which the observations are normal distribution with parameter μ, σ^2. In situation the process is in-control state, we let the parameter value $\mu = \mu_0 = 1$ and the process is out-of-control state, the parameter value $\mu = \mu_1 = \mu_0 + \delta\mu_0$ where δ is the magnitude of shift size; δ = 0.0, 0.01, 0.03, 0.05, 0.07, 0.09, 0.10, 0.20, 0.30, 0.40 and 0.50.

The results of ARL are presented in Table I – Table III. The parameter values of DGWMA Sign and DEWMA Sign control charts chosen by given $ARL_0 = 370$.

In Table I and Fig. 1, we compare between the ARL_1 of DGWMA Sign control chart and DEWMA Sign control chart. The parameter values of control charts established by setting $ARL_0 = 370$, $\lambda = 0.01$ and $\alpha = 0.01$. From the Table I and Fig. 1, we find that the DGWMA Sign chart appears as good as the DEWMA Sign control chart.

TABLE I: COMPARISON OF ARL_1 BETWEEN DEWMA SIGN AND DGWMA SIGN CHARTS GIVEN $ARL_0 = 370$, $\lambda = 0.01$ AND $\alpha = 0.01$.

Shift size δ	DEWMA Sign	DGWMA Sign
0.00	370.2283	370.0408
0.01	330.8350	295.0707*
0.03	279.9827	171.6067*
0.05	225.5293	65.1504*
0.07	181.7790	39.2231*
0.09	149.3987	24.2500*
0.10	126.7071	19.9202*
0.20	109.1409	15.5716*
0.30	96.2375	13.1070*
0.40	86.3759	11.4048*
0.50	77.8103	10.0694*

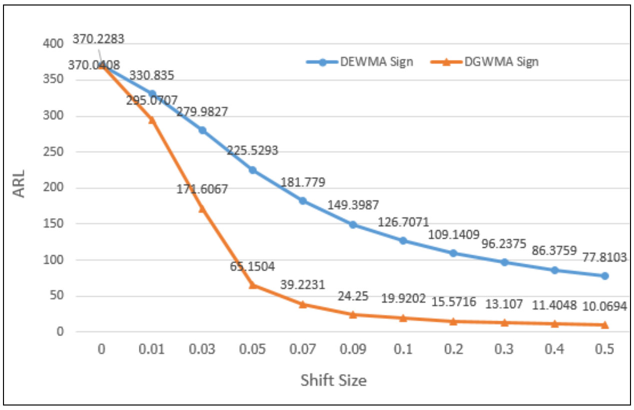

Fig. 1. Comparison of ARL_1 between DEWMA Sign and DGWMA Sign Charts given $ARL_0 = 370$, $\lambda = 0.01$ and $\alpha = 0.01$.

In Table II and Fig. 2, we compare between the ARL_1 of DGWMA Sign control chart and DEWMA Sign control chart. The parameter values of control charts established by setting $ARL_0 = 370$, $\lambda = 0.05$ and $\alpha = 0.05$. The DEWMA Sign control chart is more sensitive to small process mean shifts ($\delta < 0.03\sigma$) than DGWMA Sign control chart. In the case of shift size ($0.05 \le \delta \le 0.50$), the DGWMA Sign control chart can detect shifts more quickly than DEWMA Sign control chart.

TABLE II: COMPARISON OF ARL_1 BETWEEN DEWMA SIGN AND DGWMA SIGN CHARTS GIVEN $ARL_0 = 370$, $\lambda = 0.05$ AND $\alpha = 0.05$.

Shift size δ	DEWMA Sign	DGWMA Sign
0.00	370.2283	370.5029
0.01	329.4340*	349.5188
0.03	268.7887*	295.7543
0.05	224.6173	172.3599*
0.07	172.7890	76.5630*
0.09	134.2875	50.3408*
0.10	125.8182	21.1424*
0.20	108.2517	12.2639*
0.30	95.3286	10.6752*
0.40	87.5668	7.8213*
0.50	75.7214	6.5377*

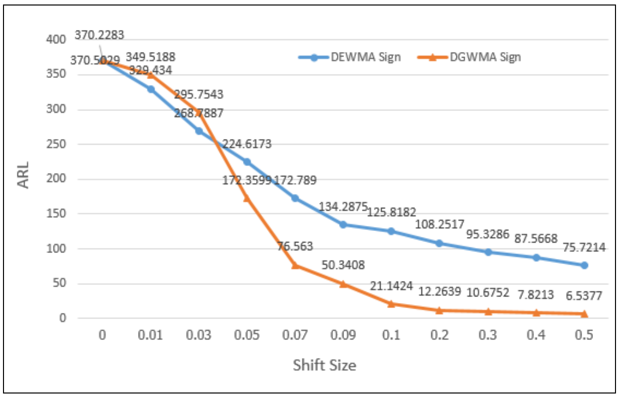

Fig. 2. Comparison of ARL_1 between DEWMA Sign and DGWMA Sign Charts given $ARL_0 = 370$, $\lambda = 0.05$ and $\alpha = 0.05$.

In Table III and Fig. 3, we compare between the ARL_1 of DGWMA Sign control chart and DEWMA Sign control chart. The parameter values of control charts established by setting $ARL_0 = 370$, $\lambda = 0.10$ and $\alpha = 0.10$. The DEWMA Sign control chart is more sensitive than DGWMA Sign control chart for detecting small shifts in the mean of a process when the shifts are less than 0.03σ. In the case of shift sizes are between 0.05 and 0.50 ($0.05 \le \delta \le 0.50$), the DGWMA Sign control chart can detect shifts more quickly than DEWMA Sign control chart.

TABLE III: COMPARISON OF ARL_1 BETWEEN DEWMA SIGN AND DGWMA SIGN CHARTS GIVEN $ARL_0 = 370$, $\lambda = 0.10$ AND $\alpha = 0.10$.

Shift size δ	DEWMA Sign	DGWMA Sign
0.00	370.2283	370.3411
0.01	328.9452*	358.8931
0.03	275.1738*	328.4747
0.05	223.4182	218.5277*
0.07	179.8490	126.8356*
0.09	145.6997	59.6207*
0.10	124.7273	39.1346*
0.20	107.3418	19.6625*
0.30	95.3376	11.6880*
0.40	84.3819	7.1402*
0.50	73.9243	5.9089*

V. CONCLUSION

A comparison between the ARL_1 of DGWMA Sign control chart and DEWMA Sign control chart in the case of one-sided shift, it has been shown that the DGWMA Sign control chart is the best control chart in the sense that it has minimized the ARL_1 when the process has a small shift.

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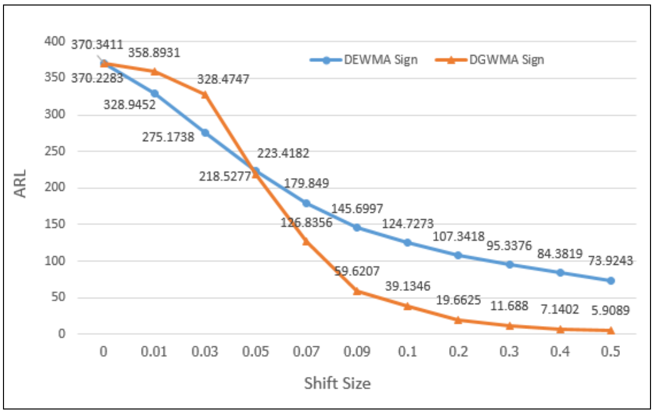

Fig. 3. Comparison of ARL_1 between DEWMA Sign and DGWMA Sign Charts given ARL_0 = 370, λ = 0.10 and α = 0.10.

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