New Criteria on Delay-range-dependent Robust Stability for LPD Discrete-time System with Interval Discrete and Distributed Time-varying Delays and Nonlinear Perturbations

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Abstract—This paper focuses on problem of delay-range-dependent robust stability for linear parameter dependent (LPD) discrete-time system with interval discrete and distributed time-varying delays and nonlinear perturbations. Based on the combination of mixed model transformation, various forms of inequalities, utilization of zero equation and a new parameter dependent Lyapunov-Krasovskii functional, new delay-range-dependent asymptotic stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs) for the system. Numerical example are given finally to demonstrate the effectiveness and less conservativeness of the proposed stability criteria.

Index Terms—parameter dependent Lyapunov-Krasovskii functional, linear matrix inequality (LMI), LPD discrete-time system, interval time-varying delay.

I. INTRODUCTION

Discrete dynamical systems with state delays have been studied rather extensively in the past several years and are used as models to describe in the fields of demography, ecology, economics, engineering, evolutionary biology, finance, mathematics, and physics [1]-[19]. In recent years, the problems of various stability analysis of continuous-time and discrete-time systems subject to time-invariant parametric uncertainty have received considerable attention. An important class of linear time-invariant parametric uncertain system is linear parameter dependent (LPD) system in which the uncertain state matrices are in the polytope consisting of all convex combination of known matrices. Most of stability criteria have been obtained via Lyapunov theory approaches in which parameter dependent Lyapunov functions have been employed [1], [7], [8], [14], [15], [17]. These conditions are always expressed in terms of LMIs which can be solved numerically by using available tools such as LMI toolbox in MATLAB.

In this paper, delay-range-dependent robust stability criteria for LPD discrete-time systems with mixed interval time-varying delays and nonlinear perturbations are studied. Based on the combination of model transformation, utilization of zero equation and parameter dependent Lyapunov functional, new delay-range-dependent robust stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to illustrate the resulting criterion outperforms the existing stability condition.

We introduce some notations, definitions and lemmas that will be used throughout the paper. $\mathbb{R}^n$ denotes the set of non-negative integer numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the vector norm $\| \cdot \|$; $\| x \|$ denotes the Euclidean vector norm of $x \in \mathbb{R}^n$, that is $\| x \|^2 = x^T x$; $\mathbb{R}^{n \times r}$ denotes the space of all real matrices of $(n \times r)$-dimensions; $A^T$ denotes the transpose of the matrix $A$; $A$ is symmetric if $A = A^T$; $I$ denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of $A$; $\lambda_{\text{max}}(A) = \max \{ \Re \lambda : \lambda \in \sigma(A) \}$; $\lambda_{\text{min}}(A) = \min \{ \Re \lambda : \lambda \in \sigma(A) \}$; $\lambda_{\text{max}}(A) = \max \{ \lambda_{\text{max}}(A_i) : i = 1, 2, \ldots, N \}$; $\lambda_{\text{min}}(A) = \min \{ \lambda_{\text{min}}(A_i) : i = 1, 2, \ldots, N \}$; Matrix $A$ is called semi-positive definite ($A \geq 0$) if $x^T A x \geq 0$, for all $x \in \mathbb{R}^n$; $A$ is positive definite ($A > 0$) if $x^T A x > 0$ for all $x \neq 0$; Matrix $B$ is called semi-negative definite ($B \leq 0$) if $x^T B x \leq 0$, for all $x \in \mathbb{R}^n$; $B$ is negative definite ($B < 0$) if $x^T B x < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $A \geq B$ means $A - B \geq 0$; $*$ represents the elements below the main diagonal of a symmetric matrix.

II. PRELIMINARIES

Consider the following LPD discrete-time system with interval time-varying delays and nonlinear perturbations of the form

$$x(k + 1) = A(\alpha)x(k) + B(\alpha)x(k - h(k)) + C(\alpha) \sum_{i=1}^{\infty} \delta(i)x(k - i) + f(k, x(k))$$

$$+ g(k, x(k - h(k)))$$

$$x(s) = \phi(s), \quad s \in \{-h_2, -h_2 + 1, \ldots, 0\},$$

where $k \in \mathbb{Z}^+$, $x(k) \in \mathbb{R}^n$ is the state variable and $\phi(s)$ is an initial value at $s$. $A(\alpha), B(\alpha), C(\alpha) \in \mathbb{R}^{n \times n}$ are uncertain matrices belonging to the polytope of the form

$$A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i, \quad B(\alpha) = \sum_{i=1}^{N} \alpha_i B_i, \quad C(\alpha) = \sum_{i=1}^{N} \alpha_i C_i,$$

$$\sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0, \quad A_i, B_i, C_i \in \mathbb{R}^{n \times n}, \quad i = 1, \ldots, N.$$
\[ f(k, x(k)) \text{ and } g(k, x(k - h(k))) \text{ are the nonlinear perturbations with respect to current state } x(k) \text{ and discrete delay state } x(k - h(k)), \text{ respectively, and are bounded in magnitude:} \]

\[
f^T(k, x(k)) f(k, x(k)) \leq a^2 x^T(k) x(k), \quad (3)\]

\[
g^T(k, x(k - h(k))) g(k, x(k - h(k))) \leq \beta^2 x^T(k - h(k)) x(k - h(k)), \quad (4)\]

where \( \alpha \) and \( \beta \) are given positive real constants. In addition, we assume that the discrete time-varying delay \( h(k) \) is upper and lower bounded. It satisfies the following assumption of the form

\[ 0 < h_1 \leq h(k) \leq h_2, \]

where \( h_1 \) and \( h_2 \) are known positive integers. There exist one constant scalar \( \delta \) such that the function \( \delta(i) \) satisfies

\[ \sum_{i=1}^{\infty} \delta(i) = w < \infty. \quad (5) \]

**Definition 1.** [15] The system (1) is said to be robustly stable if there exists a positive definite function \( V(k) : \mathbb{Z}^+ \rightarrow \mathbb{R} \) such that

\[ \Delta V(k) = V(k+1) - V(k) < 0, \]

along any trajectory of solution of system (1) with (2).

**Lemma 2.** [4] For any constant matrix \( W \in M^{n \times n}, W = W^T > 0 \), two integers \( r_M \) and \( r_m \), satisfying \( r_M \geq r_m \), vector function \( x : [r_m, r_M] \rightarrow \mathbb{R}^n \), the following inequality holds:

\[
\left( \sum_{i=r_m}^{r_M} x(i) \right)^T W \left( \sum_{i=r_m}^{r_M} x(i) \right) \leq (r_M - r_m + 1) \sum_{i=r_m}^{r_M} x^T(i) W x(i).
\]

**Lemma 3.** [16] Let \( Q \in R^{n \times n} \) be a positive-definite matrix, \( X_i \in R^n, i = 1, 2, \ldots, \) If the sums concerned are well defined, then

\[
\sum_{i=k-M}^{k-N} \sum_{j=1}^{k-N} X_j^T Q \left[ \sum_{i=k-M}^{k-N} \sum_{j=1}^{k-N} X_j \right] \leq \frac{(M-N)^2}{2} \sum_{i=k-M}^{k-N} \sum_{j=1}^{k-N} X_j^T Q X_j.
\]

**Lemma 4.** [19] Let \( M \in R^{n \times n} \) be a positive-definite matrix, \( X_i \in R^n \) and \( a_i \geq 0, i = 1, 2, \ldots, \). If the sums concerned are well defined, then

\[
\left[ \sum_{i=k-N}^{k-1} X_i \right]^T M \left[ \sum_{i=k-N}^{k-1} X_i \right] \leq N \sum_{i=k-N}^{k-1} X_i^T M X_i,
\]

\[
\left[ \sum_{i=k-N}^{k-1} X_i \right]^T M \left[ \sum_{i=k-N}^{k-1} X_i \right] \leq \frac{(N)^2}{2} \sum_{i=k-N}^{k-1} X_i^T M X_i,
\]

\[
\leq \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} a_i X_i^T M X_i.
\]

**III. MAIN RESULTS**

In this section, we derive a new delay-rage-dependent criterion for the system (1) using the Lyapunov functional method combining with linear matrix inequality technique. We introduce the following notations for later use.

\[
L_w(\alpha) = \sum_{i=1}^{N} a_i L_i^w, \quad M(\alpha) = \sum_{i=1}^{N} a_i M_i,
\]

\[
Z_1(\alpha) = \sum_{i=1}^{N} a_i Z_1^i, \quad Z_2(\alpha) = \sum_{i=1}^{N} a_i Z_2^i,
\]

\[
P_k(\alpha) = \sum_{i=1}^{N} a_i P_k^i, \quad Q_k(\alpha) = \sum_{i=1}^{N} a_i Q_k^i,
\]

\[
N \sum_{i=1}^{N} a_i = 1, \quad \alpha \geq 0,
\]

\[
L_i^w, M_i, Z_i^1, Z_i^2, P_i^k, Q_i^k, \in M^{n \times n},
\]

\[
w = 1, 2, \ldots, 6, \quad k = 1, 2, \ldots, 18,
\]

\[
s = 1, 2, \ldots, 19, \quad i = 1, 2, \ldots, N.
\]

\[
\prod_{i,j} = \left[ \sum_{i,j}^{n,m} \right]_{27 \times 27}, \quad (6)
\]

where \( \sum_{i,j}^{m,n} = \sum_{i,j}^{m,n} \).
\[\begin{align*}
\Sigma_{i,j}^{1,2} &= -G_1^i - L_i T B_1^i + A_i T L_2^i + B_1^i T L_3^i - L_4^i, \\
\Sigma_{i,j}^{1,2} &= -G_2^i - L_i T A_2^i + A_i T L_4^i + B_1^i T L_6^i - L_6^i, \\
\Sigma_{i,j}^{1,2} &= L_i T, \\
\Sigma_{i,j}^{1,2} &= L_i T C_j, \\
\Sigma_{i,j}^{2,2} &= P_1^i - L_i T - L_2^i + h^2_1 Q_1^i + r^2 h_2 Q_6^i + h_3 Q_8^i \\
&\quad + r^2 h_3 Q_1^i + p^2 Q_1^i + r^2 p^2 Q_5^i + h_1 P_5^i \\
&\quad + h_2 P_1^{11} + h_r P_1^{12} + rh_2 P_1^{13} + h_2 P_1^{14} \\
&\quad + h_2 P_1^{15} + r^2 p_1^{16} + \frac{h_4}{4} p_1^{17} + \frac{(h_2 - h_1)^2}{4} p_1^{18} \\
&\quad + h_2^2 Z_i^2 + (r h_2)^2 Z_i^2, \\
\Sigma_{i,j}^{2,5} &= -G_1^i - L_i T B_3^i - L_3^i, \\
\Sigma_{i,j}^{2,8} &= -G_2^i - L_i T A_3^i - L_4^i, \\
\Sigma_{i,j}^{2,21} &= -G_1^i - L_i T B_4^i - L_5^i, \\
\Sigma_{i,j}^{2,22} &= -G_2^i - L_i T A_5^i - L_6^i, \\
\Sigma_{i,j}^{2,25} &= L_i T, \\
\Sigma_{i,j}^{2,26} &= L_i T, \\
\Sigma_{i,j}^{2,27} &= L_i T C_j, \\
\Sigma_{i,j}^{3,3} &= -P_2^i + P_6^i - P_1^{18}, \\
\Sigma_{i,j}^{3,23} &= -P_2^i + P_6^i + p_1^{18}, \\
\Sigma_{i,j}^{4,4} &= -P_4^i - P_6^i, \\
\Sigma_{i,j}^{5,5} &= L_i T B_2^i + B_2 T L_2^i - P_8^i + P_2^i + e_2^2 T^2 I, \\
\Sigma_{i,j}^{5,8} &= L_i T A_2^i + B_2 T L_3^i, \\
\Sigma_{i,j}^{5,21} &= -L_i T B_1^i - B_1 T L_4^i, \\
\Sigma_{i,j}^{5,22} &= L_i T A_1^i + B_1 T L_5^i, \\
\Sigma_{i,j}^{5,24} &= L_i T, \\
\Sigma_{i,j}^{5,25} &= L_i T, \\
\Sigma_{i,j}^{5,26} &= L_i T, \\
\Sigma_{i,j}^{5,27} &= L_i T C_j, \\
\Sigma_{i,j}^{6,6} &= -P_3^i + P_7^i, \\
\Sigma_{i,j}^{7,7} &= -P_5^i - P_7^i, \\
\Sigma_{i,j}^{8,8} &= L_i T A_2^i + A_2 T L_4^i - P_9^i - P_1^17, \\
\Sigma_{i,j}^{8,21} &= -L_i T B_1^i - A_1 T L_5^i, \\
\Sigma_{i,j}^{8,22} &= L_i T A_1^i + A_2 T L_5^i, \\
\Sigma_{i,j}^{8,25} &= L_i T, \\
\Sigma_{i,j}^{8,26} &= L_i T, \\
\Sigma_{i,j}^{8,27} &= L_i T C_j, \\
\Sigma_{i,j}^{9,9} &= -Q_1^i, \\
\Sigma_{i,j}^{9,15} &= -Q_2^i, \\
\Sigma_{i,j}^{10,10} &= -Q_3^i, \\
\Sigma_{i,j}^{10,16} &= -Q_4^i, \\
\Sigma_{i,j}^{11,11} &= -Q_5^i, \\
\Sigma_{i,j}^{11,17} &= -Q_6^i, \\
\Sigma_{i,j}^{12,12} &= -Q_7^i, \\
\Sigma_{i,j}^{12,18} &= -Q_8^i, \\
\Sigma_{i,j}^{13,13} &= -Q_9^i, \\
\Sigma_{i,j}^{13,19} &= -Q_1^i, \\
\Sigma_{i,j}^{14,14} &= -Q_6^i, \\
\Sigma_{i,j}^{14,20} &= -Q_7^i, \\
\Sigma_{i,j}^{15,15} &= -Q_8^i, \\
\Sigma_{i,j}^{16,16} &= -Q_9^i, \\
\Sigma_{i,j}^{17,17} &= -Q_1^i, \\
\Sigma_{i,j}^{18,18} &= -Q_6^i, \\
\Sigma_{i,j}^{19,19} &= -Q_7^i, \\
\Sigma_{i,j}^{20,26} &= -Q_8^i, \\
\Sigma_{i,j}^{21,21} &= -L_i T B_1^i - B_1^2 T L_j^i - Z_1^i, \\
\Sigma_{i,j}^{21,22} &= L_i^5 T A_2^i - B_1^2 T L_j^6, \\
\Sigma_{i,j}^{21,25} &= L_i^5 T, \\
\Sigma_{i,j}^{21,26} &= L_i^5 T C_j, \\
\Sigma_{i,j}^{22,22} &= L_i^5 T A_3^i + A_2^2 T L_j^6 - Z_2^i, \\
\Sigma_{i,j}^{22,25} &= L_i^5 T C_j, \\
\Sigma_{i,j}^{22,26} &= L_i^5 T, \\
\Sigma_{i,j}^{22,27} &= L_i^5 T C_j, \\
\Sigma_{i,j}^{23,23} &= -P_1^i, \\
\Sigma_{i,j}^{24,24} &= -P_2^i, \\
\Sigma_{i,j}^{25,25} &= -e_1 I, \\
\Sigma_{i,j}^{26,26} &= -e_2 I, \\
\Sigma_{i,j}^{27,27} &= \frac{P_1^{10}}{2}, \\
G_1^i &= P_1^3 J, \\
G_2^i &= P_1^3 W, \\
\rho &= h_2 - h_1, \\
\alpha(k) &= h(k) - h_1, \\
\beta(k) &= h_2 - h(k), \\
\psi(k) &= \frac{1}{\alpha(k)} \left[ \prod_{i=k-h(k)}^{k-h_1-1} x(i) \right], \\
\phi(k) &= \frac{1}{\beta(k)} \left[ \prod_{i=k-h_2}^{k-h(k)-1} x(i) \right], \\
other\text{terms are 0.}
\end{align*}\]

**Theorem 5.** The system (1) is robustly stable, if there exist positive definite symmetric matrices \(M_i, Z_1^i, Z_2^i, P_{sk}, Q_{sk}^i, k = 1, 2, \ldots, s, s = 1, 2, \ldots, 19, i = 1, 2, \ldots, N\), any appropriate dimensional matrices \(M_i', N_i', L_i', r = 1, 2, 3, 4, l = 1, 2, \ldots, 6, i = 1, 2, \ldots, N\) and positive real constants \(e_1, e_2\) satisfying the following LMIs:

\[
\Pi_{i,j} < -1, \quad i = 1, 2, \ldots, N, \quad (7)
\]

\[
\Pi_{i,j} + \Pi_{j,i} < \frac{2}{(N-1)I}, \quad i = 1, \ldots, N - 1, \quad j = i + 1, \ldots, N, \quad (8)
\]
Consider the following Lyapunov-Krasovskii function for system (21)-(22) of the form

\[ V(k) = \sum_{i=1}^{8} V_i(k), \quad (23) \]

where

\[ V_1(k) = x^T(k)P_1(\alpha)x(k), \]
\[ V_2(k) = \sum_{i=k-h_1}^{k-1} x^T(i)P_2(\alpha)x(i) + \sum_{i=k-rh_1}^{k-1} x^T(i)P_3(\alpha)x(i) + \sum_{i=k-h_2}^{k-1} x^T(i)P_4(\alpha)x(i) + \sum_{i=k-rh_2}^{k-1} x^T(i)P_5(\alpha)x(i) + \sum_{i=k-h_3}^{k-1} x^T(i)P_6(\alpha)x(i) + \sum_{i=k-rh_3}^{k-1} x^T(i)P_7(\alpha)x(i) + \sum_{i=k-rh_4}^{k-1} x^T(i)P_8(\alpha)x(i), \]
\[ V_3(k) = h_1 \sum_{j=-h_1}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_1(\alpha) & Q_2(\alpha) \\ Q_3(\alpha) & Q_4(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} + rh_1 \sum_{j=-r_h_1}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_5(\alpha) & Q_6(\alpha) \\ Q_7(\alpha) & Q_8(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} + rh_2 \sum_{j=-r_h_2}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_9(\alpha) & Q_{10}(\alpha) \\ Q_{11}(\alpha) & Q_{12}(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} + \rho \sum_{j=-h_3}^{-h_1-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_{13}(\alpha) & Q_{14}(\alpha) \\ Q_{15}(\alpha) & Q_{16}(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} + \rho \sum_{j=-h_3}^{-h_1-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \begin{bmatrix} Q_{17}(\alpha) & Q_{18}(\alpha) \\ Q_{19}(\alpha) & Q_{20}(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}, \]

**Proof.** From model transformation method, we rewrite the system (1) in the following system:

\[ x(k + 1) = x(k) + y(k), \quad (13) \]
\[ y(k) = \left[ A(\alpha) - I \right] x(k) + B(\alpha)x(k - h(k)) + C(\alpha) \sum_{i=1}^{\infty} \delta(i)x(k - i) + f(k, x(k)) + g(k, x(k - h(k))), \quad (14) \]

In order to improve the bound of the discrete delay \( h(k) \) in (1), let us decompose constant matrix \( A(\alpha) \) and \( B(\alpha) \) as

\[ A(\alpha) = A_1(\alpha) + A_2(\alpha), \quad (15) \]
\[ B(\alpha) = B_1(\alpha) + B_2(\alpha), \quad (16) \]

where \( A_1(\alpha) = \sum_{i=1}^{N} \alpha_1 A_1^T, A_2(\alpha) = \sum_{i=1}^{N} \alpha_2 A_2^T, B_1(\alpha) = \sum_{i=1}^{N} \alpha_1 B_1^T, B_2(\alpha) = \sum_{i=1}^{N} \alpha_2 B_2^T, i = 1, 2, 3, ..., N \in \mathbb{R}^{n \times n} \) are constant matrices. By utilizing the following zero equations, we have

\[ x(k) - x(k - h(k)) = \sum_{i=k-h}^{k-1} y(i) = 0, \quad (17) \]
\[ x(k) - x(k - rh(k)) = \sum_{i=k-rh}^{k-1} y(i) = 0. \quad (18) \]

By utilizing the following zero equations, we have

\[ Jx(k) - Jx(k - h(k)) - J \sum_{i=k-h}^{k-1} y(i) = 0, \quad (19) \]
\[ Wx(k) - Wx(k - rh(k)) - W \sum_{i=k-rh}^{k-1} y(i) = 0, \quad (20) \]

where \( J, W \in \mathbb{R}^{n \times n} \) will be chosen to guarantee the asymptotic stability of the system (1)-(2). By (19)-(20), the system (13)-(14) can be represented by the form

\[ x(k + 1) = x(k) + y(k) + Jx(k) - Jx(k - h(k)) - Wx(k - rh(k)) - W \sum_{i=k-rh}^{k-1} y(i), \]
\[ y(k) = \left[ A_1(\alpha) + B_1(\alpha) - I \right] x(k) + A_2(\alpha)x(k - h(k)) + B_2(\alpha)x(k - h(k)) + C(\alpha) \sum_{i=1}^{\infty} \delta(i)x(k - i) + f(k, x(k)) + g(k, x(k - h(k))). \]
\[
+ r p \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\
\begin{bmatrix} Q_{16}(a) & Q_{17}(a) \\ * & Q_{18}(a) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix},
\]

\[
V_4(k) = \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{10}(\alpha) y(i) \\
+ \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{11}(\alpha) y(i) \\
+ \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{12}(\alpha) y(i) \\
+ \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{13}(\alpha) y(i)
\]

\[
V_6(k) = h_1 \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{14}(\alpha) y(i) \\
+ h_2 \sum_{j=-r_h+1}^{k-1} \sum_{i=1}^{k-1} y^T(i) P_{15}(\alpha) y(i)
\]

From Lemma 2.4, it follows form (24)-(27) that

\[
\Delta V(k) \leq \xi^T(k) \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \prod_{i,j} \xi(k),
\]

where \( \xi^T(k) = \begin{bmatrix} x^T(k) \\ y^T(k) \\ x^T(k-h) \\ x^T(k-h_1) \\ x^T(k-h_2) \\ x^T(k-r_h_1) \\ x^T(k-r_h_2) \\ x^T(k-r_h) \end{bmatrix} \)

By (28)-(30), if the conditions (7)-(12) are true, then

\[
\Delta V(k) < -\omega \|x\|^2,
\]

where \( \omega > 0 \). This means that system (1) is robustly stable. The proof of theorem is complete.

IV. NUMERICAL EXAMPLES

Example 6. Consider the system

\[
x(k+1) = Ax(k) + Bx(k-h(k)) + f(k, x(k)) + g(k, x(k-h(k))).
\]

with the following parameters which is considered in [13] and [18]:

\[
A = \begin{bmatrix} 0.80 & 0.05 \\ 0.09 & 0.90 \end{bmatrix}, \quad B = \begin{bmatrix} -0.10 & 0 \\ -0.20 & -0.10 \end{bmatrix},
\]

\[
\alpha \geq 0, \quad \beta \geq 0.
\]

Decompose matrix A and B as follows : \( A = A_1 + A_2 \), \( B = B_1 + B_2 \), where

\[
A_1 = \begin{bmatrix} -0.5 & -0.1 \\ -0.02 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.3 & 0.1 \\ 0.07 & 1.2 \end{bmatrix},
\]

\[
B_1 = \begin{bmatrix} -0.5 & -0.1 \\ -0.3 & -0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}.
\]

By using the LMI Toolbox in MATLAB (with accuracy 0.01) for applying Theorem 5 to system (32) with (33)-(34), one can obtain the maximum upper bounds of the time delay under different values of \( h_1 \) as shown in Table I. We can see that our results in applying Theorem 5 are much less conservative than in [13] and [18].
Example 7. Consider the system

\[ x(k+1) = Ax(k) + Bx(k - h(k)), \]

having the following parameters:

\[ A = \begin{bmatrix} 0.80 & 0 \\ 0.05 & 0.90 \end{bmatrix}, \quad B = \begin{bmatrix} -0.10 & 0 \\ -0.20 & -0.10 \end{bmatrix}. \]

Decompose matrix \( A \) and \( B \) as follows: \( A = A_1 + A_2, \) \( B = B_1 + B_2, \) where

\[ A_1 = \begin{bmatrix} -0.4 & -0.1 \\ -0.02 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.2 & 0.1 \\ 0.07 & 1.2 \end{bmatrix}, \]

\[ B_1 = \begin{bmatrix} -0.5 & -0.1 \\ -0.3 & -0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}. \]

For given \( h_1 \), we calculate the allowable maximum value of \( h_2 \) that guarantees the asymptotic stability of system (35) with (36). By using different methods, the calculated results are presented in Table II. From the table, we can see that applying Theorem 5 in this paper provides the less conservative results.

V. CONCLUSION

The problem of delay-range-dependent robust stability for LPD discrete-time system with interval discrete and distributed time-varying delays and nonlinear perturbations was studied. By utilizing the inequalities, mixed model transformation, utilization of zero equation and a new parameter dependent Lyapunov-Krasovskii functional, delay-range-dependent stability criteria have been derived and formulated in terms of LMIs for the system. Numerical example are given finally to demonstrate the effectiveness and less conservativeness of the proposed stability criteria.

Competing interests

I declare that I have no competing interests.

TABLE I

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<th>Method</th>
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TABLE II

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REFERENCES