New Criteria on Delay-range-dependent Robust Stability for LPD Discrete-time System with Interval Discrete and Distributed Time-varying Delays and Nonlinear Perturbations

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Abstract—**This paper focuses on problem of delay-range-dependent robust stability for linear parameter dependent (LPD) discrete-time system with interval discrete and distributed time-varying delays and nonlinear perturbations. Based on the combination of mixed model transformation, various forms of inequalities, utilization of zero equation and a new parameter dependent Lyapunov-Krasovskii functional, new delay-range-dependent asymptotic stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs) for the system. Numerical example are given finally to demonstrate the effectiveness and less conservativeness of the proposed stability criteria.**

Index Terms—**parameter dependent Lyapunov-Krasovskii functional, linear matrix inequality (LMI), LPD discrete-time system, interval time-varying delay.**

I. Introduction

DIscrete dynamical systems with state delays have been studied rather extensively in the past several years and are used as models to describe in the fields of demography, ecology, economics, engineering, evolutionary biology, finance, mathematics, and physics [1]-[19]. In recent years, the problems of various stability analysis of continuous-time and discrete-time systems subject to time-invariant parametric uncertainty have received considerable attention. An important class of linear time-invariant parametric uncertain system is linear parameter dependent (LPD) system in which the uncertain state matrices are in the polytope consisting of all convex combination of known matrices. Most of stability criteria have been obtained via Lyapunov theory approaches in which parameter dependent Lyapunov functions have been employed [1], [7], [8], [14], [15], [17]. These conditions are always expressed in terms of LMIs which can be solved numerically by using available tools such as LMI toolbox in MATLAB.

In this paper, delay-range-dependent robust stability criteria for LPD discrete-time systems with mixed interval time-varying delays and nonlinear perturbations are studied. Based on the combination of model transformation, utilization of zero equation and parameter dependent Lyapunov functional, new delay-range-dependent robust stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to illustrates the resulting criterion outperforms the existing stability condition.

We introduce some notations, definitions and lemmas that will be used throughout the paper. $\mathbb{Z}^+$ denotes the set of non-negative integer numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in \mathbb{R}^n$, that is $\|x\|^2 = x^T x$; $\mathbb{R}^{n\times r}$ denotes the space of all real matrices of $(n \times r)$-dimensions; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\max}(A(\alpha)) = \max\{\lambda_{\max}(A_i) : i = 1, 2, ..., N\}$; $\lambda_{\min}(A(\alpha)) = \min\{\lambda_{\min}(A_i) : i = 1, 2, ..., N\}$; Matrix A is called semi-positive definite ($A \geq 0$) if $x^T Ax \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $x^T Ax > 0$ for all $x \neq 0$; Matrix B is called semi-negative definite ($B \leq 0$) if $x^T Bx \leq 0$, for all $x \in \mathbb{R}^n$; B is negative definite ($B < 0$) if $x^T Bx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $A \geq B$ means $A - B \geq 0$; $*$ represents the elements below the main diagonal of a symmetric matrix.

II. Preliminaries

Consider the following LPD discrete-time system with interval time-varying delays and nonlinear perturbations of the form

$$\begin{aligned} x(k+1) &= A(\alpha)x(k) + B(\alpha)x(k-h(k)) \\ &\quad + C(\alpha)\sum_{i=1}^{\infty}\delta(i)x(k-i) + f(k, x(k)) \\ &\quad + g(k, x(k-h(k))), \qquad (1) \\ x(s) &= \phi(s), \quad s \in \{-h_2, -h_2+1, \ldots, 0, \}, \qquad (2) \end{aligned}$$

where $k \in \mathbb{Z}^+$, $x(k) \in \mathbb{R}^n$ is the state variable and $\phi(s)$ is a initial value at s. $A(\alpha)$, $B(\alpha)$, $C(\alpha) \in \mathbb{R}^{n\times n}$ are uncertain matrices belonging to the polytope of the form

$$A(\alpha) = \sum_{i=1}^{N}\alpha_i A_i, \quad B(\alpha) = \sum_{i=1}^{N}\alpha_i B_i, \quad C(\alpha) = \sum_{i=1}^{N}\alpha_i C_i,$$

$$\sum_{i=1}^{N}\alpha_i = 1, \quad \alpha_i \geq 0, \quad A_i, B_i, C_i \in \mathbb{R}^{n\times n}, \quad i = 1, ..., N.$$

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$f(k, x(k))$ and $g(k, x(k-h(k)))$ are the nonlinear perturbations with respect to current state $x(k)$ and discrete delay state $x(k-h(k))$, respectively, and are bounded in magnitude:

$$f^T(k, x(k)) f(k, x(k)) \leq \alpha^2 x^T(k) x(k), \tag{3}$$

$$\begin{aligned} & g^T(k, x(k-h(k))) g(k, x(k-h(k))) \\ & \quad \leq \beta^2 x^T(k-h(k)) x(k-h(k)), \end{aligned} \tag{4}$$

where α and β are given positive real constants. In addition, we assume that the discrete time-varying delay $h(k)$ is upper and lower bounded. It satisfies the following assumption of the form

$$0 < h_1 \leq h(k) \leq h_2,$$

where h_1 and h_2 are known positive integers. There exist one constant scalar δ such that the function $\delta(i)$ satisfies

$$\sum_{i=1}^{\infty} \delta(i) = w < \infty. \tag{5}$$

Definition 1. [15] The system (1) is said to be robustly stable if there exists a positive definite function $V(k) : \mathbb{Z}^+ \to \mathbb{R}$ such that

$$\Delta V(k) = V(k+1) - V(k) < 0,$$

along any trajectory of solution of system (1) with (2).

Lemma 2. [4] For any constant matrix $W \in M^{n \times n}, W = W^T > 0$, two integers r_M and r_m satisfying $r_M \geq r_m$, vector function $x : [r_m, r_M] \to \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned} & \Big(\sum_{i=r_m}^{r_M} x(i) \Big)^T W \Big(\sum_{i=r_m}^{r_M} x(i) \Big) \\ & \leq (r_M - r_m + 1) \sum_{i=r_m}^{r_M} x^T(i) W x(i). \end{aligned}$$

Lemma 3. [16] Let $Q \in R^{n \times n}$ be a positive-definite matrix, $X_i \in R^n, i = 1, 2, \ldots$. If the sums concerned are well defined, then

$$\begin{aligned} & \Big[\sum_{i=k-M}^{k-N-1} \sum_{j=i}^{k-N-1} X_j \Big]^T Q \Big[\sum_{i=k-M}^{k-N-1} \sum_{j=i}^{k-N-1} X_j \Big] \\ & \leq \frac{(M-N)^2}{2} \sum_{i=k-M}^{k-N-1} \sum_{j=i}^{k-N-1} X_j^T Q X_j. \end{aligned}$$

Lemma 4. [19] Let $M \in R^{n \times n}$ be a positive-definite matrix, $X_i \in R^n$ and $a_i \geq 0, i = 1, 2, \ldots$. If the sums concerned are well defined, then

$$\Big[\sum_{i=k-N}^{k-1} X_i \Big]^T M \Big[\sum_{i=k-N}^{k-1} X_i \Big] \leq N \sum_{i=k-N}^{k-1} X_i^T M X_i,$$

$$\begin{aligned} & \Big[\sum_{i=-N}^{-1} \sum_{i=k+j}^{k-1} X_j \Big]^T M \Big[\sum_{i=-N}^{-1} \sum_{i=k+j}^{k-1} X_j \Big] \\ & \leq \frac{(N)^2}{2} \sum_{i=-N}^{-1} \sum_{i=j+k}^{k-1} X_j^T M X_j, \end{aligned}$$

$$\begin{aligned} & \Big[\sum_{i=1}^{\infty} a_i X_i \Big]^T M \Big[\sum_{i=1}^{\infty} a_i X_i \Big] \\ & \leq \Big[\sum_{i=1}^{\infty} a_i \Big] \sum_{i=1}^{\infty} a_i X_i^T M X_i. \end{aligned}$$

III. Main results

In this section, we derive a new delay-rage-dependent criterion for the system (1) using the Lyapunov functional method combining with linear matrix inequality technique. We introduce the following notations for later use.

$$L_w(\alpha) = \sum_{i=1}^{N} \alpha_i L_i^w, \quad M(\alpha) = \sum_{i=1}^{N} \alpha_i M_i,$$

$$Z_1(\alpha) = \sum_{i=1}^{N} \alpha_i Z_i^1, \quad Z_2(\alpha) = \sum_{i=1}^{N} \alpha_i Z_i^2,$$

$$P_k(\alpha) = \sum_{i=1}^{N} \alpha_i P_i^k, \quad Q_s(\alpha) = \sum_{i=1}^{N} \alpha_i Q_i^s,$$

$$\sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0,$$

$$L_i^w, M_i, Z_i^1, Z_i^2, P_i^k, Q_i^s, \in M^{n \times n},$$

$$w = 1, 2, .., 6, \quad k = 1, 2, ..., 18,$$

$$s = 1, 2, ..., 19, \quad i = 1, 2, ..., N.$$

$$\prod\nolimits_{i,j} = \left[\Sigma_{i,j}^{m,n} \right]_{27 \times 27}, \tag{6}$$

where $\Sigma_{i,j}^{m,n} = \Sigma_{i,j}^{n,mT}$, $m, n = 1, 2, 3, ..., 27$, $i, j = 1, 2, 3, ..., N$,

$$\begin{aligned}
\Sigma_{i,j}^{1,1} &= G_i^1 + G_i^2 + {G_i^1}^T + {G_i^2}^T - {L_i^1}^T A_j^1 + {L_i^1}^T B_j^1 \\
&\quad - {L_i^1}^T + {A_i^1}^T L_j^1 + {B_i^1}^T L_j^1 - L_i^1 + P_i^2 + P_i^3 \\
&\quad + P_i^4 + P_i^5 + P_i^8 + P_i^9 + h_1^2 Q_i^1 + r^2 h_1^2 Q_i^4 \\
&\quad + h_2^2 Q_i^7 + r^2 h_2^2 Q_i^{10} + \rho^2 Q_i^{13} + r^2 \rho^2 Q_i^{16} + h_1 M_i^1 \\
&\quad + N_i^1 + {N_i^1}^T + h_2 M_i^2 + N_i^2 + {N_i^2}^T + r h_1 M_i^3 \\
&\quad + N_i^3 + {N_i^3}^T + r h_2 M_i^4 + N_i^4 + {N_i^4}^T - h_1^2 P_i^{17} \\
&\quad + \epsilon_1 \alpha^2 I + w P_i^{19}, \\
\Sigma_{i,j}^{1,2} &= P_i^1 + {G_i^1}^T + {G_i^2}^T - {L_i^1}^T + {A_i^1}^T L_j^2 + {B_i^1}^T L_j^2 \\
&\quad - L_i^2 + h_1^2 Q_i^2 + r^2 h_1^2 Q_i^5 + h_2^2 Q_i^8 + r^2 h_2^2 Q_i^{11} \\
&\quad + \rho^2 Q_i^{14} + r^2 \rho^2 Q_i^{17}, \\
\Sigma_{i,j}^{1,3} &= -N_i^1, \\
\Sigma_{i,j}^{1,4} &= -N_i^2, \\
\Sigma_{i,j}^{1,5} &= -G_i^1 + {L_i^1}^T B_j^2 + {A_i^1}^T L_j^3 + {B_i^1}^T L_j^3 - L_i^3, \\
\Sigma_{i,j}^{1,6} &= -N_i^3, \\
\Sigma_{i,j}^{1,7} &= -N_i^4, \\
\Sigma_{i,j}^{1,8} &= -G_i^2 + {L_i^1}^T A_j^2 + {A_i^1}^T L_j^4 + {B_i^1}^T L_j^4 - L_i^4 \\
&\quad + h_1 P_i^{17},
\end{aligned}$$

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\begin{aligned}
\Sigma_{i,j}^{1,21} &= -G_i^1 - L_i^{1^T} B_j^1 + A_i^{1^T} L_j^5 + B_i^{1^T} L_j^5 - L_i^5,\\
\Sigma_{i,j}^{1,22} &= -G_i^2 + L_i^{1^T} A_j^2 + A_i^{1^T} L_j^6 + B_i^{1^T} L_j^6 - L_i^6,\\
\Sigma_{i,j}^{1,25} &= L_i^{1^T},\\
\Sigma_{i,j}^{1,26} &= L_i^{1^T},\\
\Sigma_{i,j}^{1,27} &= L_i^{1^T} C_j,\\
\Sigma_{i,j}^{2,2} &= P_i^1 - L_i^{2^T} - L_i^2 + h_1^2 Q_i^3 + r^2 h_1^2 Q_i^6 + h_2^2 Q_i^9\\
&\quad + r^2 h_2^2 Q_i^{12} + \rho^2 Q_i^{15} + r^2\rho^2 Q_i^{18} + h_1 P_i^{10}\\
&\quad + h_2 P_i^{11} + r h_1 P_i^{12} + r h_2 P_i^{13} + h_1^2 P_i^{14}\\
&\quad + h_2^2 P_i^{15} + \rho^2 P_i^{16} + \frac{h_1^4}{4} P_i^{17} + \frac{(h_2-h_1)^2}{4} P_i^{18}\\
&\quad + h_2^2 Z_i^1 + (r h_2)^2 Z_i^2\\
\Sigma_{i,j}^{2,5} &= -G_i^1 + L_i^{2^T} B_j^2 - L_i^3,\\
\Sigma_{i,j}^{2,8} &= -G_i^2 + L_i^{2^T} A_j^2 - L_i^4,\\
\Sigma_{i,j}^{2,21} &= -G_i^1 + L_i^{2^T} B_j^1 - L_i^5,\\
\Sigma_{i,j}^{2,22} &= -G_i^2 + L_i^{2^T} A_j^2 - L_i^6,\\
\Sigma_{i,j}^{2,25} &= L_i^{2^T},\\
\Sigma_{i,j}^{2,26} &= L_i^{2^T},\\
\Sigma_{i,j}^{2,27} &= L_i^{2^T} C_j,\\
\Sigma_{i,j}^{3,3} &= -P_i^2 + P_i^6 - P_i^{18},\\
\Sigma_{i,j}^{3,23} &= P_i^{18},\\
\Sigma_{i,j}^{4,4} &= -P_i^4 - P_i^6,\\
\Sigma_{i,j}^{5,5} &= L_i^{3^T} B_j^2 + B_i^{2^T} L_j^3 - P_i^8 - P_i^{18} + \epsilon_2 \beta^2 I,\\
\Sigma_{i,j}^{5,8} &= L_i^{3^T} A_j^2 + B_i^{2^T} L_j^4,\\
\Sigma_{i,j}^{5,21} &= -L_i^{3^T} B_j^1 + B_i^{2^T} L_j^5,\\
\Sigma_{i,j}^{5,22} &= L_i^{3^T} A_j^2 + B_i^{2^T} L_j^6,\\
\Sigma_{i,j}^{5,24} &= P_i^{18},\\
\Sigma_{i,j}^{5,25} &= L_i^{3^T},\\
\Sigma_{i,j}^{5,26} &= L_i^{3^T},\\
\Sigma_{i,j}^{5,27} &= L_i^{3^T} C_j,\\
\Sigma_{i,j}^{6,6} &= -P_i^3 + P_i^7,\\
\Sigma_{i,j}^{7,7} &= -P_i^5 - P_i^7,\\
\Sigma_{i,j}^{8,8} &= L_i^{4^T} A_j^2 + A_i^{2^T} L_j^4 - P_i^9 - P_i^{17},\\
\Sigma_{i,j}^{8,21} &= -L_i^{4^T} B_j^1 + A_i^{2^T} L_j^5,\\
\Sigma_{i,j}^{8,22} &= L_i^{4^T} A_j^2 + A_i^{2^T} L_j^6,\\
\Sigma_{i,j}^{8,25} &= L_i^{4^T},\\
\Sigma_{i,j}^{8,26} &= L_i^{4^T},\\
\Sigma_{i,j}^{8,27} &= L_i^{4^T} C_j,\\
\Sigma_{i,j}^{9,9} &= -Q_i^1,\\
\Sigma_{i,j}^{9,15} &= -Q_i^2,\\
\Sigma_{i,j}^{10,10} &= -Q_i^4,\\
\Sigma_{i,j}^{10,16} &= -Q_i^5,\\
\Sigma_{i,j}^{11,11} &= -Q_i^7,\\
\Sigma_{i,j}^{11,17} &= -Q_i^8,
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{i,j}^{12,12} &= -Q_i^{10},\\
\Sigma_{i,j}^{12,18} &= -Q_i^{11},\\
\Sigma_{i,j}^{13,13} &= -Q_i^{13},\\
\Sigma_{i,j}^{13,19} &= -Q_i^{14},\\
\Sigma_{i,j}^{14,14} &= -Q_i^{16},\\
\Sigma_{i,j}^{14,20} &= -Q_i^{17},\\
\Sigma_{i,j}^{15,15} &= -Q_i^3 - P_i^{14},\\
\Sigma_{i,j}^{16,16} &= -Q_i^6,\\
\Sigma_{i,j}^{17,17} &= -Q_i^9 - P_i^{15},\\
\Sigma_{i,j}^{18,18} &= -Q_i^{12},\\
\Sigma_{i,j}^{19,19} &= -Q_i^{15} - P_i^{16},\\
\Sigma_{i,j}^{21,21} &= -L_i^{5^T} B_j^1 - B_i^{1^T} L_j^5 - Z_i^1,\\
\Sigma_{i,j}^{21,22} &= L_i^{5^T} A_j^2 - B_i^{1^T} L_j^6,\\
\Sigma_{i,j}^{21,25} &= L_i^{5^T},\\
\Sigma_{i,j}^{21,26} &= L_i^{5^T},\\
\Sigma_{i,j}^{21,27} &= L_i^{5^T} C_j,\\
\Sigma_{i,j}^{22,22} &= L_i^{6^T} A_j^2 + A_i^{2^T} L_j^6 - Z_i^2,\\
\Sigma_{i,j}^{22,25} &= L_i^{6^T},\\
\Sigma_{i,j}^{22,26} &= L_i^{6^T},\\
\Sigma_{i,j}^{22,27} &= L_i^{6^T} C_j,\\
\Sigma_{i,j}^{23,23} &= -P_i^{18},\\
\Sigma_{i,j}^{24,24} &= -P_i^{18},\\
\Sigma_{i,j}^{25,25} &= -\epsilon_1 I,\\
\Sigma_{i,j}^{26,26} &= -\epsilon_2 I,\\
\Sigma_{i,j}^{27,27} &= -\frac{P_i^{19}}{w},\\
G_i^1 &= P_i^1 J, \quad G_i^2 = P_i^1 W,\\
\rho &= h_2 - h_1,\\
\alpha(k) &= h(k) - h_1,\\
\beta(k) &= h_2 - h(k),\\
\psi(k) &= \frac{1}{\alpha(k)}\left[\sum_{i=k-h(k)}^{k-h_1-1} x(i)\right],\\
\phi(k) &= \frac{1}{\beta(k)}\left[\sum_{i=k-h_2}^{k-h(k)-1} x(i)\right],
\end{aligned}
$$

other terms are 0.

Theorem 5. The system (1) is robustly stable, if there exist positive definite symmetric matrices $M_i, Z_i^1, Z_i^2, P_i^k, Q_i^s, k = 1,2,...,18,\ s = 1,2,...,19,$ $i = 1,2,...,N$, any appropriate dimensional matrices M_i^r, N_i^r, L_i^l, $r = 1,2,3,4$, $l = 1,2,...,6$, $i = 1,2,...,N$ and positive real constants ϵ_1, ϵ_2 satisfying the following LMIs:

$$\prod_{i,i} < -I, \quad i = 1,2,...,N, \tag{7}$$

$$\prod_{i,j} + \prod_{j,i} < \frac{2}{(N-1)} I,$$
$$i = 1,...,N-1, j = i+1,...,N, \tag{8}$$

$$\begin{bmatrix} M_i^1 & N_i^1 \\ * & P_i^{10} \end{bmatrix} \geq 0, \quad i = 1, 2, ..., N, \tag{9}$$

$$\begin{bmatrix} M_i^2 & N_i^2 \\ * & P_i^{11} \end{bmatrix} \geq 0, \quad i = 1, 2, ..., N, \tag{10}$$

$$\begin{bmatrix} M_i^3 & N_i^3 \\ * & P_i^{12} \end{bmatrix} \geq 0, \quad i = 1, 2, ..., N, \tag{11}$$

$$\begin{bmatrix} M_i^4 & N_i^4 \\ * & P_i^{13} \end{bmatrix} \geq 0, \quad i = 1, 2, ..., N. \tag{12}$$

Proof. From model transformation method, we rewrite the system (1) in the following system:

$$x(k+1) = x(k) + y(k), \tag{13}$$

$$\begin{aligned} y(k) &= \Big[A(\alpha) - I\Big]x(k) + B(\alpha)x(k - h(k)) \\ &\quad + C(\alpha)\sum_{i=1}^{\infty} \delta(i)x(k-i) + f(k, x(k)) \\ &\quad + g(k, x(k - h(k))). \end{aligned} \tag{14}$$

In order to improve the bound of the discrete delay $h(k)$ in (1), let us decompose constant matrix $A(\alpha)$ and $B(\alpha)$ as

$$A(\alpha) = A_1(\alpha) + A_2(\alpha), \tag{15}$$

$$B(\alpha) = B_1(\alpha) + B_2(\alpha), \tag{16}$$

where $A_1(\alpha) = \sum_{i=1}^{N} \alpha_i A_i^1, A_2(\alpha) = \sum_{i=1}^{N} \alpha_i A_i^2, B_1(\alpha) = \sum_{i=1}^{N} \alpha_i B_i^1$ and $B_2(\alpha) = \sum_{i=1}^{N} \alpha_i B_i^2, A_i^1, A_i^2, B_i^1, B_i^2$, $i = 1, 2, 3, ..., N \in \mathbb{R}^{n \times n}$ are constant matrices. By utilizing the following zero equations, we have

$$x(k) - x(k - h(k)) - \sum_{i=k-h(k)}^{k-1} y(i) = 0, \tag{17}$$

$$x(k) - x(k - rh(k)) - \sum_{i=k-rh(k)}^{k-1} y(i) = 0. \tag{18}$$

By utilizing the following zero equations, we have

$$Jx(k) - Jx(k - h(k)) - J\sum_{i=k-h(k)}^{k-1} y(i) = 0, \tag{19}$$

$$Wx(k) - Wx(k - rh(k)) - W\sum_{i=k-rh(k)}^{k-1} y(i) = 0, \tag{20}$$

where $J, W \in R^{n \times n}$ will be chosen to guarantee the asymptotic stability of the system (1)-(2). By (19)-(20), the system (13)-(14) can be represented by the form

$$\begin{aligned} x(k+1) &= x(k) + y(k) + Jx(k) - Jx(k - h(k)) \\ &\quad - J\sum_{i=k-h(k)}^{k-1} y(i) + Wx(k) \\ &\quad - Wx(k - rh(k)) - W\sum_{i=k-rh(k)}^{k-1} y(i), \end{aligned} \tag{21}$$

$$\begin{aligned} y(k) &= \Big[A_1(\alpha) + B_1(\alpha) - I\Big]x(k) \\ &\quad + A_2(\alpha)x(k - rh(k)) + B_2(\alpha)x(k - h(k)) \\ &\quad - B_1(\alpha)\sum_{i=k-h(k)}^{k-1} y(i) + A_2(\alpha)\sum_{i=k-rh(k)}^{k-1} y(i) \\ &\quad + C(\alpha)\sum_{i=1}^{\infty} \delta(i)x(k-i) + f(k, x(k)) \\ &\quad + g(k, x(k - h(k))). \end{aligned} \tag{22}$$

Consider the following Lyapunov-Krasovskii function for system (21)-(22) of the form

$$V(k) = \sum_{i=1}^{8} V_i(k), \tag{23}$$

where

$$\begin{aligned} V_1(k) &= x^T(k)P_1(\alpha)x(k), \\ V_2(k) &= \sum_{i=k-h_1}^{k-1} x^T(i)P_2(\alpha)x(i) \\ &\quad + \sum_{i=k-rh_1}^{k-1} x^T(i)P_3(\alpha)x(i) \\ &\quad + \sum_{i=k-h_2}^{k-1} x^T(i)P_4(\alpha)x(i) \\ &\quad + \sum_{i=k-rh_2}^{k-1} x^T(i)P_5(\alpha)x(i) \\ &\quad + \sum_{i=k-h_2}^{k-h_1-1} x^T(i)P_6(\alpha)x(i) \\ &\quad + \sum_{i=k-rh_2}^{k-rh_1-1} x^T(i)P_7(\alpha)x(i) \\ &\quad + \sum_{i=k-h(k)}^{k-1} x^T(i)P_8(\alpha)x(i) \\ &\quad + \sum_{i=k-rh(k)}^{k-1} x^T(i)P_9(\alpha)x(i), \end{aligned}$$

$$\begin{aligned} V_3(k) &= h_1 \sum_{j=-h_1}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\ &\quad \begin{bmatrix} Q_1(\alpha) & Q_2(\alpha) \\ * & Q_3(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\ &\quad + rh_1 \sum_{j=-rh_1}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\ &\quad \begin{bmatrix} Q_4(\alpha) & Q_5(\alpha) \\ * & Q_6(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\ &\quad + h_2 \sum_{j=-h_2}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\ &\quad \begin{bmatrix} Q_7(\alpha) & Q_8(\alpha) \\ * & Q_9(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\ &\quad + rh_2 \sum_{j=-rh_2}^{-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\ &\quad \begin{bmatrix} Q_{10}(\alpha) & Q_{11}(\alpha) \\ * & Q_{12}(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \\ &\quad + \rho \sum_{j=-h_2}^{-h_1-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}^T \\ &\quad \begin{bmatrix} Q_{13}(\alpha) & Q_{14}(\alpha) \\ * & Q_{15}(\alpha) \end{bmatrix} \begin{bmatrix} x(i) \\ y(i) \end{bmatrix} \end{aligned}$$

$$+r\rho\sum_{j=-rh_2}^{-rh_1-1}\sum_{i=k+j}^{k-1}\begin{bmatrix}x(i)\\y(i)\end{bmatrix}^T \begin{bmatrix}Q_{16}(\alpha) & Q_{17}(\alpha)\\ * & Q_{18}(\alpha)\end{bmatrix}\begin{bmatrix}x(i)\\y(i)\end{bmatrix},$$

$$\begin{aligned} V_4(k) &= \sum_{j=-h_1+1}^{0}\sum_{i=k-1+j}^{k-1} y^T(i)P_{10}(\alpha)y(i) \\ &\quad + \sum_{j=-h_2+1}^{0}\sum_{i=k-1+j}^{k-1} y^T(i)P_{11}(\alpha)y(i) \\ &\quad + \sum_{j=-rh_1+1}^{0}\sum_{i=k-1+j}^{k-1} y^T(i)P_{12}(\alpha)y(i) \\ &\quad + \sum_{j=-rh_2+1}^{0}\sum_{i=k-1+j}^{k-1} y^T(i)P_{13}(\alpha)y(i) \\ &\quad + \sum_{i=1}^{\infty}\delta(i)\sum_{j=k-i}^{k-1} x_j^T M(\alpha)x_j, \\ V_5(k) &= h_1\sum_{j=-h_1}^{-1}\sum_{i=k+j}^{k-1} y^T(i)P_{14}(\alpha)y(i) \\ &\quad + h_2\sum_{j=-h_2}^{-1}\sum_{i=k+j}^{k-1} y^T(i)P_{15}(\alpha)y(i) \\ &\quad + \rho\sum_{j=-h_2}^{-h_1-1}\sum_{i=k+j}^{k-1} y^T(i)P_{16}(\alpha)y(i), \\ V_6(k) &= h_2\sum_{j=-h_2}^{-1}\sum_{i=k+j}^{k-1} y^T(i)Z_1(\alpha)y(i) \\ &\quad + rh_2\sum_{j=-rh_2}^{-1}\sum_{i=k+j}^{k-1} y^T(i)Z_2(\alpha)y(i), \\ V_7(k) &= \frac{h_1^2}{2}\sum_{i=-h_1}^{-1}\sum_{j=i}^{0}\sum_{l=k+j}^{k-1} y^T(l)P_{17}(\alpha)y(l) \\ &\quad + \frac{1}{2}\sum_{i=k-h_2}^{k-h_1-1}\sum_{j=i}^{k-h_1-1}\sum_{l=j}^{k-h_1-1} y^T(l)P_{18}(\alpha)y(l), \\ V_8(k) &= \sum_{i=1}^{\infty}\delta(i)\sum_{j=k-i}^{k-1} x^T(j)P_{19}(\alpha)x(j). \end{aligned}$$

Evaluating the forward deference of $V(k)$, it is defined as

$$\Delta V(k)=\sum_{i=1}^{8}\Delta V_i(k). \tag{24}$$

Let us define for $i=1,2,...,8$,

$$\Delta V_i(k)=V_i(k+1)-V_i(k). \tag{25}$$

From (3) and (4), we obtain for any scalars $\epsilon_1,\epsilon_2>0$,

$$\epsilon_1\Big(\alpha^2x^T(k)x(k)-f^T(k,x(k))f(k,x(k))\Big)\geq 0, \tag{26}$$

$$\epsilon_2\Big(\beta^2x^T(k-h(k))x(k-h(k)) -g^T(k,x(k-h(k)))g(k,x(k-h(k)))\Big)\geq 0. \tag{27}$$

From Lemma 2-4, it follows form (24)-(27) that

$$\Delta V(k) \leq \xi^T(k)\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\prod\nolimits_{i,j}\xi(k), \tag{28}$$

where $\xi^T(k)=\Big[x^T(k) \quad y^T(k) \quad x^T(k-h_1) \quad x^T(k-h_2) \quad x^T(k-h(k)) \quad x^T(k-rh_1) \quad x^T(k-rh_2) \quad x^T(k-rh(k)) \quad \sum_{i=k-h_1}^{k-1}x^T(i) \quad \sum_{i=k-rh_1}^{k-1}x^T(i) \quad \sum_{i=k-h_2}^{k-1}x^T(i) \quad \sum_{i=k-rh_2}^{k-1}x^T(i) \quad \sum_{i=k-h_2}^{k-h_1-1}x^T(i) \quad \sum_{i=k-rh_2}^{k-rh_1-1}x^T(i) \quad \sum_{i=k-h_1}^{k-1}y^T(i) \quad \sum_{i=k-rh_1}^{k-1}y^T(i) \quad \sum_{i=k-h_2}^{k-1}y^T(i) \quad \sum_{i=k-rh_2}^{k-1}y^T(i) \quad \sum_{i=k-h_2}^{k-h_1-1}y^T(i) \quad \sum_{i=k-rh_2}^{k-rh_1-1}y^T(i) \quad \sum_{i=k-h(k)}^{k-1}y^T(i) \quad \sum_{i=k-rh(k)}^{k-1}y^T(i) \quad \psi(k) \quad \phi(k) \quad f(k,x(k)) \quad g(k,x(k-h(k))) \quad \sum_{i=1}^{\infty}\delta(i)x(k-i)\Big]$ and $\prod_{i,j}$ is defined in (6). The facts that $\sum_{i=1}^{N}\alpha_i=1$, we obtain the following identities :

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\prod\nolimits_{i,j}=\sum_{i=1}^{N}\alpha^2\prod\nolimits_{i,i} +\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_i[\prod\nolimits_{i,j}+\prod\nolimits_{j,i}], \tag{29}$$

$$(N-1)\sum_{i=1}^{N}\alpha_i^2-2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_j =\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[\alpha_i-\alpha_j]^2\geq 0. \tag{30}$$

By (28)-(30), if the conditions (7)-(12) are true, then

$$\Delta V(k)<-\omega\|x\|^2, \tag{31}$$

where $\omega>0$. This means that system (1) is robustly stable. The proof of theorem is complete. □

IV. Numerical Examples

Example 6. Consider the system

$$\begin{aligned} x(k+1) &= Ax(k)+Bx(k-h(k))+f(k,x(k)) \\ &\quad +g(k,x(k-h(k))), \end{aligned} \tag{32}$$

with the following parameters which is considered in [13] and [18] :

$$A=\begin{pmatrix}0.80 & 0\\ 0.05 & 0.90\end{pmatrix}, B=\begin{pmatrix}-0.10 & 0\\ -0.20 & -0.10\end{pmatrix}, \tag{33}$$

$$\alpha\geq 0, \quad \beta\geq 0. \tag{34}$$

Decompose matrix A and B as follows : $A=A_1+A_2$, $B=B_1+B_2$, where

$$A_1=\begin{bmatrix}-0.5 & -0.1\\ -0.02 & -0.3\end{bmatrix}, \quad A_2=\begin{bmatrix}1.3 & 0.1\\ 0.07 & 1.2\end{bmatrix},$$

$$B_1=\begin{bmatrix}-0.5 & -0.1\\ -0.3 & -0.4\end{bmatrix}, \quad B_2=\begin{bmatrix}0.4 & 0.1\\ 0.1 & 0.3\end{bmatrix}.$$

By using the LMI Toolbox in MATLAB (with accuracy 0.01) for applying Theorem 5 to system (32) with (33)-(34), one can obtain the maximum upper bounds of the time delay under different values of h_1 as shown in Table I. We can see that our results in applying Theorem 5 are much less conservative than in [13] and [18].

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TABLE I
UPPER BOUNDS OF TIME DELAY h_2 FOR DIFFERENT CONDITIONS FOR EXAMPLE 6.

Method	h_1	2	6	10	14
$\alpha = 0.1, \beta = 0$					
Zhang et al. (2011) [18]	h_2	10	12	14	17
Ramakrishnan and Ray (2013) [13]	h_2	13	14	16	18
This paper	h_2	21	23	23	24
$\alpha = 0, \beta = 0.1$					
Ramakrishnan and Ray (2013) [13]	h_2	11	13	15	17
This paper	h_2	19	24	23	22
$\alpha = 0.1, \beta = 0.1$					
Ramakrishnan and Ray (2013) [13]	h_2	10	11	13	15
This paper	h_2	16	19	22	21

TABLE II
COMPARISON OF THE UPPER BOUNDS TIME DELAY h_2 FOR DIFFERENT CONDITIONS.

h_1	4	10	20	30
Peng (2012) [11]	18	20	26	35
Feng et al. (2015) [2]	21	22	27	35
Kwon et al. (2013) [6]	22	23	27	36
Kim (2015) [5]	22	23	28	36
Nam et al. (2015) [10]	22	23	29	36
Hien and Trinh (2016) [3]	27	28	35	39
This paper	29	30	36	40

Example 7. Consider the system

$$x(k+1) \quad = \quad Ax(k) + Bx(k-h(k)), \tag{35}$$

having the following parameters:

$$A = \begin{bmatrix} 0.80 & 0 \\ 0.05 & 0.90 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.10 & 0 \\ -0.20 & -0.10 \end{bmatrix}. \tag{36}$$

Decompose matrix A and B as follows : $A = A_1 + A_2$, $B = B_1 + B_2$, where

$$A_1 = \begin{bmatrix} -0.4 & -0.1 \\ -0.02 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.2 & 0.1 \\ 0.07 & 1.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.5 & -0.1 \\ -0.3 & -0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}.$$

For given h_1, we calculate the allowable maximum value of h_2 that guarantees the asymptotic stability of system (35) with (36). By using different methods, the calculated results are presented in Table II. From the table, we can see that applying Theorem 5 in this paper provides the less conservative results.

V. CONCLUSION

The problem of delay-range-dependent robust stability for LPD discrete-time system with interval discrete and distributed time-varying delays and nonlinear perturbations was studied. By utilizing the inequalities, mixed model transformation, utilization of zero equation and a new parameter dependent Lyapunov-Krasovskii functional, delay-range-dependent stability criteria have been derived and formulated in terms of LMIs for the system. Numerical example are given finally to demonstrate the effectiveness and less conservativeness of the proposed stability criteria.

Competing interests

I declare that I have have no competing interests.

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