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New Exponential Stability Analysis for Certain Neutral Integro-Differential Equations with Non-Differentiable Interval Time-Varying Delays

Watcharin Chartbupapan, Kanit Mukdasai

Abstract—**The problems of delay-range-dependent exponential stability analysis for certain neutral differential equation with discrete and distributed time-varying delays are studied. The time-varying delays are continuous functions belonging to the given interval delays, which mean that the lower and upper bounds for time-varying delays are available. Based on new class of augmented Lyapunov-Krasovskii functional, descriptor model transformation, decomposition technique of coefficient constant, Leibniz-Newton formula, utilization of zero equation, improved integral inequalities and Peng-Park's integral inequality, new delay-range-dependent exponential stability criteria is derived in terms of linear matrix inequalities (LMIs) for the equation. Numerical example suggests that the results given to illustrate the effectiveness.**

Index Terms—**exponential stability analysis, neutral differential equation, linear matrix inequality (LMIs), discrete and distributed time-varying delays**

I. Introduction

IN the last few decades, the problem of various stability analysis for uncertain neutral systems with state delays has been intensively studied by several researchers [1]-[19]. Since neutral delayed systems have already been applied in many fields, such as population ecology, distributed networks containing lossless transmission lines, propagation and diffusion models, and partial element equivalent circuits in very large scale integration systems [10]. Furthermore, the delay-dependent stability analysis of certain neutral differential equations (CNDE) has been received considerable attention in recent years. Since delay-dependent stability criteria makes use of information on the length of delays. Usually, the range of delay considered in most of the existing references is from zero to an upper bound. However, the delay may vary in a range for which the lower bound is not restricted to being zero. Many researchers investigate the problem of delay-range-dependent stability criteria for differential systems with interval time-varying delays. The lower and upper bounds for the time-varying delays are available and the delay functions are necessary to be differentiable. Delay-dependent asymptotic stability criteria for CNDE with constant delays has been discussed in [4], [9], [11], [12], [15], [17] by using several model transformation method and Lyapunov-Krasovskii functional approach, while the problem of exponential stability analysis has been studied with model transformation method and Lyapunov-Krasovskii functional approach in [15]. In [2], [3], [7], the authors studied the problem of exponential stability analysis for CNDE with time-varying delays by several methods. In [2], the results are derived without the use of the model transformation method and the bounding technique, while authors have been studied by using model transformation method, radially unboundedness and Lyapunov-Krasovskii functional approach in [7]. Moreover, the authors have studied the problem of stability for systems with discrete and distributed delays such as [18] which presented some stability conditions for uncertain neutral systems with discrete and distributed delays. The robust stability of uncertain linear neutral systems with discrete and distributed delays has been studied in [6]. However, neutral and discrete time-varying delays are required to be differentiable and information on bounded derivative of time-varying delays from existing methods.

Motivated by above discussions, we aim to establish the new delay-range-dependent exponential stability criteria for certain neutral differential equation with discrete and distributed interval time-varying delays. The restrictions on the derivatives of the discrete and distributed time-varying delays for the equation are removed. Based on the combination of mixed model transformation, mixed integral inequalities, decomposition technique of coefficient constant, utilization of zero equation and new Lyapunov-Krasovskii functional, sufficient condition for exponential stability is obtained and formulated in terms of LMIs for the equation. Finally, numerical example suggests that the proposed criteria is effective.

Notation:
The following notations will be accounted in this paper: let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denotes n-dimensional Euclidean space with vector norm $||.||$ and set of $n\times m$ matrices, respectively. A matrix P is symmetric positive definite, write $P>0$, if $P^T=P$ and $x^TPx>0$ for all $x\in\mathbb{R}^n$,$x\neq 0$.

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Watcharin Chartbupapan is with the Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand. E-mail : chatbupapan@gmail.com.

Kanit Mukdasai is with the Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand. E-mail : kanit@kku.ac.th.

Correspondence should be addressed to Kanit Mukdasai, kanit@kku.ac.th.

II. Preliminaries

Consider the certain neutral integro-differential equation with interval time-varying delays of the form

$$\begin{aligned}\frac{d}{dt}[x(t)+px(t-\tau(t))] &= -ax(t)+b\tanh x(t-\sigma(t)) \\ &\quad +c\int_{t-\rho(t)}^{t} x(s)ds, \\ &\quad t\geq 0, \qquad (1)\end{aligned}$$

where a, b are positive real constants and c, p are real constants with $|p|<1$. $\tau(t)$, $\sigma(t)$ and $\rho(t)$ are neutral, discrete and distributed interval time-varying delays, respectively,

$$0\leq\tau_1\leq\tau(t)\leq\tau_2, \quad \dot{\tau}(t)\leq\tau_d \qquad (2)$$

$$0\leq\sigma_1\leq\sigma(t)\leq\sigma_2, \qquad (3)$$

$$0\leq\rho_1\leq\rho(t)\leq\rho_2, \qquad (4)$$

where τ_1, τ_2, τ_d, σ_1, σ_2, ρ_1 and ρ_2 are given positive real constants. For each solution $x(t)$ of (1), we assume the initial condition

$$x_0(t)=\phi(t), \quad t\in[-\omega,0],$$

where $\phi\in C([-\omega,0];R)$ and $\omega=\max\{\tau_2,\sigma_2,\rho_2\}$. Let $x(t,\phi)$ denote the state trajectory of (1) and $x(t,0)$ is the corresponding trajectory with zero initial condition. We consider the Leibniz-Newton formula of two forms

$$0=x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds, \qquad (5)$$

$$0=x(t)-x(t-\sigma(t))-\int_{t-\sigma(t)}^{t}\dot{x}(s)ds. \qquad (6)$$

By utilizing the following zero equations, we obtain

$$0=\epsilon_1x(t)-\epsilon_1x(t-\tau(t))-\epsilon_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds, \qquad (7)$$

$$0=\epsilon_2x(t)-\epsilon_2x(t-\sigma(t))-\epsilon_2\int_{t-\sigma(t)}^{t}\dot{x}(s)ds, \qquad (8)$$

where $\epsilon_1,\epsilon_2\in R$. By descriptor model transformation and (5)-(8), (1) can be represented by the form

$$\begin{aligned}\dot{x}(t) &= y(t)+\epsilon_1x(t)-\epsilon_1x(t-\tau(t))-\epsilon_1\int_{t-\tau(t)}^{t}y(s)ds \\ &\quad +\epsilon_2x(t)-\epsilon_2x(t-\sigma(t))-\epsilon_2\int_{t-\sigma(t)}^{t}y(s)ds, \quad (9)\\ y(t) &= -ax(t)+b\tanh x(t-\sigma(t))+c\int_{t-\rho(t)}^{t}x(s)ds \\ &\quad -py(t-\tau(t)). \qquad (10)\end{aligned}$$

For $a=a_1+a_2+a_3$, we rewrite the (9)-(10) in the following equations

$$\begin{aligned}\dot{x}(t) &= y(t)+\epsilon_1x(t)-\epsilon_1x(t-\tau(t))-\epsilon_1\int_{t-\tau(t)}^{t}y(s)ds \\ &\quad +\epsilon_2x(t)-\epsilon_2x(t-\sigma(t))-\epsilon_2\int_{t-\sigma(t)}^{t}y(s)ds, (11)\\ y(t) &= -a_1x(t)-a_2x(t-\tau(t))-a_2\int_{t-\tau(t)}^{t}y(s)ds \\ &\quad -a_3x(t-\sigma(t))-a_3\int_{t-\sigma(t)}^{t}y(s)ds \\ &\quad +b\tanh x(t-\sigma(t))+c\int_{t-\rho(t)}^{t}x(s)ds \\ &\quad -py(t-\tau(t)). \qquad (12)\end{aligned}$$

Definition 1. [8] The equation (1) is *exponentially stable*, if there exist positive real constants α, β such that for each $\phi(t)\in C([-\omega,0],R)$, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\|\leq\beta\|\phi\|e^{-\alpha t}, \quad t\geq 0.$$

Lemma 2. [16] For any constant symmetric positive definite matrix $Q\in R^{n\times n}$, $0\leq h_1\leq h(t)\leq h_2$ and vector function $\omega:[-h_2,0]\to R^n$ such that the integrations concerned are well defined, then

$$\begin{aligned}&-[h_2-h_1]\int_{-h_2}^{-h_1}\omega^T(s)Q\omega(s)ds \\ \leq\; & -\int_{-h(t)}^{-h_1}\omega^T(s)dsQ\int_{-h(t)}^{-h_1}\omega(s)ds \\ & -\int_{-h_2}^{-h(t)}\omega^T(s)dsQ\int_{-h_2}^{-h(t)}\omega(s)ds.\end{aligned}$$

Lemma 3. [16] For any constant matrices $Q_1,Q_2,Q_3\in R^{n\times n}$, $Q_1\geq 0, Q_3>0$, $\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\geq 0$, $0\leq h_1\leq h(t)\leq h_2$ and vector function $\dot{x}:[-h_2,0]\to R^n$ such that the following integration is well defined, then

$$\begin{aligned}&-[h_2-h_1]\int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds \\ &\leq \begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t-h_1}x(s)ds\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}^T \begin{bmatrix}-Q_3 & Q_3 & 0 & -Q_2^T & 0\\ * & -Q_3-Q_3 & Q_3 & Q_2^T & -Q_2^T\\ * & * & -Q_3 & 0 & Q_2^T\\ * & * & * & -Q_1 & 0\\ * & * & * & * & -Q_1\end{bmatrix} \begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t-h_1}x(s)ds\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}.\end{aligned}$$

Lemma 4. [16] Let $x(t)\in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices

$X, M_i \in R^{n\times n}, i = 1, 2, \ldots, 5$ and $0 \le h_1 \le h(t) \le h_2$,

$$-\int_{t-h_2}^{t-h_1} \dot{x}^T(s) X \dot{x}(s) ds$$
$$\le \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ * & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ * & * & -M_2 - M_2^T \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}$$
$$+[h_2 - h_1] \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3 + M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix},$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \ge 0.$$

Lemma 5. (Peng-Park's integral inequality.) [14], [13] Any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \ge 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \to \mathbb{R}^n$ such that the concerned integrations are well defined,then

$$-\tau \int_{t-\tau}^{t} \dot{x}^T(s) Z \dot{x}(s) ds \le \omega^T(t) \Omega \omega(t),$$

where $\omega(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-\tau)]^T$ and $\Omega = \begin{bmatrix} -Z & Z-S & S \\ * & -2Z+S+S^T & Z-S \\ * & * & -Z \end{bmatrix}$.

III. Main results

In this section, we give our main results. We introduce the following notations for later use:

$$\sum = \left[\Omega_{(i,j)}\right]_{26\times 26}, \tag{13}$$

where $\Omega_{(i,j)} = \Omega_{(j,i)}$,

$$\begin{aligned}
\Omega_{(1,1)} &= 2\alpha k_1 + n_9 + k_7 + n_{10} + k_8 + 2z_1 + 2z_4 \\
&\quad +2k_1\epsilon_1 + 2k_1\epsilon_2 - 2q_2a_1 + k_2\tau_2^2 + k_3\sigma_2^2 \\
&\quad +n_1\tau_2^2 - 2n_1\tau_2\tau_1 + n_1\tau_1^2 + n_2\sigma_2^2 - 2n_2\sigma_2\sigma_1 \\
&\quad +n_2\sigma_1^2 - k_4e^{(-2\alpha\tau_2)} - k_5e^{(-2\alpha\sigma_2)} + \tau_1^2 f_1 \\
&\quad +\tau_2^2 r_1 - r_3e^{(-2\alpha\tau_2)} + f_4\tau_2^2 - 2\tau_1\tau_2 + \tau_1^2 f_4 \\
&\quad +\sigma_1^2 f_7 + \sigma_2^2 r_4 + \sigma_2^2 f_{10} - 2\sigma_1\sigma_2 f_{10} + \sigma_1^2 f_{10} \\
&\quad -r_6e^{(-2\alpha\sigma_2)} + n_7\sigma_1^2 + k_6\sigma_2^2 + n_8\sigma_2^2 \\
&\quad -2n_8\sigma_1\sigma_2 + n_8\sigma_1^2 + n_{11}\rho_2^2 - 2n_{11}\rho_1\rho_2 \\
&\quad +n_{11}\rho_1^2, \\
\Omega_{(1,2)} &= q_1\epsilon_1 + q_1\epsilon_2 + \tau_1^2 f_2 + \tau_2^2 r_2 + \tau_2^2 f_5 - 2\tau_1\tau_2 f_5 \\
&\quad +\tau_1^2 f_5 + \sigma_1^2 f_8 + \sigma_2^2 r_5 + \sigma_2^2 f_{11} - 2\sigma_1\sigma_2 f_{11} \\
&\quad +\sigma_1^2 f_{11} + k_1 - q_2 - q_3a_1, \\
\Omega_{(1,3)} &= -z_1 + z_2 - k_1\epsilon_1 - q_2a_2 - q_4a_1 + k_4e^{(-2\alpha\tau_2)} \\
&\quad -s_1 + r_3e^{(-2\alpha\tau_2)}, \\
\Omega_{(1,4)} &= -z_1 + z_3 - k_1\epsilon_1 - q_2a_2 - q_5a_1, \\
\Omega_{(1,5)} &= -z_4 + z_5 - k_1\epsilon_2 - q_2a_3 - q_6a_1 + k_5e^{(-2\alpha\sigma_2)} \\
&\quad -s_2 + r_6e^{(-2\alpha\sigma_2)}, \\
\Omega_{(1,6)} &= -z_4 + z_6 - k_1\epsilon_2 - q_2a_3 - q_7a_1, \\
\Omega_{(1,7)} &= q_2b - q_8a_1, \\
\Omega_{(1,8)} &= q_2c - q_{10}a_1, \\
\Omega_{(1,9)} &= -q_2p - q_9a_1, \\
\Omega_{(1,10)} &= -r_2e^{(-2\alpha\tau_2)}, \\
\Omega_{(1,12)} &= -r_5e^{(-2\alpha\sigma_2)}, \\
\Omega_{(1,17)} &= s_1, \\
\Omega_{(1,19)} &= s_2, \\
\Omega_{(2,2)} &= k_9 + n_3\tau_1^2 + k_4\tau_2^2 + n_4\sigma_1^2 + k_5\sigma_2^2 + n_5\tau_2^2 \\
&\quad -2n_5\tau_1\tau_2 + n_5\tau_1^2 + n_6\sigma_2^2 - 2n_6\sigma_1\sigma_2 \\
&\quad +n_6\sigma_1^2 + \tau_1^2 f_3 + \tau_2^2 r_3 + \tau_2^2 f_6 - 2\tau_1\tau_2 f_6 \\
&\quad +\tau_1^2 f_6 + \sigma_1^2 f_9 + \sigma_2^2 f_{12} - 2\sigma_1\sigma_2 f_{12} + \sigma_1^2 f_{12} \\
&\quad -2q_3 + \sigma_2^2 r_6, \\
\Omega_{(2,3)} &= -q_1\epsilon_1 - q_3a_2 - q_4, \\
\Omega_{(2,4)} &= -q_1\epsilon_1 - q_3a_2 - q_5, \\
\Omega_{(2,5)} &= -q_1\epsilon_2 - q_3a_3 - q_6, \\
\Omega_{(2,6)} &= -q_1\epsilon_2 - q_3a_3 - q_7, \\
\Omega_{(2,7)} &= q_3b - q_8, \\
\Omega_{(2,8)} &= q_3c - q_{10}, \\
\Omega_{(2,9)} &= -q_3p - q_9, \\
\Omega_{(3,3)} &= -2z_2 - 2q_4a_2 - 2k_4e^{(-2\alpha\tau_2)} + 2s_1 + 2m_1 \\
&\quad -2m_2 + \tau_2m_3 - \tau_1m_3 + \tau_2m_5 - \tau_1m_5 \\
&\quad -2r_3e^{(-2\alpha\tau_2)} - 2f_6e^{(-2\alpha\tau_2)}, \\
\Omega_{(3,4)} &= -z_2 - z_3 - q_4a_2 - q_5a_2, \\
\Omega_{(3,5)} &= -q_4a_3 - q_6a_2, \\
\Omega_{(3,6)} &= -q_4a_3 - q_7a_2, \\
\Omega_{(3,7)} &= q_4b - q_8a_2, \\
\Omega_{(3,8)} &= q_4c - q_{10}a_2, \\
\Omega_{(3,9)} &= -q_4p - q_9a_2, \\
\Omega_{(3,10)} &= r_2e^{(-2\alpha\tau_2)}, \\
\Omega_{(3,11)} &= -r_2e^{(-2\alpha\tau_2)} - f_5e^{(-2\alpha\tau_2)}, \\
\Omega_{(3,14)} &= f_5e^{(-2\alpha\tau_2)}, \\
\Omega_{(3,16)} &= -m_1 + m_2 + \tau_2m_4 - \tau_1m_4 + f_6e^{(-2\alpha\tau_2)}, \\
\Omega_{(3,17)} &= k_4e^{(-2\alpha\tau_2)} - s_1 - m_1 + m_2 + \tau_2m_4 - \tau_1m_4 \\
&\quad +r_3e^{(-2\alpha\tau_2)} + f_6e^{(-2\alpha\tau_2)}, \\
\Omega_{(4,4)} &= -2z_3 - 2q_5a_2, \\
\Omega_{(4,5)} &= -q_5a_3 - q_6a_2, \\
\Omega_{(4,6)} &= -q_5a_3 - q_7a_2, \\
\Omega_{(4,7)} &= q_5b - q_8a_2, \\
\Omega_{(4,8)} &= q_5c - q_{10}a_2, \\
\Omega_{(4,9)} &= -q_5p - q_9a_2, \\
\Omega_{(5,5)} &= w - 2z_5 - 2q_6a_3 - 2k_5e^{(-2\alpha\sigma_2)} + 2s_2 \\
&\quad +2m_6 - 2m_7 + \sigma_2m_8 - \sigma_1m_8 + \sigma_2m_{10} \\
&\quad -\sigma_1m_{10} - 2r_6e^{(-2\alpha\sigma_2)} - 2f_{12}e^{(-2\alpha\sigma_2)},
\end{aligned}$$

$$
\begin{aligned}
\Omega_{(5,6)} &= -z_5 - z_6 - q_6a_3 - q_7a_3,\\
\Omega_{(5,7)} &= q_6b - q_8a_3,\\
\Omega_{(5,8)} &= q_6c - q_{10}a_3,\\
\Omega_{(5,9)} &= -q_6p - q_9a_3,\\
\Omega_{(5,12)} &= r_5e^{(-2\alpha\sigma_2)},\\
\Omega_{(5,13)} &= -r_5e^{(-2\alpha\sigma_2)} - f_{11}e^{(-2\alpha\sigma_2)},\\
\Omega_{(5,15)} &= f_{11}e^{(-2\alpha\sigma_2)},\\
\Omega_{(5,18)} &= -m_6 + m_7 + \sigma_2m_9 - \sigma_1m_9\\
&\quad +f_{12}e^{(-2\alpha\sigma_2)},\\
\Omega_{(5,19)} &= k_5e^{(-2\alpha\sigma_2)} - s_2 - m_6 + m_7 + \sigma_2m_9\\
&\quad -\sigma_1m_9 + r_6e^{(-2\alpha\sigma_2)} + f_{12}e^{(-2\alpha\sigma_2)},\\
\Omega_{(6,6)} &= -2z_6 - 2q_7a_3,\\
\Omega_{(6,7)} &= q_7b - q_8a_3,\\
\Omega_{(6,8)} &= q_7c - q_{10}a_3,\\
\Omega_{(6,9)} &= -q_7p - q_9a_3,\\
\Omega_{(7,7)} &= -w + 2q_8b,\\
\Omega_{(7,8)} &= q_8c + q_{10}b,\\
\Omega_{(7,9)} &= -q_8p + q_9b,\\
\Omega_{(8,8)} &= 2q_{10}c - n_{11}e^{(-2\alpha\rho_2)},\\
\Omega_{(8,9)} &= -q_{10}p + q_9c,\\
\Omega_{(9,9)} &= -k_9e^{(-2\alpha\tau_2)} + k_9\tau_d - 2q_9p,\\
\Omega_{(10,10)} &= -k_2e^{(-2\alpha\tau_2)} - r_1e^{(-2\alpha\tau_2)},\\
\Omega_{(11,11)} &= -k_2e^{(-2\alpha\tau_2)} - n_1e^{(-2\alpha\tau_2)} - r_1e^{(-2\alpha\tau_2)}\\
&\quad -f_4e^{(-2\alpha\tau_2)},\\
\Omega_{(11,17)} &= r_2e^{(-2\alpha\tau_2)} + f_5e^{(-2\alpha\tau_2)},\\
\Omega_{(12,12)} &= -k_3e^{(-2\alpha\sigma_2)} - r_4e^{(-2\alpha\sigma_2)},\\
\Omega_{(13,13)} &= -k_3e^{(-2\alpha\sigma_2)} - n_2e^{(-2\alpha\sigma_2)} - r_4e^{(-2\alpha\sigma_2)}\\
&\quad -f_{10}e^{(-2\alpha\sigma_2)},\\
\Omega_{(13,19)} &= r_5e^{(-2\alpha\sigma_2)} + f_{11}e^{(-2\alpha\sigma_2)},\\
\Omega_{(14,14)} &= -n_1e^{(-2\alpha\tau_2)} - f_4e^{(-2\alpha\tau_2)},\\
\Omega_{(14,16)} &= -f_5e^{(-2\alpha\tau_2)},\\
\Omega_{(15,15)} &= -n_2e^{(-2\alpha\sigma_2)} - f_{10}e^{(-2\alpha\sigma_2)},\\
\Omega_{(15,18)} &= -f_{11}e^{(-2\alpha\sigma_2)},\\
\Omega_{(16,16)} &= -n_9e^{(-2\alpha\tau_1)} + 2m_1 + \tau_2m_3 - \tau_1m_3\\
&\quad -f_6e^{(-2\alpha\tau_2)},\\
\Omega_{(17,17)} &= -k_7e^{(-2\alpha\tau_2)} - k_4e^{(-2\alpha\tau_2)} - 2m_2 + \tau_2m_5\\
&\quad -\tau_1m_5 - r_3e^{(-2\alpha\tau_2)} - f_6e^{(-2\alpha\tau_2)},\\
\Omega_{(18,18)} &= -n_{10}e^{(-2\alpha\sigma_1)} + 2m_6 + \sigma_2m_8 - \sigma_1m_8\\
&\quad -f_{12}e^{(-2\alpha\sigma_2)},\\
\Omega_{(19,19)} &= -k_8e^{(-2\alpha\sigma_2)} - k_5e^{(-2\alpha\sigma_2)} - 2m_7 + \sigma_2m_{10}\\
&\quad -\sigma_1m_{10} - r_6e^{(-2\alpha\sigma_2)} - f_{12}e^{(-2\alpha\sigma_2)},\\
\Omega_{(20,20)} &= -n_3e^{(-2\alpha\tau_1)} - f_3e^{(-2\alpha\tau_1)},\\
\Omega_{(20,22)} &= -f_2e^{(-2\alpha\tau_1)},\\
\Omega_{(21,21)} &= -n_4e^{(-2\alpha\sigma_1)} - f_9e^{(-2\alpha\sigma_1)},\\
\Omega_{(21,23)} &= -f_8e^{(-2\alpha\sigma_1)},\\
\Omega_{(22,22)} &= -f_1e^{(-2\alpha\tau_1)},\\
\Omega_{(23,23)} &= -f_7e^{(-2\alpha\sigma_1)},\\
\Omega_{(24,24)} &= -n_7e^{(-2\alpha\sigma_1)},\\
\Omega_{(25,25)} &= -k_6e^{(-2\alpha\sigma_2)},\\
\Omega_{(26,26)} &= -n_8e^{(-2\alpha\sigma_2)},
\end{aligned}
$$

other terms are 0.

Theorem 6. For given positive real constants $\alpha, \sigma_1, \sigma_2, \tau_1, \tau_2, \tau_d, \rho_1$ and ρ_2, the certain neutral integro-differential equation with interval time-varying delays (1) is exponentially stable if there exist positive real constants $r_1, r_3, r_4, r_6, f_4, f_6, f_{10}$, f_{12}, w, k_i, n_j where $i = 1, 2, \ldots, 9$, $j = 1, 2, \ldots, 11$, and real constants $s_1, s_2, r_2, r_5, f_1, f_2, f_3, f_5, f_7, f_8, f_9, f_{11}, q_k, m_k$ where $k = 1, 2, \ldots, 10$ such that the following symmetric linear matrix inequality holds:

$$\sum < 0, \tag{14}$$

$$\begin{bmatrix} k_4e^{-2\alpha\tau_2} & s_1 \\ * & k_4e^{-2\alpha\tau_2} \end{bmatrix} \geq 0, \tag{15}$$

$$\begin{bmatrix} k_5e^{-2\alpha\sigma_2} & s_2 \\ * & k_5e^{-2\alpha\sigma_2} \end{bmatrix} \geq 0, \tag{16}$$

$$\begin{bmatrix} n_5(\tau_2-\tau_1)e^{-2\alpha\tau_2} & m_1 & m_2 \\ * & m_3 & m_4 \\ * & * & m_5 \end{bmatrix} \geq 0, \tag{17}$$

$$\begin{bmatrix} n_6(\sigma_2-\sigma_1)e^{-2\alpha\sigma_2} & m_6 & m_7 \\ * & m_8 & m_9 \\ * & * & m_{10} \end{bmatrix} \geq 0, \tag{18}$$

$$\begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \geq 0, \tag{19}$$

$$\begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \geq 0, \tag{20}$$

$$\begin{bmatrix} f_4 & f_5 \\ * & f_6 \end{bmatrix} \geq 0, \tag{21}$$

$$\begin{bmatrix} f_{10} & f_{11} \\ * & f_{12} \end{bmatrix} \geq 0. \tag{22}$$

Proof. For $r_1, r_3, r_4, r_6, f_4, f_6, f_{10}$, f_{12}, w, k_i, n_j are positive real constants where $i = 1, 2, \ldots, 9$, $j = 1, 2, \ldots, 11$ and $s_1, s_2, r_2, r_5, f_1, f_2, f_3, f_5, f_7, f_8, f_9, f_{11}, q_k, m_k$ are real constants where $k = 1, 2, \ldots, 10$, consider the Lyapunov-Krasovskii functional candidate for (11)-(12) of the form

$$V(t) = \sum_{i=1}^{7} V_i(t), \tag{23}$$

where

$$
\begin{aligned}
V_1(t) &= k_1x^2(t),\\
V_2(t) &= k_2\tau_2\int_{-\tau_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds\\
&\quad +k_3\sigma_2\int_{-\sigma_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds\\
&\quad +n_1(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds\\
&\quad +n_2(\sigma_2-\sigma_1)\int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds,\\
V_3(t) &= n_3\tau_1\int_{-\tau_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds\\
&\quad +k_4\tau_2\int_{-\tau_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds\\
&\quad +n_4\sigma_1\int_{-\sigma_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds\\
&\quad +k_5\sigma_2\int_{-\sigma_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds\\
&\quad +n_5(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds\\
&\quad +n_6(\sigma_2-\sigma_1)\int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}y^2(\theta)d\theta ds,
\end{aligned}
$$

$$
\begin{aligned}
V_4(t) &= \tau_1 \int_{-\tau_1}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_1 & f_2 \\ * & f_3 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad +\tau_2 \int_{-\tau_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} r_1 & r_2 \\ * & r_3 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad +(\tau_2-\tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_4 & f_5 \\ * & f_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad +\sigma_1 \int_{-\sigma_1}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_7 & f_8 \\ * & f_9 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad +\sigma_2 \int_{-\sigma_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} r_4 & r_5 \\ * & r_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds \\
&\quad +(\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} f_{10} & f_{11} \\ * & f_{12} \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds, \\
V_5(t) &= n_7\sigma_1 \int_{-\sigma_1}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \tanh^2 x(\theta) d\theta ds \\
&\quad +k_6\sigma_2 \int_{-\sigma_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \tanh^2 x(\theta) d\theta ds \\
&\quad +n_8(\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^{t} e^{2\alpha(\theta-t)} \\
&\quad \tanh^2 x(\theta) d\theta ds, \\
V_6(t) &= n_9 \int_{t-\tau_1}^{t} e^{2\alpha(s-t)} x^2(s) ds \\
&\quad +k_7 \int_{t-\tau_2}^{t} e^{2\alpha(s-t)} x^2(s) ds \\
&\quad n_{10} \int_{t-\sigma_1}^{t} e^{2\alpha(s-t)} x^2(s) ds \\
&\quad +k_8 \int_{t-\sigma_2}^{t} e^{2\alpha(s-t)} x^2(s) ds \\
&\quad +k_9 \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)} y^2(s) ds, \\
V_7(t) &= n_{11}(\rho_2-\rho_1) \int_{-\rho_2}^{-\rho_1} \int_{t+s}^{t} e^{2\alpha(\theta-t)} x^2(\theta) d\theta ds.
\end{aligned}
$$

The time-derivative of the Lyapunov-Krasovskii functional along the trajectory of (11)-(12) is given by

$$\dot{V}(t) = \sum_{i=1}^{7} \dot{V}_i(t). \tag{24}$$

From the utilization of zero equations, the following equations are true for any real constants z_m, $m = 1, 2, ..., 6$,

$$
\begin{aligned}
&2\Big[z_1 x(t) + z_2 x(t-\tau(t)) + z_3 \int_{t-\tau(t)}^{t} y(s) ds\Big] \\
&\times\Big[x(t) - x(t-\tau(t))) - \int_{t-\tau(t)}^{t} y(s) ds\Big] = 0, \quad (25) \\
&2\Big[z_4 x(t) + z_5 x(t-\sigma(t)) + z_6 \int_{t-\sigma(t)}^{t} y(s) ds\Big] \\
&\times\Big[x(t) - x(t-\sigma(t))) - \int_{t-\sigma(t)}^{t} y(s) ds\Big] = 0. \quad (26)
\end{aligned}
$$

We obtain, for any positive real constants w,

$$0 \le w x^2(t-\sigma(t)) - w \tanh^2 x(t-\sigma(t)). \tag{27}$$

According to (24)-(27), it is straightforward to see that

$$\dot{V}(t) + 2\alpha V(t) \le \xi^T(t) \sum \xi(t), \tag{28}$$

where
$\xi = [x(t), y(t), x(t-\tau(t)), \int_{t-\tau(t)}^{t} y(s)ds, x(t-\sigma(t)),$
$\int_{t-\sigma(t)}^{t} y(s)ds, \tanh x(t-\sigma(t)), \int_{t-\rho(t)}^{t} x(s)ds, y(t-\tau(t)),$
$\int_{t-\tau(t)}^{t} x(s)ds, \int_{t-\tau_2}^{t-\tau(t)} x(s)ds, \int_{t-\sigma(t)}^{t} x(s)ds,$
$\int_{t-\sigma_2}^{t-\sigma(t)} x(s)ds, \int_{t-\tau(t)}^{t-\tau_1} x(s)ds, \int_{t-\sigma(t)}^{t-\sigma_1} x(s)ds, x(t-\tau_1),$
$x(t-\tau_2), x(t-\sigma_1), x(t-\sigma_2), \int_{t-\tau_1}^{t} y(s)ds, \int_{t-\sigma_1}^{t} y(s)ds,$
$\int_{t-\tau_1}^{t} x(s)ds, \int_{t-\sigma_1}^{t} x(s)ds, \int_{t-\sigma_1}^{t} \tanh x(s)ds,$
$\int_{t-\sigma_2}^{t} \tanh x(s)ds, \int_{t-\sigma_2}^{t-\sigma_1} \tanh x(s)ds]$

and $\sum$ is defined in (13). It is true that if conditions (14)-(22) hold, then

$$\dot{V}(t) + 2\alpha V(t) \le 0, \quad t \ge 0. \tag{29}$$

From (29), it is easy to see that

$$\|x(t,\phi)\| \le \beta\|\phi\| e^{-\alpha t}, \quad t \in R^+.$$

This means that equation (1) is exponentially stable. This completes the proof of this theorem.□

IV. Numerical Examples

In this section, numerical example is given to present the effectiveness of our main results by comparing the upper bounds of the delays and the parameter b as well as investigating the rate of convergence.

Example 7. Consider the following equation with mixed interval time-varying delays:

$$
\begin{aligned}
\frac{d}{dt}[x(t) + 0.1x(t-\tau(t))] &= -1.5x(t) \\
&\quad +b\tanh x(t-\sigma(t)) \\
&\quad +0.5 \int_{t-\rho(t)}^{t} x(s) ds. \quad (30)
\end{aligned}
$$

Decompose constants a as following $a = a_1 + a_2 + a_3$, respectively, where

$$a_1 = 0.5, \ a_2 = 0.5, \ a_3 = 0.5.$$

Solving the LMI (14) when $b = 0.2, \alpha = 0.4, \tau(t) = 0.1 + \frac{\sin^2(t)}{2}$, $\sigma(t) = 0.2 + \frac{\cos^2(t)}{2}$ and $\rho(t) = 0.2 + \frac{|\cos(t)|}{5}$,

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we can obtain a set of parameters guaranteeing exponential stability as follows:
$k_1 = 2.7032, \quad k_2 = 0.2344, \quad k_3 = 0.2169,$
$k_4 = 0.4090, \quad k_5 = 0.4276, \quad k_6 = 0.3210,$
$k_7 = 0.1988, \quad k_8 = 0.2886, \quad k_9 = 0.5843,$
$n_1 = 0.2663, \quad n_2 = 0.2725, \quad n_3 = 0.4393,$
$n_4 = 0.4393, \quad n_5 = 0.4855, \quad n_6 = 0.4993,$
$n_7 = 0.4513, \quad n_8 = 0.4058, \quad n_9 = 0.2142,$
$n_{10} = 0.2466, \quad n_{11} = 1.7784, \quad f_1 = 0.4641,$
$f_2 = 0.0067, \quad f_3 = 0.0266, \quad f_4 = 0.3016,$
$f_5 = 0.0391, \quad f_6 = 0.3447, \quad f_7 = 0.4656,$
$f_8 = 0.0273, \quad f_9 = 0.0335, \quad f_{10} = 0.3390,$
$f_{11} = 0.0435, \quad f_{12} = 0.3765, \quad r_1 = 0.3116,$
$r_2 = 0.0568, \quad r_3 = 0.2819, \quad r_4 = 0.2992,$
$r_5 = 0.0715, \quad r_6 = 0.3063, \quad m_1 = 0.0159,$
$m_2 = -0.0018, \quad m_3 = 0.2805, \quad m_4 = -0.1840,$
$m_5 = 0.2586, \quad m_6 = -0.0044, \quad m_7 = 0.0259,$
$m_8 = 0.2482, \quad m_9 = -0.1856, \quad m_{10} = 0.2196.$

Moreover, the upper bounds of the parameter b which guarantees the exponential and asymptotic stabilities are 1.5220 and 2.4310, respectively. The upper bounds b of this example can be found in Table I for different values of α, τ_d. The maximum upper bounds b for exponential and asymptotic stabilities of Example 7 is listed in Table II for different values of α, c.

TABLE I
UPPER BOUNDS OF b FOR EXAMPLE 7.

τ_d	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.1	2.5325	2.3401	2.1627	2.0011	1.8451
0.2	2.5102	2.3210	2.1402	1.9730	1.8049
0.3	2.4807	2.2920	2.1060	1.9354	1.7611
0.4	2.4640	2.2529	2.0690	1.8760	1.6860
0.5	2.4310	2.2112	2.0002	1.7749	1.5220

TABLE II
UPPER BOUNDS OF b FOR EXAMPLE 7.

c	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
0.5	2.4310	2.2112	2.0002	1.7749	1.5220
1	2.4243	2.2310	2.0110	1.7719	1.5170
2	2.4256	2.2210	2.0004	1.7786	1.5151
3	2.4410	2.2180	2.0001	1.7738	1.5166
4	2.4297	2.2191	2.0031	1.7831	1.5159

V. CONCLUSION

In this paper, we proposed the delay-range-dependent exponential stability criteria for certain neutral differential equation with discrete and distributed interval time-varying delays by using new class of augmented Lyapunov-Krasovskii functional, descriptor model transformation, decomposition technique of coefficient constant, Leibniz-Newton formula, utilization of zero equation, improved integral inequalities and Peng-Park's integral inequality. Finally, numerical example is given to show that the proposed criteria is effective.

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