

Development of Reliability Function in 2-Component Standby Redundant System with Priority Based on Maximum Entropy Principle

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Abstract—A 2-component standby redundant system with priority is well-known as one of the most fundamental redundant models in the reliability theory. There are many studies about the 2-component standby redundant system with priority. In late years, under the situation that only the average and variance of failure and repair time of each component are provided, the method for evaluating mean time to failure (MTTF) in the entire system has been proposed. Furthermore, by expanding the idea to evaluate MTTF under such a situation, an evaluation method for the variance of failure time in the entire system has been established. In this study, the procedure for obtaining the reliability function in the entire system by utilizing the mean and variance of system failure time is developed based on the maximum entropy principle.

Index Terms—method of Lagrange multiplier, maximum entropy principle, system reliability, 2-component standby redundant system with priority.

I. INTRODUCTION

IN modern society that various systems have become more complex and complicated with progress of technology, high reliability in various systems in the society have been required. Hence, the research and development in the reliability field is active. In the details, see textbooks in the reliability field such as Gertsbakh [1] and Birolini [2].

A 2-component redundant system with priority is well-known as one of the most fundamental redundant models in the reliability theory. There are many studies about the 2-component standby redundant system with priority. Zhang and Wang [3] and Yuan and Meng [4] have discussed a 2-component standby redundant system with priority in use. Also, Leung et al. [5] have investigated a standby redundant system with priority in repair. As their results, some important reliability indices and optimal operating policy of components in 2-component redundant system have been suggested. Note that it has been assumed in their studies that all distributions about failure and repair times of each component are respectively specified and exactly provided as respective exponential distributions.

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On the other hand, Buzacott [6] and Osaki [7], [8] have considered the 2-component redundant system consisting of priority and standby components, where the priority component is used whenever it is available. When the priority component falls in failure, it starts to be repaired immediately and the standby component starts to operate. If the standby component falls in failure during repairing the priority component, the entire system becomes down. Then, under the assumption that the respective failure and repair times of both components are explicitly provided as known probability distributions, Buzacott [6] and Osaki [7], [8] have derived individually their theoretical formulae of mean time to failure (MTTF) of the entire system.

Note that the evaluation formulae of MTTF derived by Buzacott and Osaki require any probability density functions (PDFs) and/or cumulative distribution functions (CDFs) about failure and repair times in the respective components. That is, the theoretical evaluation methods of MTTF of the 2-component standby redundant system with priority suggested by Buzacott and Osaki have required the prerequisite that the probability distributions of failure and repair times in each component are respectively specified as particular functions. However, it is considerably difficult to satisfy this prerequisite in practice. If we cannot identify explicitly the probability distributions about failure and repair times in each component, the methods mentioned above would not be applicable. In contrast, the mean and variance are the required minimum information on characterizing the probability distribution. In most practical cases, the mean and variance in a probability distribution will be at least known based on historical information. Hence, also for the probability distributions about failure and repair times in respective components, we can assume that respective the mean and variance of failure and repair times in respective components are respectively provided based on historical information.

In recent years, Takemoto and Arizono [9] have proposed the evaluation method of MTTF in the 2-component standby redundant system with priority under the limited information that only mean and variance about failure and repair times in the respective components are provided. Then, Takemoto and Arizono have considered the situation that the probability distributions about failure and repair times can be approximated as a mixed Erlang distribution. Under this situation, Takemoto and Arizono develop a new approximation procedure for evaluating the MTTF of the system by combining some results of the Markov analysis based on Erlang distributions. As the result, the approximate formula for computing the MTTF in the the 2-component standby

redundant system with priority has been constructed as the closed form under the limited information that only mean and variance about failure and repair times are provided. Furthermore, through expanding the evaluation procedure considered by Takemoto and Arizono [9], Oigawa et al. [10] have proposed the evaluation of the variance of failure time in the 2-component redundant standby system with priority in the case that only mean and variance about failure and repair times of each component are provided. It goes without saying that MTTF is effective as an index for evaluating the reliability of the system. Similarly, the variance of failure time in the 2-component standby system with priority is also an important index.

Through the above researches, we can evaluate MTTF and the variance of failure time quantitatively. Then, we can evaluate qualitatively the reliability of a 2-component standby redundant system with priority by MTTF and variance of failure time. For example, even if MTTF is large and in the case that the variance of failure time is relatively large, it is imagined to cause a problem in reliability.

In this study, we consider the reliability function in order to evaluate quantitatively the system reliability in the situation that only mean and variance about failure and repair times are provided as the limited information. Since the mean and variance of failure time in the entire system have been evaluated based on the considerations by Takemoto and Arizono [9] and Oigawa et al. [10], we consider to develop the probability density function about failure time in the entire system based on the the mean and variance of failure time in the entire system. However, there are many probability distributions with the same values in the first and second order moments. Hence, we adopt the maximum entropy principle to develop the system reliability function through identifying the the probability density function about failure time in the entire system under the constraints of the mean and variance of failure time in the entire system. The procedure based on the maximum entropy principle in the information theory is widely known as an elegant approach for deriving the probability density function under the constraints of the values of moments in the probability distribution.

II. DETAILS OF 2-COMPONENT STANDBY REDUNDANT SYSTEM WITH PRIORITY

We consider the 2-component standby redundant system composed of Component 1 having priority and Component 2 which is a standby component. In this case, the component with priority means a component which is always used whenever it is available. In contrast, the standby component means a component which stands by whenever the component with priority is available. When the priority component falls in failure, it starts to be repaired immediately and the standby component starts to operate. If the standby component falls in failure during repairing the priority component, the entire system becomes down. The outline of behavior in this system is illustrated in Figure 1. The solid line represents the operating status of each component. The dashed line represents the repair status of Component 1, the dashed-dotted line represents the standby status of Component 2. In addition, the dotted arrowhead connecting Component 1 and Component 2 represents the switching of components.

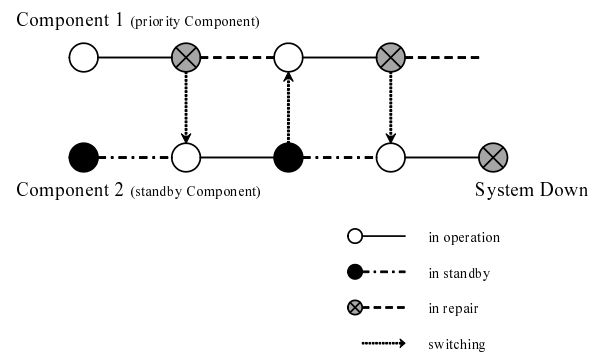


Fig. 1. 2-component standby redundant system with priority

Initially, at time $t = 0$, Component 1 is working, and Component 2 is in the standby state. At this time, the standby time of the Component 2 is not included in its lifetime. When Component 1 fails, it starts to be repaired immediately and then Component 2 starts to work. If the repair of Component 1 has been completed and Component 2 is still working, then Component 1 starts to work immediately, and Component 2 is in a standby state. However, if Component 2 fails during the repair of Component 1, the entire system goes down. The switching of components is reliably executed, and it is assumed that the switchover time is instantaneous.

III. OUTCOME OF PREVIOUS LITERATURE

For the 2-component standby redundant system of Figure 1, Buzacott [6] has derived MTTF under the situation that the failure and repair time distributions of each component are explicitly prescribed as individual specific probability distributions. On the other hand, under the situation that Component 2 has an exponential failure time distribution, Osaki [7], [8] has derived MTTF of the 2-component standby redundant system of Figure 1. All the distribution functions of the failure and repair times in each component have to be explicitly specified for evaluating MTTF by using the theoretical formulae by Osaki and Buzacott.

However, in some practical situations, it is difficult to know exactly the probability distributions of failure and repair times. On the other hand, in most practical cases, the mean and variance about failure and repair times in respective components will be at least known based on historical information.

In recent years, Takemoto and Arizono [9] and Oigawa et al. [10] have respectively considered the evaluations of mean time to failure and variance of failure time in the 2-component standby redundant system illustrated in Figure 1. In these evaluations, the followings have been assumed:

- i) The failure time distribution of Component 1 has the cumulative distribution function (CDF) $F_1(t)$ with mean $E_{F_1} = 1/\lambda_1$ and variance V_{F_1} .
- ii) The repair time of Component 1 obeys the probability distribution G_1 with mean E_{G_1} and variance V_{G_1} .
- iii) The failure time of Component 2 obeys the probability distribution F_2 with mean E_{F_2} and variance V_{F_2} .

They have considered the approximation where the repair (or failure) time distribution in Component 1 (or 2) is

transformed into a mixed Erlang distribution under the constraints of mean E_{G_1} (or E_{F_2}) and variance V_{G_1} (or V_{F_2}). Then, the system has been reconstructed by four types of tentative systems based on the combinations of respective Erlang distributions in the mixed Erlang distributions for the repair time distribution of Component 1 and the failure time distribution of Component 2. The respective tentative systems have been analyzed using Markov process theory because the state transition in the respective tentative systems can be expressed by a state transition diagram based on an exponential distribution. MTTFs in the respective tentative systems are weightedly summed up using the ratios of mixture on the mixed Erlang distribution.

As a consequence, Takemoto and Arizono [9] have defined MTTF of the entire system as follows:

$$\begin{aligned}
 MTTF &= p_1 p_2 MTTF_{(n_1, n_2)} \\
 &+ (1 - p_1) p_2 MTTF_{(n_1+1, n_2)} \\
 &+ p_1 (1 - p_2) MTTF_{(n_1, n_2+1)} \\
 &+ (1 - p_1) (1 - p_2) MTTF_{(n_1+1, n_2+1)} \equiv \mu, \quad (1)
 \end{aligned}$$

where

$$\begin{aligned}
 MTTF_{(n_1, n_2)} &= \tilde{R}_{(n_1, n_2)}(0) \\
 &= \frac{1/\lambda_1}{1 - \tilde{b}^{n_1}(0)} \sum_{j=0}^{n_2-1} \tilde{d}_{j(n_1)}(0) + \frac{\tilde{a}(0)}{1 - \tilde{b}^{n_1}(0)} \\
 &\times \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} \binom{i+k}{k} \tilde{b}^i(0) \tilde{c}^k(0) \tilde{d}_{j(n_1)}(0), \quad (2) \\
 \frac{1}{n_i + 1} &< \varphi_i^2 \leq \frac{1}{n_i}, \quad (3)
 \end{aligned}$$

$$p_i = \frac{(n_i + 1)\varphi_i^2 - \sqrt{(n_i + 1)(1 - n_i\varphi_i^2)}}{\varphi_i^2 + 1}, \quad (4)$$

$$\varphi_1 = \sqrt{V_{G_1}/E_{G_1}}, \quad (5)$$

$$\varphi_2 = \sqrt{V_{F_2}/E_{F_2}}, \quad (6)$$

$$\mu_1 = \frac{n_1 + 1 - p_1}{E_{G_1}}, \quad (7)$$

$$\lambda_2 = \frac{n_2 + 1 - p_2}{E_{F_2}}, \quad (8)$$

$$\begin{aligned}
 \tilde{R}_{(n_1, n_2)}(s) &= \tilde{u}_{(n_1)}(s) \sum_{i=0}^{n_2-1} \tilde{d}_{i(n_1)}(s) \\
 &+ \tilde{a}(s) \tilde{v}_{(n_1)}(s) \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} \binom{i+k}{k} \\
 &\times \tilde{b}^i(s) \tilde{c}^k(s) \tilde{d}_{j(n_1)}(s), \quad (9)
 \end{aligned}$$

$$\tilde{a}(s) = \frac{1}{s + \mu_1 + \lambda_2}, \quad (10)$$

$$\tilde{b}(s) = \frac{\mu_1}{s + \mu_1 + \lambda_2}, \quad (11)$$

$$\tilde{c}(s) = \frac{\lambda_2}{s + \mu_1 + \lambda_2}, \quad (12)$$

$$\tilde{d}_{0(n_1)}(s) = 1, \quad (13)$$

$$\begin{aligned}
 \tilde{d}_{j(n_1)}(s) &= \tilde{w}_{(n_1)}(s) \\
 &\times \sum_{k=0}^{j-1} \binom{n_1 - 1 + j - k}{j - k} \tilde{c}^{j-k}(s) \tilde{d}_{k(n_1)}(s), \quad (14)
 \end{aligned}$$

$$\tilde{u}_{(n_1)}(s) = \frac{\tilde{R}_1(s)}{1 - \{1 - s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}, \quad (15)$$

$$\tilde{v}_{(n_1)}(s) = \frac{1 - s\tilde{R}_1(s)}{1 - \{1 - s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}, \quad (16)$$

$$\tilde{w}_{(n_1)}(s) = \frac{\{1 - s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}{1 - \{1 - s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}. \quad (17)$$

The notations of $MTTF_{(n_1, n_2)}$, $MTTF_{(n_1+1, n_2)}$, $MTTF_{(n_1, n_2+1)}$ and $MTTF_{(n_1+1, n_2+1)}$ in equation (1) denote the mean time to failure in the respective tentative systems. Further, the symbols of p_i , φ_1 , φ_2 , μ_1 and λ_2 denote the parameters in the case of approximating the repair time distribution of Component 1 and the failure time distribution of Component 2 to the respective mixed Erlang distributions. In addition, the notations of $\tilde{R}_{(n_1, n_2)}(s)$, $\tilde{a}(s)$, $\tilde{b}(s)$, $\tilde{c}(s)$, $\tilde{d}_{j(n_1)}(s)$, $\tilde{u}_{(n_1)}(s)$, $\tilde{v}_{(n_1)}(s)$ and $\tilde{w}_{(n_1)}(s)$ mean the functions derived by Takemoto and Arizono [9] for evaluating MTTF in the 2-component standby redundant system. Further, $R_1(t) = 1 - F_1(t)$ and then, $\tilde{R}_1(s)$ denotes the Laplace transformation for $R_1(t)$.

Further, Oigawa et al. [10] have addressed the derivation of the variance of the failure time of the considered 2-component standby redundant system with priority. Denote the probability density function (PDF) of the failure time distribution of the entire system by $f(t)$. Then, the Laplace transformation for $f(t)$ is defined as:

$$\tilde{f}(s) = \int_0^\infty e^{-st} f(t) dt. \quad (18)$$

On the other hand, since the reliability function $R(t)$ of the entire system is expressed as

$$R(t) = 1 - \int_0^t f(\tau) d\tau, \quad (19)$$

the relationship between the Laplace transformations $\tilde{f}(s)$ and $\tilde{R}(s)$ can be given as $\tilde{f}(s) = 1 - s\tilde{R}(s)$. Then, by applying the final-value theorem and the L'Hopital's rule to the differential calculus forms for this relationship, the following equations can be shown:

$$\frac{d}{ds} \tilde{f}(s)|_{s=0} \equiv \tilde{f}^{(1)}(0) = -E[t] = -\mu, \quad (20)$$

$$\frac{d^2}{ds^2} \tilde{f}(s)|_{s=0} \equiv \tilde{f}^{(2)}(0) = E[t^2], \quad (21)$$

where superscript (i) of the function $\tilde{f}^{(i)}(0)$ means i -th differentiation of the function $\tilde{f}(s)$.

Therefore, the variance $V[t]$ of the failure time of the considered 2-component standby redundant system with priority can be obtained as

$$V[t] = \tilde{f}^{(2)}(0) - \left\{ \tilde{f}^{(1)}(0) \right\}^2 \equiv \sigma^2, \quad (22)$$

where

$$\begin{aligned} \tilde{f}^{(2)}(0) &= p_1 p_2 \tilde{f}_{(n_1, n_2)}^{(2)}(0) \\ &+ (1 - p_1) p_2 \tilde{f}_{(n_1+1, n_2)}^{(2)}(0) \\ &+ p_1 (1 - p_2) \tilde{f}_{(n_1, n_2+1)}^{(2)}(0) \\ &+ (1 - p_1)(1 - p_2) \tilde{f}_{(n_1+1, n_2+1)}^{(2)}(0), \quad (23) \end{aligned}$$

$$\begin{aligned} \tilde{f}_{(n_1, n_2)}^{(2)}(0) &= -2\tilde{u}_{(n_1)}^{(1)}(0) \sum_{j=0}^{n_2-1} \tilde{d}_{j(n_1)}(0) \\ &- 2\tilde{u}_{(n_1)}(0) \sum_{j=0}^{n_2-1} \tilde{d}_{j(n_1)}^{(1)}(0) \\ &+ 2\tilde{a}(0)\tilde{v}_{(n_1)}(0) \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} \binom{i+k}{k} \\ &\times \tilde{b}^i(0)\tilde{c}^k(0) \left\{ \tilde{d}_{j(n_1)}(0)\tilde{a}(0) \left(1 + i + k \right. \right. \\ &\left. \left. + \frac{h_{(n_1, n_2)}(0)}{\tilde{a}(0)} \right) - \tilde{d}_{j(n_1)}^{(1)}(0) \right\}, \quad (24) \end{aligned}$$

$$\begin{aligned} \tilde{u}_{(n_1)}^{(1)}(0) &= -\frac{V_{F1} + \left(\frac{1}{\lambda_1}\right)^2}{2 \left\{ 1 - \tilde{b}^{n_1}(0) \right\}} \\ &- \frac{\frac{1}{\lambda_1} \tilde{b}^{n_1}(0) \left\{ n_1 \tilde{a}(0) + \frac{1}{\lambda_1} \right\}}{\left\{ 1 - \tilde{b}^{n_1}(0) \right\}^2}, \quad (25) \end{aligned}$$

$$\begin{aligned} \tilde{d}_{j(n_1)}^{(1)}(0) &= \tilde{w}_{(n_1)}^{(1)}(0) \sum_{k=0}^{j-1} \binom{n_1-1+j-k}{j-k} \\ &\times \tilde{c}^{j-k}(0)\tilde{d}_{k(n_1)}(0) \\ &- \tilde{w}_{(n_1)}(0) \sum_{k=0}^{j-1} \binom{n_1-1+j-k}{j-k} \\ &\times (j-k)\tilde{a}(0)\tilde{c}^{j-k}(0)\tilde{d}_{k(n_1)}(0) \\ &+ \tilde{w}_{(n_1)}(0) \sum_{k=0}^{j-1} \binom{n_1-1+j-k}{j-k} \\ &\times \tilde{c}^{j-k}(0)\tilde{d}_{k(n_1)}^{(1)}(0), \quad (26) \end{aligned}$$

$$\begin{aligned} \tilde{w}_{n_1}^{(1)}(0) &= -\tilde{v}_{n_1}(0)h_{(n_1, n_2)}(0)\tilde{b}^{n_1}(0) \\ &- n_1\tilde{a}(0)\tilde{b}^{n_1}(0)\tilde{v}_{n_1}(0), \quad (27) \end{aligned}$$

$$\begin{aligned} h_{(n_1, n_2)}(0) &= \tilde{R}_{(n_1, n_2)}(0) \\ &+ \frac{\tilde{b}^{n_1}(0)\tilde{R}_{(n_1, n_2)}(0) + n_1\tilde{a}(0)\tilde{b}^{n_1}(0)}{1 - \tilde{b}^{n_1}(0)}, \quad (28) \end{aligned}$$

where note that the notations of $\tilde{f}_{(n_1, n_2)}^{(i)}(0)$, $\tilde{u}_{(n_1)}^{(1)}(0)$, $\tilde{d}_{j(n_1)}^{(1)}(0)$ and $\tilde{w}_{n_1}^{(1)}(0)$ are similar to the notation of $\tilde{f}^{(i)}(0)$.

Through the similar way, $\tilde{f}_{(n_1+1, n_2)}^{(2)}(0)$, $\tilde{f}_{(n_1, n_2+1)}^{(2)}(0)$ and $\tilde{f}_{(n_1+1, n_2+1)}^{(2)}(0)$ can be obtained. Remind that the concrete mathematical expressions of $\tilde{f}_{(n_1+1, n_2)}^{(2)}(0)$, $\tilde{f}_{(n_1, n_2+1)}^{(2)}(0)$ and $\tilde{f}_{(n_1+1, n_2+1)}^{(2)}(0)$ are omitted for convenience of the number of the paper.

IV. DERIVATION OF RELIABILITY FUNCTION

In this study, based on μ and σ^2 derived as equations (1) and (22), we evaluate the probability density function $f(t)$ of the failure time of the entire system. As for derivation of μ and σ^2 , we have considered that only the mean and variance of failure and repair times in each component are given as the limited information. In other words, the basic condition in this study is that the distribution of failure and repair times in each component is unknown and not specified. Hence, we do not suppose a specific distribution type to evaluate the probability density function $f(t)$ of the failure time in the entire system. Then, it is widely known that the maximum entropy principle brings an elegant solution for deriving the probability density function under the constraints of the values of moments. Therefore, we apply the maximum entropy principle to obtain the probability density function $f(t)$ of the failure time in the entire system.

Based on the maximum entropy principle, the identification problem of $f(t)$ can be formulated as follows:

$$\text{Maximize } H(f) = - \int_0^\infty f(x) \log f(x) dx, \quad (29)$$

subject to

$$\int_0^\infty f(x) dx = 1, \quad (30)$$

$$\int_0^\infty xf(x) dx = \mu, \quad (31)$$

$$\int_0^\infty (x - \mu)^2 f(x) dx = \sigma^2. \quad (32)$$

By using equations (29)–(32), a Lagrange function is defined with introducing Lagrange multipliers α_1 , α_2 and α_3 as

$$\begin{aligned} V &= H(f) - \alpha_1 \left\{ \int_0^\infty f(x) dx - 1 \right\} \\ &- \alpha_2 \left\{ \int_0^\infty xf(x) dx - \mu \right\} \\ &- \alpha_3 \left\{ \int_0^\infty (x - \mu)^2 f(x) dx - \sigma^2 \right\}. \quad (33) \end{aligned}$$

Through adopting the variational method to the Lagrange function V , we have

$$\begin{aligned} \frac{\delta V}{\delta f} &= - \left\{ \log f(x) + 1 \right\} \\ &- \alpha_1 - \alpha_2 x - \alpha_3 (x - \mu)^2 = 0. \quad (34) \end{aligned}$$

From equation (34), $f(x)$ is expressed as

$$f(x) = e^{-1-\alpha_1} e^{-\alpha_2 x} e^{-\alpha_3 (x-\mu)^2}. \quad (35)$$

At the same instant, by partially differentiating V with α_1 , α_2 and α_3 , we have equations (30)–(32). Therefore, based on equations (30)–(32) and (35), $f(x)$ is rewritten as

$$f(x) = C e^{-\frac{(x-\mu)^2}{2\delta^2}}, \quad (36)$$

where

$$C = e^{-1-\alpha_1-\alpha_3\mu^2} e^{\alpha_3\left(\frac{2\mu\alpha_3-\alpha_2}{2\alpha_3}\right)^2}, \quad (37)$$

$$m = \frac{2\mu\alpha_3 - \alpha_2}{2\alpha_3}, \quad (38)$$

$$\delta^2 = \frac{1}{2\alpha_3}. \quad (39)$$

Accordingly, the derivation of $f(x)$ becomes a problem of finding C , m and δ^2 .

Furthermore, by substituting equation (36) for equation (30), (31) and (32), the following equations can be shown:

$$C \int_0^\infty e^{-\frac{(x-m)^2}{2\delta^2}} dx = 1, \quad (40)$$

$$C \int_0^\infty x e^{-\frac{(x-m)^2}{2\delta^2}} dx = \mu, \quad (41)$$

$$C \int_0^\infty (x-\mu)^2 e^{-\frac{(x-m)^2}{2\delta^2}} dx = \sigma^2. \quad (42)$$

Then, from equations (40), we can derive as follows:

$$\frac{1}{C} = \int_0^\infty e^{-\frac{(x-m)^2}{2\delta^2}} dx. \quad (43)$$

Moreover, by multiplying both sides of equation (43) by $1/\sqrt{2\pi\delta^2}$, we obtain the following equation:

$$\frac{1}{\sqrt{2\pi\delta^2}} \frac{1}{C} = \frac{1}{\sqrt{2\pi\delta^2}} \int_0^m e^{-\frac{(x-m)^2}{2\delta^2}} dx + \frac{1}{\sqrt{2\pi\delta^2}} \int_m^\infty e^{-\frac{(x-m)^2}{2\delta^2}} dx. \quad (44)$$

The right side means the upper side probability function of the normal distribution with mean m and variance δ^2 . Through the variable transformation of $t = (x-m)/\delta$, equation (44) is converted to

$$\frac{1}{\sqrt{2\pi\delta^2}} \frac{1}{C} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{m}{\delta}}^0 e^{-\frac{t^2}{2}} dt + \frac{1}{2}. \quad (45)$$

Further, by replacing t by $\sqrt{2}\tau$, equation (45) is expressed as

$$\begin{aligned} \frac{1}{\sqrt{2\pi\delta}} \frac{1}{C} &= \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_{-\frac{m}{\sqrt{2\delta}}}^0 e^{-\tau^2} d\tau \\ &= \frac{1}{2} - \frac{1}{2} \left(\frac{2}{\sqrt{\pi}} \int_0^{-\frac{m}{\sqrt{2\delta}}} e^{-\tau^2} d\tau \right) \\ &= \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(-\frac{m}{\sqrt{2\delta}} \right). \end{aligned} \quad (46)$$

Then, the function $\operatorname{erf}(z)$ is known as the error function as follows:

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt. \quad (47)$$

The error function is frequently used in probability theory, statistics, material science, and partial differential equation. Further, the error function and confluent hypergeometric function have the following relationship:

$$\operatorname{erf}(x) = \frac{2x}{\sqrt{\pi}} {}_1F_1 \left(\frac{1}{2}; \frac{3}{2}; -x^2 \right). \quad (48)$$

Note that the confluent hypergeometric function is classified as special functions. Then, the confluent hypergeometric function is defined as

$${}_1F_1(\beta; \gamma; z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{\Gamma(\beta+n)}{\Gamma(\gamma+n)} \frac{z^n}{n!}. \quad (49)$$

Furthermore, the following Kummer transformation can be applied to be confluent hypergeometric function:

$${}_1F_1(\beta; \gamma; z) = e^{-z} {}_1F_1(\gamma - \beta; \gamma; -z). \quad (50)$$

Consequently, based on equation (48) and (50), equation (46) is transformed as follows

$$C = \frac{1}{\sqrt{\frac{\pi\delta^2}{2}} + m e^{-\frac{m^2}{2\delta^2}} {}_1F_1 \left(1; \frac{3}{2}; \frac{m^2}{2\delta^2} \right)}. \quad (51)$$

Further, as for equations (41) and (42), we have respectively the following equations:

$$C = \frac{\mu - m}{\delta^2 e^{-\frac{m^2}{2\delta^2}}}, \quad (52)$$

$$C = \frac{\mu^2 + \sigma^2 - m^2 - \delta^2}{m\delta^2 e^{-\frac{m^2}{2\delta^2}}}. \quad (53)$$

Then, based on equations (52) and (53), the following equations can be shown:

$$\delta^2 = -\mu m + \mu^2 + \sigma^2. \quad (54)$$

Accordingly, by substituting equation (54) for equation (51) and (52) (or (53)), we can obtain the following function for m :

$$\begin{aligned} (\mu - m) \left\{ \sqrt{\frac{\pi(-\mu m + \mu^2 + \sigma^2)}{2}} + m e^{-\frac{m^2}{2(-\mu m + \mu^2 + \sigma^2)}} \right. \\ \left. \times {}_1F_1 \left(1, \frac{3}{2}; \frac{m^2}{2(-\mu m + \mu^2 + \sigma^2)} \right) \right\} \\ - (-\mu m + \mu^2 + \sigma^2) e^{-\frac{m^2}{2(-\mu m + \mu^2 + \sigma^2)}} = 0. \end{aligned} \quad (55)$$

It is difficult to solve analytically equation (55) for m because that equation (55) contains the confluent hypergeometric function. However, we can derive the value of m satisfying equation (55) numerically by appropriate search algorithms such as linear search. It is clear that we can get the values of δ^2 and C by calculated m . Moreover, by substituting equation (36) for equation (19), we can derive as follows.

$$\begin{aligned} R(t) &= C \left\{ \sqrt{\frac{\pi\delta^2}{2}} - (t - m) \right. \\ &\quad \left. \times e^{-\frac{(t-m)^2}{2\delta^2}} {}_1F_1 \left(1; \frac{3}{2}; \frac{(t-m)^2}{2\delta^2} \right) \right\}. \end{aligned} \quad (56)$$

Thus, the reliability of the entire system can be evaluated concretely by substituting calculated m , δ^2 and C for equation (56).

V. NUMERICAL ANALYSIS

In this section, we investigate the validity of our proposal on evaluating the reliability function with the following procedures:

- i) The mean and variance of failure and repair time in Component 1 are respectively given. Further, the mean and variance of failure time in Component 2 are also given.
- ii) The probability distributions for the failure and repair time in Component 1 and the probability distribution for the failure time in Component 2 are individually specified as a particular probability distribution.
- iii) Through a million times of trials iterated in the Monte Carlo simulation under the combination of specified probability distributions, the reliability in the entire system on time t is estimated.
- iv) The proposed reliability function is compared with the reliability estimated through the Monte Carlo simulation.

As an example, suppose that $E_{F_1} = 1/\lambda_1 = 100.0$, $V_{F_1} = 20.0^2$, $E_{G_1} = 5.0$, $V_{G_1} = 1.5^2$, $E_{F_2} = 50.0$, and $V_{F_2} = 22.5^2$, respectively. Moreover, the failure and repair time distributions in both components are respectively defined as the log-normal and/or Weibull distributions. The log-normal distribution and Weibull distribution are well known in the reliability field.

Based on the above numerical values and the supposition for the failure and repair time distributions, the values of μ and σ^2 for the failure times in the entire system are calculated as the following numerical value:

$$(\mu, \sigma^2) = (1106.089, 239852.167).$$

Further, m , δ^2 and C are obtained as follows:

$$(m, \delta^2, C) = (1057.804, 345871, 144, 0.000704).$$

So, we can evaluate the reliability function in equation (56) by applying the above values of (m, δ^2, C) .

One of the results of comparing the simulation and the proposed reliability is shown in Figure 2. In Figure 2, the failure time distribution $F_1(t)$ and the repair time distribution $G_1(t)$ in Component 1 and the failure time distribution $F_2(t)$ in Component 2 are respectively supposed as the Weibull distribution with mean E_{F_1} and variance V_{F_1} , the log-normal distribution with mean E_{G_1} and variance V_{G_1} and the Weibull distribution with mean E_{F_2} and variance V_{F_2} .

From Figure 2, it has been confirmed that the shape of the reliability function proposed in this study resembles that of simulation result. Moreover, remark that similar results have been obtained under different conditions. Consequently, it has been confirmed that the proposed reliability function based on the maximum entropy principle is useful as an approximation method for the reliability evaluation under the condition that only the mean and variance of failure time in the entire system can be used.

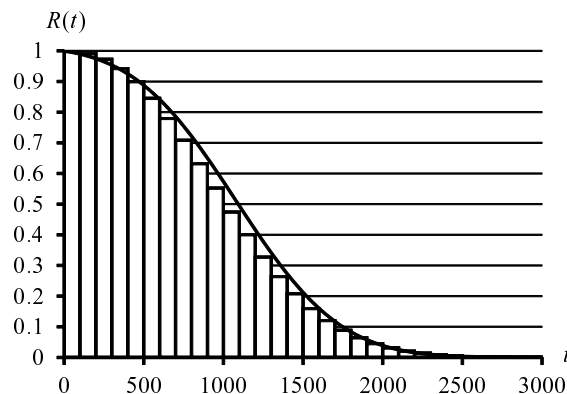


Fig. 2. Comparison of reliability based on proposed reliability function and the simulation results

VI. CONCLUSION

In this study, we have addressed the evaluation method of the reliability function in the 2-component standby redundant system with priority. Then, by utilizing the evaluation results for the mean time to failure by Takemoto and Arizono [9] and the variance of the failure time by Oigawa et al. [10], we have succeeded in deriving the evaluation method of the reliability function in the 2-component standby redundant system with priority. In concrete, by using only the mean and variance of the failure time in the entire system, the reliability function in the 2-component standby redundant system with priority has been derived based on the maximum entropy principle.

Then, it would like to be the future issue to establish the way of the optimal preventive maintenance of the system by using the reliability indices such as the reliability function.

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