Fuzzy Supply Chain Integrated Inventory Model with Quantity Discounts and Unreliable Process in Uncertain Environments

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ABSTRACT - This paper mainly focuses on the discussion of the best economic production quantity of unreliable process, uncertain environments and defective items reproduce of the production system. In today's production and manufacturing schedule, the quality of the supply chain often determines the efficiency of a company's operations. However, the traditional method of solving the problem of economic production quantities, mostly assumes that perfect production process does not appear defective items and backorder situation. This research's system is based on the production of finished goods inventory system. In the system, defective products are separated by its defectiveness. Non-repairable defective products are destroyed; the rest will be repaired and resent to the buyer. Because of the uncertain environments, fuzzy is added to the research in order to obtain a more realistic result.

Index Terms—Quantity Discounts, Uncertain Environment, Inventory Model, Unreliable Process.

I. INTRODUCTION

Inventory strategy is very important for firms, which maintain advantages in production and logistics parts. The comprehensive inventory system can achieve the best level of service and reduce manufacturing and inventory costs and maximize profits. Previous studies usually built the traditional integrated inventory model in the perfect production processes without defective products. However; in reality, the defective products are unavoidable by human errors, mechanical failures and other reasons. Therefore, this study commits to sort out what the defective rate impact of the costs for buyers and sellers and also reduce losses arising from defective products.

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Suppliers commonly used quantity discount as concessions to attract buyers. However, this study considers that quantity discount is used to compensate the buyer to purchase the loss caused by defective products.

This paper presents an integrated supply chain inventory model which includes the uncertainty of environment and quantity discounts with the consideration of minimizing the total cost of the buyer and seller. Due to the uncertainty of buyer's demand, therefore fuzzy is added to this research. Also, this paper assumes that the production process will produce a certain number of defective products, the buyer checks out the defective products, and returned to the seller to repair, and the vendor will provide a discount. All of the above are put in the research for a more realistic solution. To find the minimum total cost, must determine the optimal order quantity (Q) and delivery times per production cycle (n), so taking first and second-order partial derivative of EK (n, Q) with respect to Q and n, this paper obtains n and Q's extreme value, because delivery times (n) is an integer, this study used an interactive method to calculate the optimal solution of n and Q as well as the minimum total costs.

After obtaining the minimum total cost, this study applies 4 parameters (screening rate (X), annual demand (D), percentage of the defective products (k), production rate (P)) in doing sensitivity analysis with EK (n, Q); and shows how the effect of the 4 parameters changed. Subsequently, this paper will apply the experimental data with mathematical software for Q, D, and the EK to assess three-dimensional map.

Starting from the previous study review, Porteus (1986) was the first researcher to incorporated the impact caused by defective products into basic EOQ model. Based on the research, we acknowledge the importance of unreliable process's impact. Schwaller (1988) extended EOQ models to conform real-life environment of inventories by adding assumptions of a known proportion of defectives present in the incoming lots. Ben-Daya and Hariga (2000) considered the problem about impact of imperfections in process of a model, and assumed that the start of production facility is in perfect quality. But facilities will deteriorate over time and move to an uncontrollable state, defectives are then produced. Salameh and Jaber (2000) assumed production and inventory situation, items, or products are not in perfect quality. Defective and unwanted products can be used in other restrictive procedures, acceptance control production and inventory situation with the consideration of poor-quality items at the end should be sold out. Goyal and Cardenas-

Barron (2002) developed a model to determine the total profits per unit time and purchase products from supplies EOQ; also proposed a method to determined EPQ and defective products. Huang (2004) suggested that a model developed under JIT manufacturing environment to determine defective items held by the seller and the buyer is the best integrated inventory strategy. Huang (2004) also proposed a model that is built on examine defective products during continuous consumption of inventory, and the items that are spot out will be reimbursed.

The discussion of this paper is to carry out the changes of inventory quantity discount model and process unreliable situation, also to find out the most suitable order quantity from buyers and sellers in order to achieve a minimized total cost.

In today's highly competitive global markets, many marketing tactics and manufacturers are applying price discounts to attract consumers. Lal and Staelin (1984) developed a strategy to conducive the buyer for the best price discounts. Chakravarty and Martin (1988) provided the vendor with the means for optimally, determining both the discount price and the replenishment interval under periodic review for desired joint savings-sharing scheme between the seller and multiple-buyer(s). Munson and Rosenblatt (1988) proposed a third-level quantity discount with supply chain and fixed demand rate.

Wang (2005) extended traditional quantity discounts that are based solely on buyers' individual order size to discount policies that are based on both buyers' individual order size and their annual volume. They showed that discount policies are able to achieve nearly optimal system profit and, hence, provide effective coordination. Li and Liu (2006) developed a model that explains how to use quantity discount policy in order to achieve the supply chain coordination, considering only selling one product with multi-cycle and the probability of customer's demand for the buyer and seller system, and suggest that the combination in a mutually acceptable quantity discount profit exceeded the sum of profits from each other in the case of decentralized decision-making.

Yang *et al.*, (2010) established an inventory model for retailer in a supply chain when a supplier offers either a cash discount or a delay payment linked to ordering quantity. Lin and Lin (2014) developed a model about defective products and quantity discounts. The purpose was to find optimal pricing and ordering strategy; the analysis is based on the buyer's order quantity. Zhang and Xu (2014) proposed multiple objective decision making (MODM) model considered the bi-fuzzy environment and quantity discount policy, and quantity discount is an important factor in their study.

The past studies mainly focused on price promotions, discounts and prices strategies; this is because the above can directly affect the cost and profit. However, those studies ignored that different quantity discount policy may cause a bad influence to the profit. Therefore, this research determines quantity discounts based on detective rate. Because of the uncertain environments, fuzzy is added to the research in order to obtain a more realistic result.

II. MATERIALS AND METHODS

To establish the proposed model, the following notations are used, and some assumptions are made throughout this study.

2.1 Notations

- S_v: Set up cost for the vendor; $/time
- Q: Each time the number of transported from the buyer; pcs/times
- P: Production rate; pcs/year
- R: Recovery cost for the vendor; $/month
- L: Maintenance cost for the vendor; $/month
- n: Number of deliveries each production cycle; times
- h_v Holding cost for the vendor; $/month
- V: Warranty cost for the vendor; $/month
- Y: The percentage of defective products, as random variables
- Q_r: manufacturing cost for vendor; $/month
- S_b: Order cost for the buyer; $/time
- F: Transportation cost per shipment; $/trip
- $\widetilde{D}$: Triangular fuzzy number; $\widetilde{D} = (D - \Delta_1, D, D + \Delta_2), 0 < \Delta_1 < D, 0 < \Delta_2$, and Δ_1,Δ_2 is determine by the decision maker
- h_b: Holding cost for buyer; $/month
- d: Screening cost for buyer; $/month
- X: Screening rate; pce/year
- σ: Discount rate. ; $\sigma = m * Y * k$, punishment multiples (m) is determined by the seller themselves
- B: Purchase cost for buyer; $/month
- T: Transporting each successive time intervals
- T_c: Cycle time$T_c = n * T$.
- k: Percentage of defective products cannot be repaired percentage
- EK: Expected annual integrated total cost.

2.2 Assumptions

- This paper is based on single vender and single buyer for single item.
- The production rate is finite.
- Shortage are not allowed.
- Because of shortages are not allowed, non-defective product's production rate must higher than buyer's demand.
- Quantity discount and defective rate has direct relation.
- Returned defective product will be repaired, but not fully repaired.
- When buyer's inventory remains Q/2, all products must be inspected, defective products must be picked up and send back to the vender.
- Quantity discount have a restriction, because vender's cost can't more than buyer's purchased cost; otherwise, the vender doesn't have profits.

$$v + \frac{Q_r}{\widetilde{D}} + \sigma B < B \Rightarrow \sigma < 1 - \frac{\left(\frac{Q_r}{\widetilde{D}} + v\right)}{B}$$

- Assumed the discount rate as $\sigma = mYk$(m is a magnification, determine by vender.), the discount rate for the buyer will increase by the amount of defective products.

2.3 Vender's Cost

$$\begin{aligned} \text{Vender's cost} = {} & \text{setup cost} + \text{transportation cost} \\ & + \text{manufacturing cost} \\ & + \text{recovery cost} \\ & + \text{maintenence cost} + \text{holding cost} \end{aligned}$$

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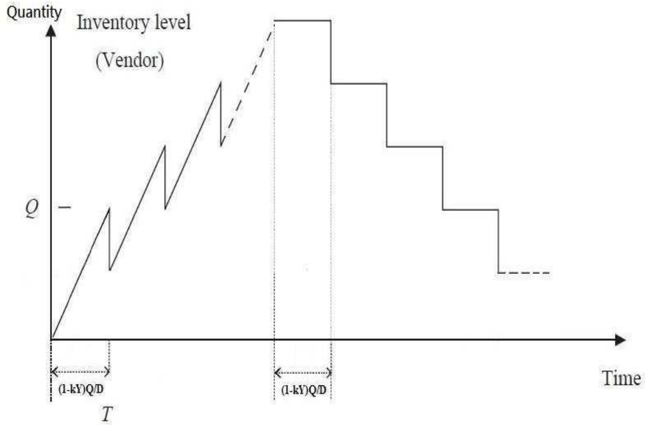

Fig.1. Schematic diagram of Vender's Cost

$$TC_V(Q,n) = S_V * \frac{\widetilde{D}}{nQ(1-kY)} + F(1+2Y-kY) * \frac{\widetilde{D}}{Q(1-kY)} + Q_r\widetilde{D} + RkY\widetilde{D} + LY\widetilde{D} + h_vQ\left[\frac{n-1}{2}+\frac{\widetilde{D}(2-n)}{2p(1-kY)}\right]$$

$$\Rightarrow TC_V(Q,n) = \widetilde{D}\left[\frac{S_V}{nQ(1-kY)}+\frac{F(1+2Y-kY)}{Q(1-kY)}+Qr+RkY+LY+\frac{h_vQ(2-n)}{2p(1-kY)}\right]+\frac{h_vQ(n-1)}{2}$$

Definition 1. From kaufmann and Gupta (1991), Zimmermann (1996), Yao and Wu (2000), for a fuzzy set $\tilde{B} \in \Omega$ and ore $[0,1]$, the α-cut of the fuzzy set $\tilde{B}$ is $\mathrm{B}(\alpha) = \{x \in \Omega | \mu_B(x) \geq \alpha\} = [B_L(\alpha), B_U(\alpha)]$, where $B_L(\alpha) = a + \alpha(b-d)$ and $B_U(\alpha) = c - \alpha(c-b)$. We can obtain the following equation. The signed distance of $\tilde{B}$ to $\tilde{0}_1$ is defined as

$d(\tilde{B},\tilde{0}_1) = \int_0^t d\{[B_L(\alpha),B_U(\alpha)],\tilde{0}_1\}d\alpha = \frac{1}{2}\int_0^1 [B_L(\alpha),B_U(\alpha)]d\alpha$.

So this equation is

$d(\tilde{B},\tilde{0}_1) = \frac{1}{2}\int_0^1 [B_L(\alpha),B_U(\alpha)]d\alpha = \frac{1}{4}(2b+a+c)$.

Streamlined distance method is used to the defuzzication of $TC_V(Q,n)$.

$$\widetilde{D} = d(\widetilde{D},\tilde{0}_1) = \frac{1}{4}[(D-\Delta_1)+2D+(D-\Delta_2)] = D+\frac{1}{4}(\Delta_2-\Delta_1)$$

$$\Rightarrow TC_V(Q,n) = \left[D+\frac{(\Delta_2-\Delta_1)}{4}\right]\left[\frac{S_V}{nQ(1-kY)}+\frac{F(1+2Y-kY)}{Q(1-kY)}+Qr+RkY+LY+\frac{h_vQ(2-n)}{2p(1-kY)}\right]+\frac{h_vQ(n-1)}{2} \quad (1)$$

Vender transportation cost's derivation:

$$TrC_V = F * \frac{Q+YQ+(1-k)YQ}{Q} * \frac{\widetilde{D}}{Q(1-kY)}$$
$$= \mathrm{F} * \frac{Q+2YQ-kYQ}{Q} * \frac{\widetilde{D}}{Q(1-kY)}$$
$$= \mathrm{F}(1+2Y-kY) * \frac{\widetilde{D}}{Q(1-kY)}$$

Vender holding cost's derivation:

$$Hc_V = \frac{h_V\left\{nQ\left[\frac{Q}{P}+T(n-1)\right]-\frac{nQ\left(\frac{nQ}{P}\right)}{2}-T[Q+2Q+\cdots(n-1)Q]\right\}}{nT}$$
$$= \frac{h_V\left\{nQ\left[\frac{Q+Tnp-Tp}{p}\right]-\frac{n^2Q^2}{2p}-\frac{n^2TQ-nTQ}{2}\right\}}{nT}$$
$$= \frac{h_V\left\{\frac{2nQ^2+2Tn^2QP-2nQTp-n^2Q^2}{2P}-\frac{n^2TpQ-nTpQ}{2p}\right\}}{nT}$$
$$= h_V\left(\frac{2Q^2-nQ^2}{2pT}+\frac{2nQ-2Q-nQ+Q}{2}\right)$$
$$= h_V\left[\frac{(2-n)Q^2}{2pT}+\frac{Q(n-1)}{2}\right]$$

$where\ \ T = \frac{Q(1-kY)}{\widetilde{D}}$

$$\Rightarrow HC_V = h_vQ\left[\frac{n-1}{2}+\frac{\widetilde{D}(2-n)}{2p(1-kY)}\right]$$

2.4 Buyer's Cost

$$\text{Buyer's cost} = \text{order cost} + \text{screening cost} + \text{purchase cost} + \text{warranty cost} + \text{holding cost}$$

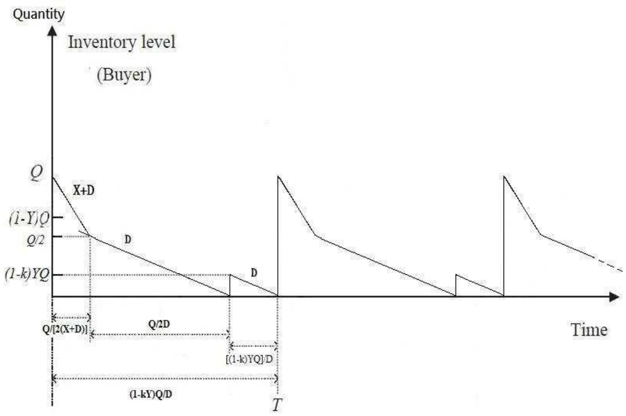

Fig.2. Schematic diagram of Buyer's Cost

$$TC_B(Q,n) = S_B * \frac{\widetilde{D}}{nQ(1-kY)} + dX + B\widetilde{D}(1-\sigma) + V\widetilde{D} + \frac{h_BQ}{4}\left[\frac{\widetilde{D}}{(X+\widetilde{D})(1-kY)}+\frac{2(1-k)^2Y^2-Y+1}{1-kY}\right]$$

$$\Rightarrow TC_B(Q,n) = \widetilde{D}\left[\frac{S_B}{nQ(1-kY)}+B(1-\sigma)+V+\frac{h_BQ}{4(X+\widetilde{D})(1-kY)}\right]+dX+\frac{h_BQ[2(1-k)^2Y^2-Y+1]}{4(1-kY)}$$

Streamlined distance method is used to the defuzzication of $TC_B(Q,n)$.

$$\widetilde{D} = d(\widetilde{D},\tilde{0}_1) = \frac{1}{4}[(D-\Delta_1)+2D+(D-\Delta_2)] = D+\frac{1}{4}(\Delta_2-\Delta_1)$$

$$\Rightarrow TC_B(Q,n) = \left[D + \frac{(\Delta_2 - \Delta_1)}{4}\right]\left\{\frac{S_B}{nQ(1-kY)} + B(1-\sigma) + V + \frac{h_B Q}{4\left[X + D + \frac{(\Delta_2-\Delta_1)}{4}\right](1-kY)}\right\} + dX + \frac{h_B Q[2(1-k)^2Y^2 - Y + 1]}{4(1-kY)} \tag{2}$$

Buyer holding cost's derivation:

$$Hc_B = \frac{h_B}{T}\left\{\frac{1}{2}\left[YQ * \frac{Q}{2(X+\widetilde{D})}\right] + \frac{Q}{2}(1-Y)\left[\frac{Q}{2(X+\widetilde{D})} + \frac{Q}{2\widetilde{D}}\right] + \frac{YQ}{2}(1-k) * \frac{YQ(1-k)}{\widetilde{D}}\right\}$$

$$= \frac{h_B Q^2}{2T}\left[\frac{Y}{2(X+\widetilde{D})} + \frac{(1-Y)}{2(X+\widetilde{D})} + \frac{(1-Y)}{2\widetilde{D}} + \frac{2Y^2(1-k)^2}{2\widetilde{D}}\right]$$

$$= \frac{h_B Q^2}{2T}\left[\frac{Y+1-Y}{2(X+\widetilde{D})} + \frac{(1-Y)+2Y^2(1-k)^2}{2\widetilde{D}}\right]$$

$$= \frac{h_B Q^2}{4T}\left[\frac{1}{X+\widetilde{D}} + \frac{2Y^2(1-k)^2 - Y + 1}{\widetilde{D}}\right]$$

$where\ T = \frac{Q(1-kY)}{\widetilde{D}}$

$$\Rightarrow HC_B = \frac{h_B Q}{4}\left[\frac{\widetilde{D}}{(X+\widetilde{D})(1-kY)} + \frac{2(1-k)^2Y^2 - Y + 1}{1-kY}\right]$$

2.5 Solving Procedure

$EK(Q,n) = TC_V + TC_B$

$$S_V * \frac{\widetilde{D}}{nQ(1-kY)} + F(1+2Y-kY) * \frac{\widetilde{D}}{Q(1-kY)} + Q_r\widetilde{D} + RkY\widetilde{D} + LY\widetilde{D} + h_v Q\left[\frac{n-1}{2} + \frac{\widetilde{D}(2-n)}{2p(1-kY)}\right] + S_B * \frac{\widetilde{D}}{nQ(1-kY)} + dX + B\widetilde{D}(1-\sigma) + V\widetilde{D} + \frac{h_B Q}{4}\left[\frac{\widetilde{D}}{(X+\widetilde{D})(1-kY)} + \frac{2(1-k)^2Y^2-Y+1}{1-kY}\right] \tag{3}$$

By taking second-order partial deviation of $EK(Q,n)$, so taking the derivative of $EK(Q,n)$with respect to Q, which is a convex function in Q for $Q > 0$.

$$\frac{\partial EK(Q,n)}{\partial \mathrm{Q}} = -S_V * \frac{\widetilde{D}}{nQ^2(1-kY)} - S_B * \frac{\widetilde{D}}{nQ^2(1-kY)} - F(1+2Y-kY) * \frac{\widetilde{D}}{Q^2(1-kY)} + H_v + H_B \tag{4}$$

$$where\ H_v = h_v\left[\frac{n-1}{2} + \frac{\widetilde{D}(2-n)}{2p(1-kY)}\right]$$

$$H_B = \frac{h_B}{4}\left[\frac{\widetilde{D}}{(X+\widetilde{D})(1-kY)} + \frac{2(1-k)^2Y^2 - Y + 1}{1-kY}\right]$$

$$let\ \frac{\partial EK(Q,n)}{\partial \mathrm{Q}} = 0$$

$$Q^* = \sqrt{\frac{\widetilde{D}[S_V + S_B + nF(1+2Y-kY)]}{n(1-kY)(H_V+H_B)}}$$

$$Q^* = \sqrt{\frac{\left[D + \frac{(\Delta_2-\Delta_1)}{4}\right][S_V + S_B + nF(1+2Y-kY)]}{n(1-kY)(H_V+H_B)}} = \sqrt{\frac{(4D + \Delta_2 - \Delta_1)[S_V + S_B + nF(1+2Y-kY)]}{4n(1-kY)(H_V+H_B)}} \tag{5}$$

Then, take the derivation of $EK(Q,n)$ with respect to Q in order to understand the effect of m in $EK(Q,n)$.

$$\frac{\partial EK(Q,n)}{\partial \mathrm{n}} = -S_V * \frac{\widetilde{D}}{n^2Q(1-kY)} - S_B * \frac{\widetilde{D}}{n^2Q(1-kY)} + h_v\left[\frac{Q}{2} - \frac{Q\widetilde{D}}{2p(1-kY)}\right] \tag{6}$$

$$let\ \frac{\partial EK(Q,n)}{\partial \mathrm{n}} = 0$$

$$n^* = \sqrt{\frac{\widetilde{D}(S_V+S_B)}{h_v Q(1-kY)\left[\frac{Q}{2} - \frac{Q\widetilde{D}}{2p(1-kY)}\right]}} = \sqrt{\frac{2p\widetilde{D}(1-kY)(S_V+S_B)}{h_v Q^2(1-kY)[p(1-kY)-\widetilde{D}]}}$$

Streamlined distance method is used to the defuzzication of n^*.

$$\widetilde{D} = d(\widetilde{D}, \tilde{0}_1) = \frac{1}{4}[(D-\varDelta_1) + 2D + (D-\varDelta_2)] = D + \frac{1}{4}(\varDelta_2 - \varDelta_1)$$

$$n^* = \sqrt{\frac{2p\left[D + \frac{(\Delta_2-\Delta_1)}{4}\right](1-kY)(S_V+S_B)}{h_v Q^2(1-kY)\left\{p(1-kY) - \left[D + \frac{(\Delta_2-\Delta_1)}{4}\right]\right\}}} = \sqrt{\frac{2p(4D+\Delta_2-\Delta_1)(1-kY)(S_V+S_B)}{h_v Q^2(1-kY)[4p(1-kY) - 4D - \Delta_2 + \Delta_1]}} \tag{7}$$

Meanwhile, take the second-order partial derivative of $EK(Q,n)$ with respect to n.

$$\frac{\partial EK(Q,n)}{\partial \mathrm{n}^2} = (S_V + S_B) * \frac{2\widetilde{D}}{Qn^3(1-kY)} > 0 \tag{8}$$

The result of the equation proves that there is a minimum to the solution.

2.6 Algorithm

1. Set n = 1.
2. Substitute $\mathrm{n} = 1$ into equation (5) to evaluate Q_1.
3. Substitute Q_1 into equation (4) to evaluate EK.
4. Set $\mathrm{n} = \mathrm{n} + 1$, substitute $\mathrm{n} = \mathrm{n} + 1$ into equation (5) to evaluate Q_2, and repeat step 2 to step 3 to get EK.
5. Substitute Q_2 into equation (4) to evaluate EK.
6. If $EK' < EK$, return to step 4; otherwise EK is the optimal solution.

III. RESULTS

This research presents a detailed numerical example to illustrate the results of the proposed models:
$\mathrm{D} = 5000\mathrm{pieces/year}$, $S_V = 3000\$/setup$, $S_b = 300\$/cycle$, $\mathrm{X} = 1000\mathrm{pieces/year}$, $H_V = 1\$/piece, H_b = 4\$/piece, P = 8000 pieces/year, V = 1.5\$/piece, d = 0.5\$/piece, Y = 0.01, Q_r = 10\$/piece, B = 25\$/piece, K = 0.3$, $F = 800, \sigma = mYk = 0.3, R = 2\$/piece, M = 100$
The solution to the minimum cost required multiple (Δ_1, Δ_2), take note that (Δ_1, Δ_2) were determine by the decision maker in order to handle uncertain problems. All of the results are provided in Table I.

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TABLE I
NUMERICAL EXAMPLE RESULTS

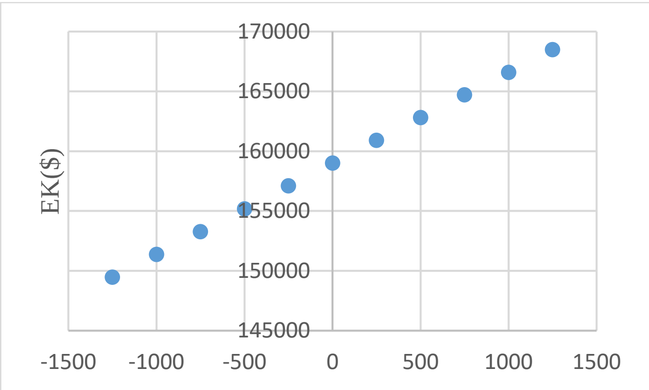
$(\tilde{D}-\Delta_1,\tilde{D},\tilde{D}+\Delta_2)$	n^*	Q^*	EK($)	V_Q (%)	V_W(%)
$(4750, 5000, 5500)$	2	4633.53	160892.4	0.5858	1.1963
$(4500, 5000, 6000)$	2	4660.36	162793.9	1.1682	2.3923
$(4250, 5000, 6500)$	2	4687.03	164694.8	1.7471	3.5879
$(4000, 5000, 7000)$	2	4713.54	166595.2	2.3227	4.7832
$(3750, 5000, 7500)$	2	4739.91	168495.1	2.8950	5.9782
$(2500, 5000, 7500)$	2	4606.55	158990.3	0	0
$(2500, 5000, 6250)$	2	4469.09	149471.4	-2.9839	-5.9871
$(3000, 5000, 6000)$	2	4496.93	151376.4	-2.3796	-4.7890
$(3500, 5000, 5750)$	2	4524.59	153280.8	-1.7791	-3.5912
$(4000, 5000, 5500)$	2	4552.08	155184.5	-1.1824	-2.3937
$(4500, 5000, 5250)$	2	4579.40	157087.7	-0.5894	-1.1967

(1) When $\Delta_1 < \Delta_2$, then $d(\tilde{D},\tilde{0}_1) > D$; so that $V_Q > 0$, $V_W > 0$. When $(\Delta_2 - \Delta_1)$ decreases, both V_Q and V_W will decrease as well. The smaller $(\Delta_2 + \Delta_1)$ is in this fuzzy model, the more similar to the tradition model.

(2) When $\Delta_1 > \Delta_2$, then $d(\tilde{D},\tilde{0}_1) < D$; so that $V_Q < 0$, $V_W < 0$. When $(\Delta_2 - \Delta_1)$ increases, both V_Q and V_W will increase as well.

(3) When $\Delta_1 = \Delta_2 = \mathbf{2500}$, then $\boldsymbol{d(\tilde{D},\tilde{0}_1) = D = 5000}$. In this case, this fuzzy model will be exactly the same compare to the traditional models; both $\boldsymbol{V_Q}$ ***and*** $\boldsymbol{V_W}$ will equal to 0.

(4) The mathematical relationship diagram of **EK** and $(\Delta_2 - \Delta_1)$ is shown in figure 3.



$(\Delta_2 - \Delta_1)$

Fig.3. The mathematical relationship diagram of **EK** and $(\Delta_2 - \Delta_1)$

(5) Figure 4 is the comparison diagram that compares $\boldsymbol{V_Q}$ and $\boldsymbol{V_W}$ to $(\Delta_2 - \Delta_1)$. It shows that V_W's slope is larger than V_Q's, which represents the range of V_W's change is larger than V_Q's.

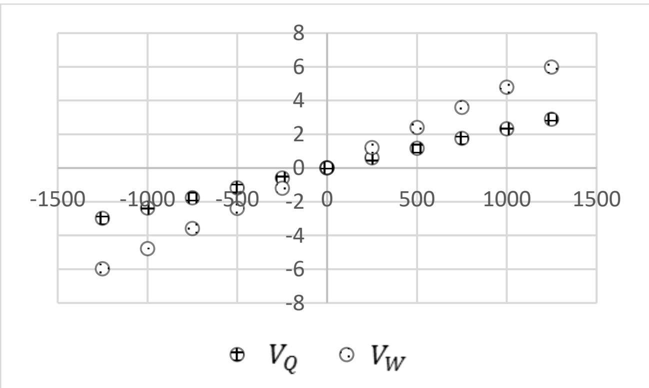


Fig. 4. The comparison diagram that compares $\boldsymbol{V_Q}$ and $\boldsymbol{V_W}$ to $(\Delta_2 - \Delta_1)$

IV. CONCLUSION

In the competitive global market, both pricing strategy and the quality often determine the orientation of the customers. Lowering the price is not a good strategy for the suppliers, it might decrease profit or increase defective products. Therefore, a well-designed quantity discount policy is crucial. This research incorporated quantity discount, unreliable process and uncertain environments into integrated inventory model. Through the sensitive analysis in this study, it shows that if $(\Delta_2 - \Delta_1)$ increases, both V_Q and V_W will increase as well. Also the smaller $(\Delta_2 + \Delta_1)$ is in the fuzzy model, the more similar it gets to the tradition model. In future, studies may include more manufacturers, then production scheduling will be added into the model to simulate a more realistic situation.

REFERENCES

[1] M.F. Yang, Y. Lin*, W.F. Kao and L.H. Ho (2016, Mar). An Integrated Multi-echelon Logistics Model with Uncertain Delivery Lead Time and Quality Unreliability, Mathematical Problems in Engineering DOI: 10.1155/2016/8494268

[2] Hsin-I Hsiao, Mengru Tu*, M.F. Yang, Wei-Chung Tseng (2017, Jul), "Deteriorating inventory model for Ready-to-eat food under fuzzy environment," International Journal of Logistics Research and Applications (Accepted, Article in press). DOI: 10.1080/13675567.2017.1351532

[3] C.Y. Chiu,Y. Lin, and M.F.Yang (2014, Jul). Applying Fuzzy Multi objective Integrated Logistics Model to Green Supply Chain Problems. Journal of Applied Mathematics, Volume 2014, Article ID 767095, 12 pages. (SCI, 104/247,MATHEMATICS, APPLIED). NSC 100-2410-H-019-004.

[4] M.F. Yang*, T.S. Huang and Y. Lin," Variable lead time fuzzy inventory system under backorder rate with allowed shortage", International Journal of Innovative Computing Information and Control, Vol.6 (2010), No. 11, 5015-5034.

[5] Ben-Daya, M. and M. Hariga, 2000. Economic lot scheduling problem with imperfect production process. Journal of the Operational Research Society, 51:875-881. DOI: 10.1057/palgrave.jors.2600974.

[6] Chakravarty, A.K. and G.E. Martin, 1988. An optimal joint buyer-seller discount pricing model. Computers and Operations Research, 15:271-281.

[7] Huang, C.K., 2004. An optimal policy for a single vendor single-buyer integrated production-inventory. problem with process unreliability consideration. Int. J. Product. Econom., 91: 91-98. DOI: 10.1016/S0925-5273(03)00220-2

[8] Li, J. and L. Liu, 2006. Supply chain coordination with quantity discount policy. Int. J. Product. Econom., 101: 89-98. DOI: 10.1016/j.ijpe.2005.05.008

[9] Lin, Y.J. and C.H. Ho, 2011. Integrated inventory model with quantity discount and price-sensitive demand. TOP, 19: 177-188. DOI: 10.1007/s11750-009-0132-1

[10] Porteus, E.L., 1986. Optimal lot sizing, process quality improvement and setup cost reduction. Operat. Res., 34: 137-144. DOI: 10.1287/opre.34.1.137

[11] Salameh, M.K. and M.Y. Jaber, 2000. Economic production quantity model for items with imperfect quality. Int. J. Product. Econom., 64: 59-64. DOI: 10.1016/S0925-5273(99)00044-4

[12] Schwaller, R.L., 1988. EOQ under inspection costs. Product. Inventory Manage., 29: 22-24.

[13] Wang, Q., 2005. Discount pricing policies and the coordination of decentralized distribution systems. Decision Sci., 36: 627-646. DOI: 10.1111/j.1540-5414.2005.00105.x

[14] Zhang, Z. and J. Xu, 2014. Applying nonlinear MODM model to supply chain management with quantity discount policy under complex fuzzy environment. J. Indust. Eng. Manage., 7: 660-680. DOI: 10.3926/jiem.1079

[15] J. S. Yao and K. Wu (2000), Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems, Vol. 116, pp. 275-288

[16] A. Kaufmann and M. M. Gupta (1991), Introduction to Fuzzy Arithmetic: Theory and Applications, Van Nostrand Reinhold, New York.

[17] H. J. Zimmermann (1996), Fuzzy Set Theory and its Application, 3rd edition, Kluwer Academic Publishers, Dordrecht.

[18] Lal, R. and R. Staelin, 1984. An approach for developing an optimal

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discount pricing policy. Manage. Sci., 30: 1524-1539. DOI: 10.1287/mnsc.30.12.1524

[19] Goyal, S.K. and L.E. Cardenas-Barron, 2002. Note on: Economic production quantity model for items with imperfect quality-a practical approach. Int. J. Product. Econom., 77: 85-87. DOI: 10.1016/S0925-5273(01)00203-1