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Bi-Objective Optimization Model for Harvesting of Sugarcane with Fixed and Variable Costs of Harvesting

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Abstract—**In this research, the purpose was to find the optimal quantity and optimal time for harvesting of sugarcane in order to maximize the revenue and minimize the gathering cost. The government determines the sugarcane price which is based on weight and sweetness. The cost of production was separated into two parts: a fixed cost and a variable cost. The ε-constraints method was applied to solve a bi-objective mathematical model in which maximizing the revenue was the main objective. The optimal harvesting policies for sugarcane in four regions of Thailand were calculated from the model for crop years 2012/13, 2013/14 and 2014/15. The optimal revenues calculated for the sugarcane harvesting were compared with the actual revenue received by the sugarcane farmers in the four regions for the three crop years.**

Index Terms—**Bi-objective mathematical model, sugarcane harvesting, optimal harvesting policy, ε-constraints method.**

I. INTRODUCTION

THE sugar industry is of great importance to the Thai economy because sugar is one of the top five agricultural products that the country exports. Since 1992/93 the Commercial Cane Sugar (C.C.S.) System has been used as the main sugarcane trading system in Thailand [1]. In this system, the Royal Thai government determines the price of sugarcane for each of the four regions in the country based on two main factors, quality and weight. The quality is considered as a sweetness or C.C.S., where C.C.S. means the percentage of sucrose produced from a tonne of sugarcane. The government determines the price of sugarcane in each region by combining sweetness and weight, where a "standard sweetness" is counted as 10 C.C.S. Generally, the level of sweetness of sugarcane will increase with time, whereas the weight will decrease with time. Therefore, the price of the sugarcane usually first increases with time as the sweetness increases, reaches a maximum and then decreases with time as the weight decreases. Before the sugarcane is harvested, a sugar factory will carry out a randomized check of C.C.S. value. If the value lies above a standard value of 10 C.C.S. then that factory will accept the sugarcane immediately. However, the farmers might delay harvesting if they expect that they will obtain more revenue by waiting for their sugarcane to reach its maximum value.

Many researchers have studied planning models for sugarcane farming. For example, in 2012 Gomes [2] studied a bi-objective mathematical model for choosing sugarcane varieties which have a harvest biomass residual for use in electicity generation. The bi-objective optimization model studied was to maximize the revenue from sale of the generated electricity and to minimize the cost of harvesting the residual biomass. In 2013, Yano and Matsui [3] proposed an interactive decision making method based on random, fuzzy, multi-objective linear programming and applied the method to a crop planning problem in which a farmer of agricultural company wants to maximize total profit and minimize working time. In 2016, Sungnul et.al [4] studied a multi-objective optimization model to find an optimal time of harvesting for sugarcane growers in the North-Eastern region of Thailand. The aim of this work was to help farmers to find the optimal harvesting time in order to maximize the revenue and to minimize the cost. Sungnul et al. used the ε-constraints method [6] to solve the mathematical model by choosing the revenue as the objective function and the costs as constraints. In 2017, Sungnul et al. [5] extended the work in [4] to find the optimal harvesting times for all of the four regions of Thailand.

The main purpose of this paper is to find the optimal harvesting policies for sugarcane in the four regions of Thailand for crop years 2012/13, 2013/14 and 2014/15. The optimization problem is to maximize the revenue and minimize the gathering cost, where the gathering cost is assumed to consist of a fixed cost and a variable cost.

II. BI-OBJECTIVE OPTIMIZATION MODEL

A. The ε-constraints method

The ε-constraints method [6] consists in reformulating a multi-objective problem by choosing the most important objective to optimize while maintaining other objectives as constraints. A multi-objective optimization problem can be stated as follows :

$$\begin{aligned} \text{Min} \quad & f_1(\vec{x}), \\ \text{such that} \quad & f_r(\vec{x}) \le \varepsilon_r; \quad r = 2,3,...,m, \\ & \vec{x} \in X, \end{aligned} \tag{1}$$

where ε_r is an upper bound on objective r and X is the set of feasible solutions for the problem.

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For example, if the original optimization problem has two objectives f_1 and f_2 to be minimized, then if one of them, e.g., f_1, is selected as the main objective to be minimized then an upper bound ε_2 is chosen and $f_2 \leq \varepsilon_2$ is used as a constraint on the minimization of f_1. We can then formulate the original optimization problem in the form $\hat{P}_1(\varepsilon_2)$, where f_1 is selected as the objective to be optimized and f_2 is used as a constraint or in the form $\hat{P}_2(\varepsilon_1)$, where f_2 is selected as the objective to be optimized and f_1 is used as a constraint. That is, we have:

$$\begin{array}{rlrl} & \hat{P}_1(\varepsilon_2) & & \hat{P}_2(\varepsilon_1) \\ \text{Min} & f_1(\vec{x}), & \text{Min} & f_2(\vec{x}), \\ \text{subject to} & f_2(\vec{x}) \leq \varepsilon_2, & & f_1(\vec{x}) \leq \varepsilon_1, \\ & \vec{x} \in X & & \vec{x} \in X. \end{array} \tag{2}$$

Two important theorems for bi-objective optimization problems are as follows [6]:

Theorem 1 $\vec{x}^*$ is an efficient solution of the bi-objective problem (1) if and only if $\exists\ \varepsilon_2$ such that $\vec{x}^*$ solves $\hat{P}_1(\varepsilon_2)$ or $\exists\ \varepsilon_1$ such that $\vec{x}^*$ solves $\hat{P}_2(\varepsilon_1)$.

Theorem 2 If $\vec{x}^*$ solves either $\hat{P}_1(\varepsilon_2)$ or $\hat{P}_2(\varepsilon_1)$ and if this solution is unique then $\vec{x}^*$ is an efficient solution of (1).

However, it is necessary to select suitable bounds for ε_1 or ε_2, since if the bounds are not properly selected then the feasible regions for the constraints for both problems $\hat{P}_1(\varepsilon_2)$ and $\hat{P}_2(\varepsilon_1)$ can be empty and the solutions do not exist. Therefore, during construction of the efficient frontier it is necessary to initially determine a set of values for ε_1 or ε_2 such that the constraint regions for either $\hat{P}_1(\varepsilon_2)$ or $\hat{P}_2(\varepsilon_1)$ or both are not empty.

One method for obtaining suitable values for the $\varepsilon_r(j)$ for an objective j in a bi-objective problem is as follows. For each objective function f_j, we select p equally-spaced choices for $\varepsilon_r(j)$ between a lower bound LB_j and an upper bound UB_j for the gathering cost f_j in area j as shown in Eq. (3)[6].

$$\varepsilon_{r+1}(j) = \varepsilon_r(j) + \Delta\varepsilon(j), \qquad r = 1, 2, ..., p, \tag{3}$$

where $\Delta\varepsilon(j) = \frac{UB_j - LB_j}{p-1}$ is a distance between the values $\varepsilon_r(j)$, $\varepsilon_1(j) = LB_j$ and $\varepsilon_p(j) = UB_j$.

Then, if there exists an optimal solution for either $\hat{P}_1(\varepsilon_2)$ or $\hat{P}_2(\varepsilon_1)$ or both, then we select the solution from solutions of $\hat{P}_1(\varepsilon_2)$ with minimum f_1 value and minimum $\varepsilon_r(2)$ or from solutions of $\hat{P}_2(\varepsilon_1)$ with minimum f_2 value and minimum $\varepsilon_r(1)$.

The method proposed in this study is to find the optimal quantity of sugarcane harvested in each area in order to maximize the farmers revenue and to minimize the gathering cost.

B. Revenue from sugarcane selling [4]

There are two main factors for determining sale price of sugarcane. These factors are weight and C.C.S. or sweetness.

1) Revenue from weight of sugarcane :
A survey found that sugarcane at the sugar factory is classified into 2 types; a) fresh sugarcane and b) fired sugarcane. Farmers who sell fired sugarcane will be deducted 20 baht/tonne from a basic sugarcane price determined by the government each year. The factory will share the total amount of deducted money to farmers who sell fresh sugarcane at a rate not exceeding 70 baht/tonne of fresh sugarcane delivered, thus increasing the price of the fresh sugarcane above the basic price. The price of fired sugarcane based on weight is given by

$$PW(B) = P_w - 20, \tag{4}$$

where P_w is the basic price of sugarcane (baht/tonne) based on weight set by the government and $PW(B)$ is the price of fired sugarcane, where we use B to denote fired sugarcane.
The actual price received by the farmer for fresh sugarcane (we use A to denote fresh sugarcane) based on weight adjusted for the extra deductions from the fired sugarcane is then given by

$$PW_j(A) = P_w + \frac{20a_j(B)}{a_j(A)}, \tag{5}$$

where $a_j(A)$ is the total amount of fresh sugarcane (tonnes) from planted area j and $a_j(B)$ is the total amount of fired sugarcane (tonnes) from planted area j.

2) Revenue from weight with C.C.S. :
The price per tonne based on C.C.S for sugarcane from region j harvested in time period k is given by

$$PC_{j,k} = P_c(1 + 0.06y_{j,k}), \tag{6}$$

where P_c is the price of sugarcane with 10 C.C.S. based on weight per tonne determined by the government, $y_{j,k} = \text{C.C.S.} - 10$ (C.C.S. is the average C.C.S. from sugarcane in planted area j which is harvested at time k), and the factor 0.06 is the rate of change of price per 1 C.C.S. from the base level of 10.
Therefore, the revenue $[RV_{j,k}(A)]$ (baht/tonne) from selling fresh sugarcane from planted area j which is harvested at time k is determined by adding Equation (5) and Equation (6). We obtain

$$RV_{j,k}(A) = PW_j(A) + PC_{j,k}. \tag{7}$$

Similary, we obtain the revenue $[RV_{j,k}(B)]$ (baht/tonne) from selling fired sugarcane by adding Equation (4) and Equation (6).

$$RV_{j,k}(B) = PW(B) + PC_{j,k}. \tag{8}$$

C. Gathering Cost of Production

The gathering cost of production can be separated into two parts; a) fixed cost and b) variable cost. The total gathering cost $[GC_{j,k}(i)]$ (baht/tonne) of sugarcane production from planted area j which is harvested at time k is given by

$$GC_{j,k}(i) = CF_j + CT_j + CV_{j,k}(i), \quad i = A, B, \tag{9}$$

where CF_j and CT_j are the fixed costs (baht/tonne) for average cost of production on the farms and transportation cost to the factory for sugarcane produced in area j, respectively. $CV_{j,k}(i)$ is the variable cost (baht/tonne) which consists of maintenance and fuel cost $(CM_j(i))$ as shown in Equation (10).

$$CV_{j,k}(i) = [a_0 + a_1\cos(k\omega) + b_1\sin(k\omega)]CM_j(i) \tag{10}$$

where a_0, a_1, b_1 and ω are constants based on real data [7].

D. Bi-objective Mathematical Model of Sugarcane

The maximum revenue from the sugarcane selling is considered as the objective function and the minimum of gathering cost of sugarcane production becomes a constraint. As explained in the Results and Discussions section, we consider the two types of sugarcane fresh $i = A$ and fired $i = B$ for four main regions of Thailand ($j = 1, 2, 3, 4$) and twelve harvesting periods each year ($k = 1, ..., 12$).

Therefore, our bi-objective model is as shown in Equations (11)–(15).

$$\text{Maximize} \quad \sum_{k=1}^{12} RV_{j,k}(i)\alpha_{j,k}a_j(i); \quad j = 1, 2, 3, 4; \quad i = A, B, \tag{11}$$

$$\text{subject to} \quad \sum_{k=1}^{12} GC_{j,k}(i)\alpha_{j,k}a_j(i) \leq \varepsilon_r; \quad j = 1, 2, 3, 4; \quad i = A, B, \tag{12}$$

$$\sum_{k=1}^{12} \alpha_{j,k} = 1; \qquad j = 1, 2, 3, 4, \tag{13}$$

$$\alpha_{j,k} \in [0, 1]; \quad k = 1, 2, ..., 12; \quad j = 1, 2, 3, 4, \tag{14}$$

$$y_{j,k} > -4 \tag{15}$$

The objective function in Equation (11) maximizes the revenue from the sugarcane selling. The constraint in Equation (12) represents the second objective of the problem which is to minimize the gathering cost of production with an upper bound given by ε_r, where $\alpha_{j,k}$ is the percentage of the amount of sugarcane from planted area j which is harvested at time k, and $a_j(i)$ is the total amount of fresh (i=A) and fired (i=B) sugarcane from planted area j.

The condition in Equation (13) on $\alpha_{j,k}$ is the condition that the total amount of sugarcane harvested from planted area j at all times is 1 (i.e., 100 %), and the condition in Equation (14) is the condition that the fraction $\alpha_{j,k}$ harvested in time k in area j is in the range $0 \leq \alpha_{j,k} \leq 1$.

The constraint Equation (15) means that the C.C.S. of sugarcane from planted area j which is harvested at time k has to be greater than 6 ($y_{j,k} = C.C.S._{j,k} - 10$).

E. Data and Equipment Used

The mathematical model in the bi-objective optimization problems was solved by the ε-constraints method discussed in previous sections [6] using the GNU linear programming kit package GLPK. The real data used in the model was for crop years 2012/13, 2013/14 and 2014/15 and was obtained from the Office of the Cane and Sugar Board [7].

III. Results and Discussion

The bi-objective mathematical models for harvesting fresh and fired sugarcane are described by the objective function Equation (11) subject to constraints Equation (12)-(15). The harvest time in each crop year was divided into 12 intervals ($k = 1, 2, ..., 12$). For example, the harvest time for crop year 2012/13 was 15 November 2012 to 16 May 2013, and therefore $k = 1$ represented the time between 15-30 November 2012 and $k = 12$ represented the time between 1-16 May 2013. The results for the optimal quantities of fresh and fired sugarcane harvested in each region of Thailand in time period k for the three crop years 2012/13, 2013/14 and 2014/15 are shown in TABLES (I)-(III), respectively.

TABLE I
Percentage of Optimal Quantity (P.O.Q.) in Optimal Time for Crop Year 2012/13

Area	Optimal time		P.O.Q.	
	Fresh	Fired	Fresh	Fired
Northern	16-31 Mar 2013	16-31 Mar 2013	50.75	49.45
	1-15 Apr 2013	1-15 Apr 2013	49.25	50.55
Central	16-31 Mar 2013	16-31 Mar 2013	51.15	50.53
	1-15 Apr 2013	1-15 Apr 2013	48.85	49.47
Eastern	16-30 Apr 2013	1-15 Apr 2013	47.67	51.82
	1-16 May 2013	16-30 Apr 2013	52.33	48.18
North-Eastern	16-30 Apr 2013	1-15 Apr 2013	49.08	54.26
	1-16 May 2013	16-30 Apr 2013	50.92	45.74

TABLE II
Percentage of Optimal Quantity (P.O.Q.) in Optimal Time for Crop Year 2013/14

Area	Optimal time		P.O.Q.	
	Fresh	Fired	Fresh	Fired
Northern	16-31 Mar 2014	1-15 Mar 2014	50.73	49.98
	1-15 Apr 2014	16-30 Apr 2014	49.27	50.02
Central	16-31 Mar 2014	16-31 Mar 2014	50.58	51.13
	1-15 Apr 2014	1-15 Apr 2014	49.42	48.87
Eastern	1-15 Mar 2014	1-15 Mar 2014	48.78	45.75
	16-31 Mar 2014	16-31 Mar 2014	51.22	54.25
North-Eastern	1-15 Mar 2014	16-31 Mar 2014	50.43	52.58
	1-16 May 2014	1-16 May 2014	49.57	47.42

TABLE III
Percentage of Optimal Quantity (P.O.Q.) in Optimal Time for Crop Year 2014/15

Area	Optimal time		P.O.Q.	
	Fresh	Fired	Fresh	Fired
Northern	1-15 Apr 2015	16-31 Mar 2015	58.65	63.59
	16-30 Apr 2015	1-15 Apr 2015	41.35	36.41
Central	16-31 Mar 2015	16-31 Mar 2015	52.80	50
	1-15 Apr 2015	1-15 Apr 2015	47.20	50
Eastern	16-31 Mar 2015	16-31 Jan 2015	55.26	53.24
	1-15 Apr 2015	1-15 Apr 2015	44.74	46.76
North-Eastern	16-30 Apr 2015	1-15 Apr 2015	100	50.67
		16-30 Apr 2015		49.33

For example, in TABLE I the optimal quantity of fresh sugarcane which should be harvested in the North-Eastern region and delivered into the factory in 16-30 April 2013 is 49.08% and the remaining 50.92% should be harvested and delivered into the factory in 1-16 May 2013. Similarly, the optimal quantity of fired sugarcane which should be harvested and delivered into the factory in 1-15 April 2013 is 54.26% and the remaining 45.74% of the fired sugarcane should be harvested and delivered into the factory in 16-30 April 2013.

IV. Conclusion

In this research, a bi-objective mathematical model was developed to obtain the optimal fresh and fired sugarcane harvesting policies in the four main sugar-producing areas of Thailand (Northern, Central, Eastern and North-Eastern) for crop years 2012/13, 2013/14 and 2014/15. The two objectives in the model were to maximize the revenue and minimize the gathering cost. The ε-constraints method was used. In this model, the main factors in determining the optimal quantity of sugarcane harvested in each region in each period were the the variable production costs. Fig.1-Fig.4 give a comparison between the actual profits obtained from the Office of the Cane and Sugar Board [7] and the optimal profits computed from the bi-objective optimization model. We can conclude that the optimal profit computed from the bi-objective optimization model are higher than the actual profit computed from the bi-objective optimization model for both fresh and fired sugarcane. As can be seen from Fig. 1-Fig. 4, there was an actual loss (negative profit) from sugarcane production in all regions for crop year 2014/15, whereas the optimal policy for the Northern, Central and North-Eastern regions from our model showed a profit. However, for the Eastern region our model did not have a feasible solution for any value of ε for crop year 2014/15 because the cost exceeded the revenue. Therefore, the optimal harvesting policies for the farmer to harvest the sugarcane suggested by our model would have given the farmers greater profit than their actual profit.

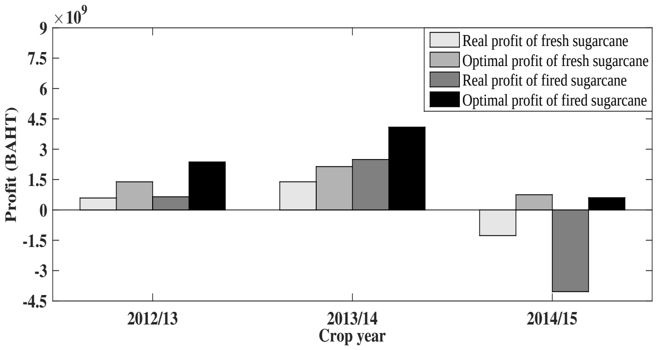

Fig. 1. Comparison of actual profit and optimal profit of sugarcane harvested in Northern Region

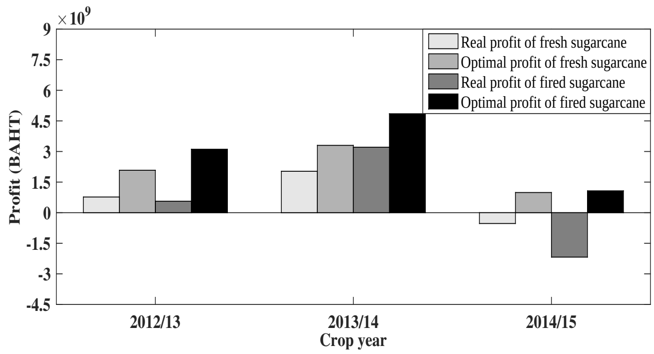

Fig. 2. Comparison of actual profit and optimal profit of sugarcane harvested in Central Region

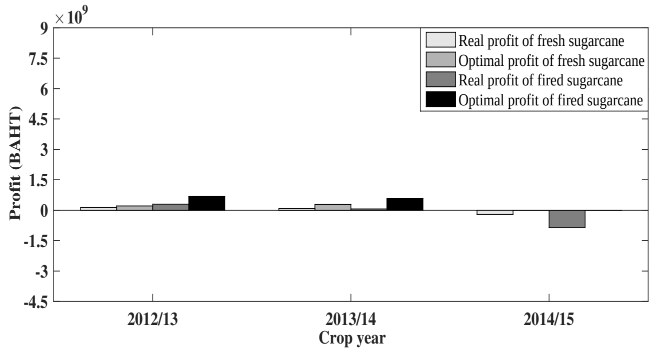

Fig. 3. Comparison of actual profit and optimal profit of sugarcane harvested in Eastern Region

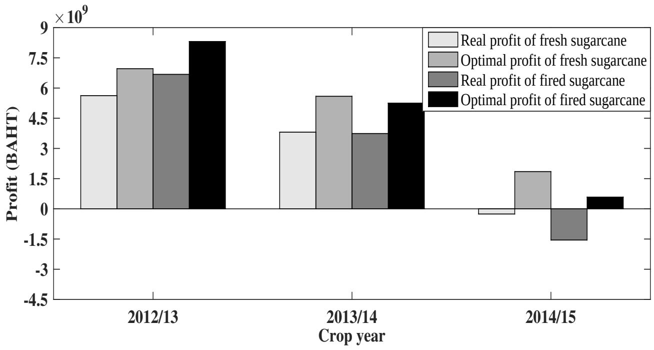

Fig. 4. Comparison of actual profit and optimal profit of sugarcane harvested in North-Eastern Region

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