An Integrated Inventory Model for Perishable Goods Considering Carbon Tax

Min-Der Ko, Wu-Hsun Chung, and Yeng-Ju Lin

Abstract— Nowadays, alleviating the effect of carbon emissions to the environment is a significant issue in the global supply chain. It is especially important for the food industry, because the energy consumption of perishable goods is quite high with the use of cold-chain transport and storage. Therefore, in this paper, a single-producer and single-buyer integrated inventory model is proposed with the consideration of carbon tax for the perishable goods, and the objective is to minimize the total cost. In order to make model closer to the practical use in a real world, the cost of the raw materials for the producer and the backlogging cost for the buyer are considered. Analytical results of numerical example are presented to illustrate the solution procedure.

Index Terms— Integrated inventory, perishable goods, carbon tax

I. INTRODUCTION

To prevent global warming and climate change from worsening, governments and firms should take immediate action to reduce carbon emissions. With the enactment of “Kyoto Protocol” in 1997, it makes developed countries have the legal obligation to control greenhouse gas and reduce emission. Since then, mechanisms are designed by the European Union and many governments to inhibit the amount of carbon emissions, such as carbon offset, cap-and-trade, carbon taxes and carbon caps [1]. Under such situations, firms may have to redesign their supply chain or make different decisions to optimize their strategies.

In recent years, with the awareness of environment protect, there are more and more studies on supply chain under emission regulations. These activities of supply chain include procurement, production, transportation, and inventory management etc. Mingzhou Jin et al [2] investigated freight transportation modes under three different carbon policies with the use of the integrated assessment models. Choudhary et al [1] incorporated various carbon emission parameters into traditional integrated forward/reverse logistics to minimize both the total cost and the carbon footprint. Hammad et al [3] proposed a multi-echelon production-inventory model considering lead time constraints, different manufacturing facilities and different external suppliers under the case of carbon emissions cap.

Choi [4] illustrate how the carbon taxation scheme affect the optimal decision of sourcing management in fashion apparel industry by combining quick response (QR) system to enhance environmental sustainability.

However, currently researches about carbon tax are relatively few on the supply chain of perishable items. With the gradually increase of the requirements on food safety and quality for the consumers, the cost of business investment on keeping the goods fresh also increased. Besides food, drugs, human blood and some electronic products are perishable goods. Most goods will deteriorate over time during storage and transportation, and their original value will decline or be lost. The reservation period of these goods is short, and the management of these perishable products needs quick turnover time and high flexibility. Thus, the difficulty of inventory control increases. In addition, the retailers should not only balance the correspondence between the high demand rate and high inventory level to increase potential market profits, but also control inventory level to reduce the rising costs caused by deterioration [5].

Many researchers considered various factors and complicated circumstances to simulate the reality. Yang and Tseng [6] combined the traditional deterioration model and quality prediction model, so that vendor can have an approach to quantify quality and remaining value easily. Lin and Lin [7] proposed a coordinated inventory system between supplier and buyer for deteriorating items and permit the completed back-order in this model. Bhaba and Bingqing [8] extended the Lin and Lin’s model by considering single-producer multi-buyer with raw materials storage costs.

Broekmeulen and Donselaar [9] suggested a replenishment policy for perishable products which takes into account the age of inventories due to the presence of new information technologies like RFID. For the manufacturer, Luong and Karim [10] presented EPQ model facing the random breakdown, uncertain repair time of the machine that produce the product and deterioration of product over time. These phenomena usually happened may cause the possibility of shortage.

The special characteristics of agri-food supply chains (ASC) are the perishability of their products and the high uncertainty of demand. Moreover, there are carbon emissions exist in an ASC from different sources involved storage, distribution and disposal. So Galal and El-Kilany [11] used modeling and simulation in a two-echelon ASC on various costs, emissions and service level. Tiwari et al [12] incorporated the carbon trading and carbon pricing policies in integrated supply chain in a case study of perishable goods. Simultaneously, using the cloud computing technology (CCT)
to minimize and monitor the carbon emission. Hua et al [14] proposed a perishable inventory control based on EOQ model with freshness-dependent demand under three different carbon policies to analyze the impact on inventory decisions.

The purpose of this study is to develop a integrated model that takes into account the link between perishable goods and carbon tax. The construction of this model is based on EOQ model with progressive carbon taxation [13] and integrated inventory model for perishable goods [8]; then to minimize both the total costs and carbon emissions. Sensitive analysis of the carbon tax is also performed to investigate the effect of carbon tax on the entire inventory model.

II. NOTATIONS AND ASSUMPTIONS

To establish the model proposed, the following notations and assumptions are used.

A. Notations

n
The replenishment cycle for the buyer (the production cycle for the producer), a decision variable
T
The length of shortage time per cycle time for the buyer, a decision variable
t
The time when replenishment begins at buyer’s level in cycle i
s
The time when shortage begins at buyer’s level in cycle i
p
The time when production begins at producer’s level in cycle i
H
Holding cost of goods($/unit/unit-time)
β
The carbon tax charged for per unit carbon emission
γ
The amount of carbon emissions per perishable goods
K
Fixed carbon emissions of holding inventory

Buyer side

D
The demand rate (units/year)
θb
The deterioration rate of the goods kept by the buyer
A∞
The ordering cost of finished goods of the buyer per cycle ($/cycle)
cb
The cost of finished goods per deteriorated unit at the buyer’s level ($/unit)
πb
The backlogging cost of the buyer ($/unit/unit-time)
lb
The level of inventory for the buyer per cycle (unit)
Wb
The number of deteriorated finished goods units of the buyer per cycle
TCP
The buyer’s total cost

Producer side

P
The production rate (units/year)
M
The usage rate for producing finished goods of raw materials
θt
The deterioration rate of the finished goods kept by the producer
θt′
The deterioration rate of the raw materials kept by the producer
θt′
The deterioration rate of the goods transported between producer and buyer
At
The setup cost of finished goods of the producer per cycle ($/cycle)
Ar
The ordering cost of raw materials of the producer per cycle ($/cycle)

cAr
The cost of finished goods per deteriorated unit at the producer’s level ($/unit)
cAr
The cost of raw materials per deteriorated unit at the producer’s level ($/unit)
Qt
The order quantity of raw material by the producer per batch (unit)
lb
The level of finished goods inventory for the producer per cycle (unit)
l′b
The level of raw material inventory for the producer per batch (unit)
Wb
The number of deteriorated finished goods units of the producer per cycle
Wb
The number of deteriorated raw material units of the producer per batch
TCP
The producer’s cost of finished goods
TCP
The producer’s cost of raw materials

B. Assumptions

1. A single item, a single producer and a single buyer are considered.
2. The production rate greater than the demand rate, P>D.
3. The lower bound of P equals to D(1+θt′+θt+θt,t).
4. The buyer’s shortage is allowed where either raw materials or finished goods is not allowed for the producer.
5. The Buyer’s replenishment information is available to the producer.
6. The buyer’s replenishment cycle equals to the producer’s production cycle.
7. A lot-for-lot policy is applied in the model between the producer and the buyer.
8. In every replenishment cycle, the deterioration rates of items are equal.
9. The holding cost of finished goods and raw materials are equal.
10. The carbon emissions for holding inventory units of the finished goods is $g o+t gl$, as well as the raw materials.

III. MODEL FORMULATION

Fig.1 depicts inventory levels of products at both producer and buyer. According Bhaba and Bingqing’s model [8], the buyer’s instantaneous inventory level is lb and instantaneous shortage level is l′b at time t are calculated as follow

\[
\frac{dlb(t)}{dt} + \theta l′b(t) = -D, \\
\frac{dl′b(t)}{dt} = -D, \\
t_{i-1} \leq t \leq s_i, i = 1, 2, ..., n
\]

With the initial condition l′b(s_i) = 0, the inventory level l′b(t) and the shortage level l′b(t) can be solved as

\[
l′b(t) = e^{-\theta t} \int_t^{s_i} e^{\theta u} Du = \frac{D}{-\theta} (e^{\theta (s_i - t)} - 1), \\
l′b(t) = D(s_i - t), \quad t_{i-1} \leq t \leq s_i, i = 1, 2, ..., n
\]

Therefore, the total amount of inventory carried by the buyer is formulated as follow
\[ I_b = \int_{t_{i-1}}^{t_i} \left( \frac{D}{\theta_b} e^{\theta_b T (1-f)} - \frac{D}{\theta_b} T (1-f) \right) dt \]
\[ I_b = \int_{t_i}^{t_{i+1}} \left( \frac{D}{2} T^2 f^2 \right) dt \]

(3)

Then, the number of deteriorating items equals to \( I_b(t_{i-1}) \) minus the total demand from \( t_{i-1} \) to \( t_i \), which is given by

\[ W_b = I_b(t_{i-1}) - \int_{t_{i-1}}^{t_i} Ddu \]

\( = \frac{P}{\theta_b} \left( e^{\theta_b T (1-f)} - 1 \right) - DT \left( 1 - f \right) \)

(4)

The buyer’s cost is composed of ordering cost, holding cost, backlogging cost, deterioration cost, and the cost of carbon emissions. Thus, the total cost \( TC_b \) of the buyer in unit time is given by

\[ TC_b = \frac{1}{T} \left\{ A_b + HI_b + \pi_b I_b + c_b^d W_b + \beta (g_0 + g_1 t_b + \gamma W_b) \right\} \]

(5)

For the producer, the instantaneous finished goods inventory level \( I_t(t) \) can be calculated as

\[ \frac{dI_t(t)}{dt} + \theta_t \frac{dI_t(t)}{dt} = P, \quad t_{i-1} \leq t \leq t_i, \quad i = 1, 2, ..., n \]
\[ I_t(t) = 0, \quad t_{i-1} \leq t \leq t_i, \quad i = 1, 2, ..., n \]

(6)

According to equation (6), \( I_t(t) \) can be solved as follow

\[ I_t(t) = e^{-\theta_t t} \int_{t_{i-1}}^{t} e^{\theta_t u} Pdu = \frac{P}{\theta_t} \left( 1 - e^{\theta_t (t_{i-1} - t)} \right), \]
\[ t_{i-1} \leq t \leq t_i, \quad i = 1, 2, ..., n \]

(7)

The product quantity received by the buyer at the beginning of each cycle equals to the delivery quantity of the producer minus the number of deteriorated goods during the transportation from the producer to buyer, so the inventory level of the producer and the buyer can be shown by the function as follow

\[ (1 - \theta_b) \frac{P}{\theta_t} (1 - e^{\theta_t (t_{i-1} - t)} - \frac{D}{\theta_b} (e^{\theta_b T (1-f)} - 1) + DT f) \]

(8)

Then the producer’s total production time in cycle \( i \), \( T_i \), can be presented as

\[ T_i = -\frac{1}{\theta_t} \ln \left( 1 - \frac{D \theta_t}{\theta_b (1-\theta_a) \theta_t} \left( e^{\theta_b T (1-f)} - 1 \right) + \theta_b T f \right) \]

(9)

The inventory of finished goods \( I_f \) carried by the producers in cycle \( i \) is

\[ I_f = \frac{P}{\theta_t} (T + \frac{1}{\theta_t} (e^{-\theta_t T} - 1)) \]

(10)

And the number of deteriorated finished goods for the producer per cycle is equal to the total production quantity minus the original delivery quantity as follow

\[ W_f = \left[ T_i P - I_f(t_i) \right] \]

\[ = \frac{P}{\theta_t} \ln (1 - \frac{D \theta_t}{\theta_b (1-\theta_a) \theta_t} \left( e^{\theta_b T (1-f)} - 1 \right) + \theta_b T f) \]

(11)

The cost of producer’s finished goods consists of the setup cost, holding cost, deterioration cost, and the cost of carbon emissions, which can be given by

\[ TC_f = \frac{1}{T} \left\{ A_f + HI_f + c_f^d W_f + \beta (g_0 + g_1 t + \gamma W_f) \right\} \]

(12)

Finally, for the raw materials, the usage of raw materials for producing per production cycle can be described as \( MDT \). So, the order amount of raw material by the producer for per batch is

\[ Q_r = \sum_{i=1}^{n} MDT / (1 - \theta_r)^i \]

(13)

Then, the number of deteriorated raw material units for the producer in each batch is equal to the original ordering quantity minus the used quantity

\[ W_r = \sum_{i=1}^{n} MDT / (1 - \theta_r)^i - \sum_{i=1}^{n} MDT \]

(14)

And the amount of raw material inventory carried by the producer in each batch is

\[ I_r = \sum_{i=1}^{n} MDT / (1 - \theta_r)^i \times ( 1 - \theta_r)^{i-1} \]

(15)

The cost of producer’s raw materials consist of the setup cost, holding cost, and deterioration cost, and the cost of carbon emissions, which can be given by

\[ TC_r = \frac{1}{T} \left\{ A_r + HI_r + c_r^d W_r + \beta (g_0 + g_1 t + \gamma W_r) \right\} \]

(16)
Hence, the system’s joint total cost function $TC(n, T, f) = TC^*(T, f) + TC(T, n, f)$, composed of the producer’s and buyer’s costs, which is given by

$$
TC = \frac{1}{T} \left\{ A_b + (H + \beta) \left[ D \left( e^{\beta T(1-f)} - 1 \right) + \frac{D}{\theta_b} T (1-f) \right] + \beta \theta_b + n_b D \frac{T}{2} f^2 + (c^d + \beta) \left[ D \left( e^{\beta T(1-f)} - 1 \right) - DT (1-f) \right] \right\}
$$

$$
+ \left( c^d + \beta \right) \left[ \frac{\theta_b}{\gamma} \left( 1 - \frac{D \theta_1}{\theta_2 (1-\gamma)} \right) \left( e^{\beta T(1-f)} - 1 \right) + \theta_b T f \right] \right\}
$$

$$
+ \left( \frac{c^d + \beta}{\gamma} \right) \left[ - \frac{\theta_2}{\gamma-1} (1-e^{-\theta_1 f}) \right] + \frac{\theta_b}{\gamma} (1-e^{-\theta_1 f}) \right) \right\}
$$

$$
\left( \frac{c^d + \beta}{\gamma} \left[ \sum_{i=1}^{n} \frac{MDT}{\gamma-1} (1-\alpha_i) \right] + \beta \theta_0 \right)
$$

$$
+ \left( \frac{c^d + \beta}{\gamma} \left[ \sum_{i=1}^{n} \frac{MDT}{\gamma-1} - MDT \right] \right) \right\}
$$

(17)

Firstly, taking the first-order and second-order partial derivatives of $TC(n, T, f)$ with respect to $n$, for $k(n)=(1-\theta_n)^n$, we obtain

$$
\frac{\partial TC(n, T, f)}{\partial n} = \frac{1}{n^2} \left[ \frac{DM}{\theta_2 (1-\gamma)} \left[ (c^d + \beta) \left( \log(k(n)) + 1 \right) \right] - k(n) (c^d + \beta + \gamma) \right] + \frac{A_b + \beta \theta_0}{T}
$$

(18)

$$
\frac{\partial^2 TC(n, T, f)}{\partial n^2} = \frac{1}{n^3} \left[ \frac{DM}{\theta_2 (1-\gamma)} \left[ (c^d + \beta) \left( \log(k(n)) + 1 \right) \right] - k(n) (c^d + \beta + \gamma) \right] + \frac{2k(n) (c^d + \beta + \gamma)}{T} > 0
$$

(19)

Therefore, for fixed $T$ and $f$, $TC(n, T, f)$ is strongly concave on $n>0$. Thus, determining the optimal number of production cycles per batch $n^*$, is simplified to obtain a global optimum.

Similarly, by taking the first-order and second-order partial derivatives of $TC(n, T, f)$ with respect to $T$ and $f$ separately, we have

$$
\frac{\partial TC(n, T, f)}{\partial T} = \frac{D \times P \times \theta_1 \times (c^d + \beta) \times \left( f - e^{-\theta_1 T (1-f)} \right) \times (f-1)}{T \times \left( D \times \theta_1 \times \left( e^{-\theta_1 T (1-f)} + T \times \theta_1 \times (f-1) \right) + P \times \theta_1 \times (\theta_2 - 1) \right) - \left( A_b + A_1 + A_3 \times \beta \theta_0 \right)}
$$

$$
= \frac{D \left[ \theta_1 (c^d + \beta) + H \times \beta \theta_0 \right] \left( e^{-\theta_1 T (1-f)} (T \theta_1 (f-1) + 1) - 1 \right) + P \times \left( c^d + \beta \right) \times \log \left( \frac{D \times \theta_1 \times \left( e^{-\theta_1 T (1-f)} + T \times \theta_1 \times (f-1) \right)}{P \times \theta_1 \times (\theta_2 - 1)} + 1 \right)}{T \times \theta_1}
$$

$$
+ P \left( \frac{\theta_1 (c^d + \beta) + H \times \beta \theta_0 \right] \left( e^{-\theta_1 T (1-f)} (T \theta_1 (f-1) + 1) - 1 \right) + P \times \left( c^d + \beta \right) \times \log \left( \frac{D \times \theta_1 \times \left( e^{-\theta_1 T (1-f)} + T \times \theta_1 \times (f-1) \right)}{P \times \theta_1 \times (\theta_2 - 1)} + 1 \right)}{T \times \theta_1}
$$

(20)

Consequently, we know $\frac{\partial^2 TC(n, T, f)}{\partial T^2} > 0$ and $\frac{\partial^2 TC(n, T, f)}{\partial f^2} > 0$, so $TC(n, T, f)$ is strongly concave on $T$ and $f$ Thus there exist an unique replenishment time value of $T$ and the optimal shortage time $f$, which minimizes $TC(n, T, f)$. Finally, the value of $T$ and $f$ can be found by equating (20) as well as (21) to be zero.

IV. SOLUTION PROCEDURE

To summarize the arguments above, we establish the following algorithm to obtain the optimal values of $T^*, f^*, n^*$, $TC(n, T, f)^*$, where the other parameters are given.

Algorithm

Step 1: Initialize system parameter.

Step 2: Set $T \in [0, a], f \in [0, 1], n \in [1, \infty]$, initialize $T=0, f=0$, and $n=1$.

Step 3: Determine the $T$ by taking first-order partial derivative of with respect to $T$.

Step 4: Take $T$ into $TC(n, T, f)$ and take first-order partial derivative of with respect to $f$.

Step 5: Take $f$ into $TC$.

Step 6: Set $n=n+1$, repeat steps 2-5 to obtain $TC(n, T[n], f[n])$.

Step 7: If $TC(n-1, T\left[n-1\right], f\left[n-1\right]) \leq TC(n, T\left[n\right], f\left[n\right])$, go to step 7. if not, go to step 8 and stop.

Step 8: Set $TC(n, T, f)^* = TC(n-1, T\left[n-1\right], f\left[n-1\right])$. so $(n^*, T^*, f^*)$ is an optimal solution in this case, go to next case and repeat step 3-8.

Step 9: Output $n^*, T^*, f^*$ and $TC(n, T, f)^*$

V. NUMERICAL EXAMPLE

To illustrate the proposed solution procedure, consider an inventory situation with the following parametric values partially adopted in Bhaba, Bingqing [8], and G.W. Hua et al [14].

<table>
<thead>
<tr>
<th>Common</th>
<th>Buyer</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>H=2</td>
<td>D=200</td>
<td>P=300</td>
</tr>
<tr>
<td>γ=0.2</td>
<td>A_c=100</td>
<td>M=10</td>
</tr>
<tr>
<td>g=5</td>
<td>c^d=12</td>
<td>A_v=1000 \cdot A_{v}=3000</td>
</tr>
<tr>
<td>g=1</td>
<td>\theta_b=0.02</td>
<td>c^d_{vr}=10 \cdot c^d_{vr}=3</td>
</tr>
<tr>
<td>β = 1</td>
<td>π_0=12</td>
<td>\theta_r=0.005 \cdot \theta_r=0.015</td>
</tr>
<tr>
<td></td>
<td>\theta_a=0.01</td>
<td></td>
</tr>
</tbody>
</table>

Applying the equation and algorithm already given in this article, the optimal solutions are shown in Table 2.
The findings of the numerical result indicated that the minimum cost is $52664.21 with the optimal production cycles per batch is 18 in the integrated chain. And the relationship between the carbon emissions and total cost can be seen clearly in Fig 2.

Table 2. The optimal solutions for the example

<table>
<thead>
<tr>
<th>n</th>
<th>T</th>
<th>f</th>
<th>TC(n,T,f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12.77</td>
<td>0.2444</td>
<td>59796.78</td>
</tr>
<tr>
<td>12</td>
<td>14.07</td>
<td>0.2449</td>
<td>57619.73</td>
</tr>
<tr>
<td>13</td>
<td>15.41</td>
<td>0.2470</td>
<td>55964.12</td>
</tr>
<tr>
<td>14</td>
<td>16.79</td>
<td>0.2491</td>
<td>54725.11</td>
</tr>
<tr>
<td>15</td>
<td>18.20</td>
<td>0.2513</td>
<td>53826.35</td>
</tr>
<tr>
<td>16</td>
<td>19.66</td>
<td>0.2536</td>
<td>53210.52</td>
</tr>
<tr>
<td>17</td>
<td>21.15</td>
<td>0.2560</td>
<td>52834.35</td>
</tr>
<tr>
<td>18</td>
<td>22.69</td>
<td>0.2584</td>
<td>52664.21</td>
</tr>
<tr>
<td>19</td>
<td>24.63</td>
<td>0.2609</td>
<td>52948.03</td>
</tr>
<tr>
<td>20</td>
<td>25.88</td>
<td>0.2635</td>
<td>52842.35</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

Under the pressure of global warming, the awareness of environmental protection has been growing recently. All members of the supply chain management make efforts to control carbon emissions. The deteriorating inventory model of this paper proposed takes both buyer and producer into account, which extends the concept of Hua et al’s model [14]. In this research, we considered the raw material and allowed shortage on the buyer’s side, which make the model closer to the real world. According to the numerical results of the study, by applying this model, we can obtain the optimal cycle time and the fraction of shortage time to formulate an optimized production and replenishment policy to minimize the joint cost. With the number of production cycles increase, as well as the carbon emissions increase, but it does exist the minimum total cost for integrated system.

Future research on this issue can make further analysis to examine the value of other factor, such as carbon tax, affect the size of the joint cost. Or consider other scenarios for the models to discuss change in different situations. In this way can urge companies in a supply chain become more eco-friendly without extra cost.

REFERENCES


