Proceedings of the International MultiConference of Engineers and Computer Scientists 2019
IMECS 2019, March 13-15, 2019, Hong Kong

Biquadratic Phase-Compensating System With Robust Stability

Tian-Bo Deng

***Abstract*—A new method is proposed in this paper for the design of an allpass biquadratic (biquad) phase-compensating system whose stability-margin satisfies an arbitrarily preset stability criterion. The design technique utilizes the generalized stability-triangle (GST) proposed by the author and the bilinear (BL) phase-error function of the allpass biquad system. Based on the GST condition, it is shown that the original biquad-system coefficients can be converted to the functions of other two new variables, while the two variables have no limits (bounds) on their values to satisfy the GST condition. Based on the above function conversions, the design technique further employs a nonlinear optimization method to find the optimum values of the two variables to approximate a prescribed ideal phase, while the resulting values of the two variables never violate the GST condition. That is, the resulting allpas biquad system can always satisfy a prescribed stability margin and the given ideal phase can also be best approximated in the minimax sense. An example is included in the paper to demonstrate the guaranteed stability margin.**

***Index Terms*—Bilinear (BL) error function, biquadratic (biquad) system, stability, generalized stability-triangle (GST), stability margin.**

I. Introduction

DIGITAL phase-compensating systems are the basic circuits that are required in the phase-compensations of signal processing sytems and digital communication systems when the systems have phase distortions, and such phase distortions need to be calibrated. The design of a finite-impulse-response phase system needs to approximate both the amplitude and phase characteristics [1]-[6], while the design of a recursive allpass one only needs to elaborate its phase approximation [7]-[9]. However, the latter needs to ensure the system stability. Instead of just guaranteeing the stability, it is also preferred to let a recursive system have an extra stability margin because some coefficient-value perturbations such as coefficient-quantization errors may cause instability. In [8], a new stability triangle called generalized stability-triangle (GST) is proposed, which allows the system designer to add a preset margin to the system stability during the system design [9].

This paper formalizes an alternative method for the design of an allpass biquadratic (biquad) system approximating a prescribed phase characteristic. The mathematic model of the biquad system is the second-order rational function that has two independent coefficients. However, its phase is nonlinearly related to the two coefficients. This forces the phase-approximation problem to be a nonlinear minimization one. To express the phase as an explicit function of the two coefficients, this paper first shows the detailed expression of the phase error, which is referred to as bilinear (BL) error. This BL error can be viewed as a special case of the higher-order system [7]. Then, the BL error is employed together with the GST condition in designing an allpass biquad system. It is demonstrated through a biquad-system example that not only a preset stability margin is guaranteed, but also a high-accuracy design can be achieved.

Manuscript received Sept. 4, 2018; revised Sept. 21, 2018. This work was supported by JSPS KAKENHI (Grant Number 16K06368).

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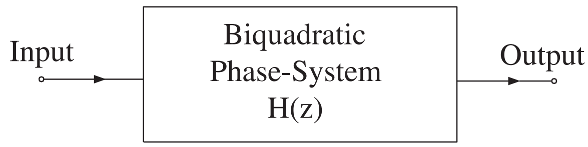

Fig. 1. Biquad phase-compensating system.

II. Bilinear Error

Suppose that we are given the desired phase $\Theta_d(\omega)$ and it is approximated uing the allpass biquad-system function

$$\begin{aligned} H(z) &= \frac{c_2 + c_1 z^{-1} + z^{-2}}{1 + c_1 z^{-1} + c_2 z^{-2}} \\ &= z^{-2} \cdot \frac{1 + c_1 z + c_2 z^2}{1 + c_1 z^{-1} + c_2 z^{-2}} \\ &= z^{-2} \cdot \frac{C(z^{-1})}{C(z)} \end{aligned} \tag{1}$$

where $C(z)$ denotes the denominator of the biquad-system function, and this denominator is the second-order polynomial

$$C(z) = 1 + c_1 z^{-1} + c_2 z^{-2}.$$

Fig. 1 illustrates the block-diagram of this biquad system.

To formalize the approximation problem, we give the detailed derivation of the BL phase error as follows. This BL error is to be minimized in the design.

By writing the frequency response of the quadratic polynomial $C(z)$ as

$$\begin{aligned} C(\omega) &= 1 + c_1 e^{-j\omega} + c_2 e^{-j2\omega} \\ &= \sum_{k=0}^{2} c_k e^{-jk\omega} \end{aligned}$$

with $c_0 = 1$, one can readily obtain its complex-conjugate expression

$$C^*(\omega) = \sum_{k=0}^{2} c_k e^{jk\omega}.$$

Using $C(\omega)$ and $C^*(\omega)$ yields the frequency response of the biquad-system $H(z)$ as

$$H(\omega) = e^{-j2\omega} \cdot \frac{C^*(\omega)}{C(\omega)}.$$

ISBN: 978-988-14048-5-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Let the phase of $C^*(\omega)$ be $\beta(\omega)$, i.e.,

$$\beta(\omega) = \angle C^*(\omega). \tag{2}$$

The overall phase of $H(\omega)$ can be expressed as

$$\begin{aligned}\Theta(\omega) &= 2\beta(\omega) - 2\omega \\ &= 2\left[\beta(\omega) - \omega\right].\end{aligned}$$

In the extreme case (approximation error equal to zero), i.e.,

$$\Theta(\omega) = \Theta_d(\omega)$$

we have

$$2\left[\beta(\omega) - \omega\right] = \Theta_d(\omega).$$

Equivalently,

$$\beta(\omega) = \omega + \Theta_d(\omega)/2.$$

This means that the ideal phase for $\beta(\omega)$ should be

$$\beta_d(\omega) = \omega + \Theta_d(\omega)/2. \tag{3}$$

To simplify the notations, we denote

$$e_\beta(\omega) = \beta(\omega) - \beta_d(\omega). \tag{4}$$

Consequently, the phase error can be expressed as

$$\begin{aligned}e_\Theta(\omega) &= \Theta(\omega) - \Theta_d(\omega) \\ &= 2[\beta(\omega) - \omega] - \Theta_d(\omega) \\ &= 2\left[\beta(\omega) - \left(\omega + \frac{\Theta_d(\omega)}{2}\right)\right] \\ &= 2e_\beta(\omega).\end{aligned} \tag{5}$$

Thus, it suffices to minimize $e_\beta(\omega)$ in order to minimize $e_\Theta(\omega)$. To minimize $e_\beta(\omega)$, the cost function (BL phase error) is derived as follows.

Let

$$C^*(\omega) = C_\mathrm{R} + jC_\mathrm{I}$$

with

$$\begin{aligned}C_\mathrm{R} &= \sum_{k=0}^{2} c_k \cos(k\omega) \\ C_\mathrm{I} &= \sum_{k=0}^{2} c_k \sin(k\omega).\end{aligned}$$

Hence, $e_\beta(\omega)$ can be further detailed as

$$\begin{aligned}e_\beta(\omega) &= \beta(\omega) - \beta_d(\omega) \\ &= \tan^{-1}\frac{C_\mathrm{I}}{C_\mathrm{R}} - \beta_d(\omega).\end{aligned}$$

When $e_\beta(\omega)$ is small enough, it is possible to use the approximation

$$\begin{aligned}e_\beta(\omega) &\approx \tan(e_\beta(\omega)) \\ &= \tan(\beta(\omega) - \beta_d(\omega)) \\ &= \frac{\tan\beta - \tan\beta_d}{1 + \tan\beta \cdot \tan\beta_d} \\ &= \frac{C_\mathrm{I}/C_\mathrm{R} - \sin\beta_d/\cos\beta_d}{1 + C_\mathrm{I}/C_\mathrm{R} \cdot \sin\beta_d/\cos\beta_d} \\ &= \frac{C_\mathrm{I}\cos\beta_d - C_\mathrm{R}\sin\beta_d}{C_\mathrm{R}\cos\beta_d + C_\mathrm{I}\sin\beta_d} \\ &= \frac{\sum_{k=0}^{2} c_k[\sin(k\omega)\cos(\beta_d) - \sin(\beta_d)\cos(k\omega)]}{\sum_{k=0}^{2} c_k[\cos(k\omega)\cos(\beta_d) + \sin(k\omega)\sin(\beta_d)} \\ &= \frac{\sum_{k=0}^{2} c_k \sin(k\omega - \beta_d)}{\sum_{k=0}^{2} c_k \cos(k\omega - \beta_d)}\end{aligned} \tag{6}$$

where $\beta_d(\omega)$ is shortened to β_d. The above derivation result can be viewed as a special case of the derivation in [7], and this BL expression explictly relates the phase error to the two independent coefficients c_1 and c_2.

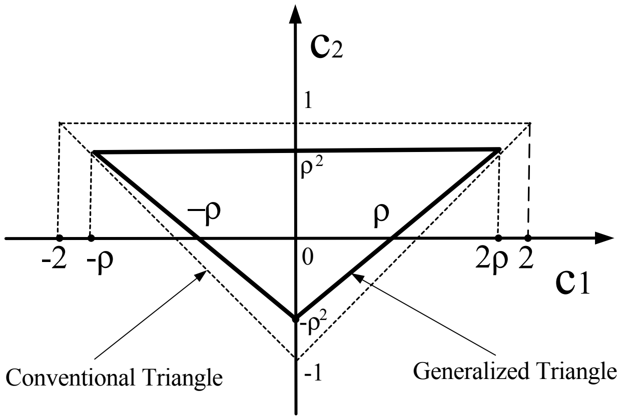

Fig. 2. The GST embedded inside the CST.

III. GST-Based Minimax Design

Fig. 2 illustrates the generalized stability-triangle (GST) derived in [8], which enables the designer to design a second-order recursive system with a preset stability margin.

In Fig. 2, the proposed GST is embedded inside the conventional stability-triangle (CST). If the system coefficients c_1 and c_2 satisfy the condition

$$\begin{cases}|c_2| \le \rho^2 \\ |c_1| \le \rho + \rho^{-1}c_2\end{cases} \tag{7}$$

the two poles p_1 and p_2 of the biquad system meet the condition

$$|p_i| \le \rho,\ i = 1, 2$$

where ρ represents the upper bound on the radii of the two poles, and $0 < \rho < 1$.

ISBN: 978-988-14048-5-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

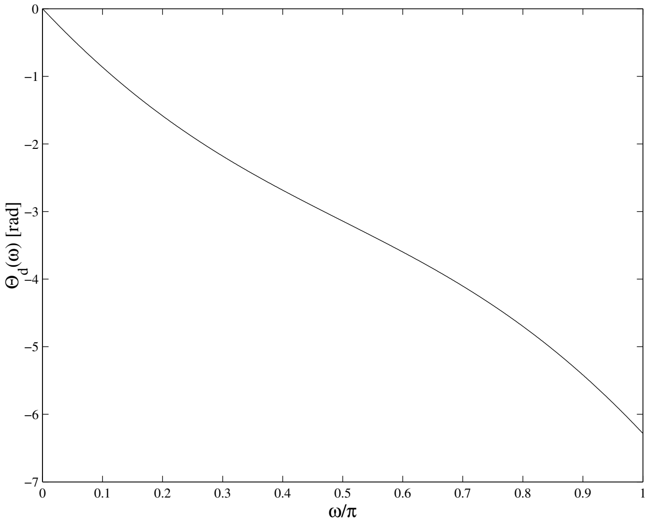

Fig. 3. Desired phase for the BL phase-error-based design.

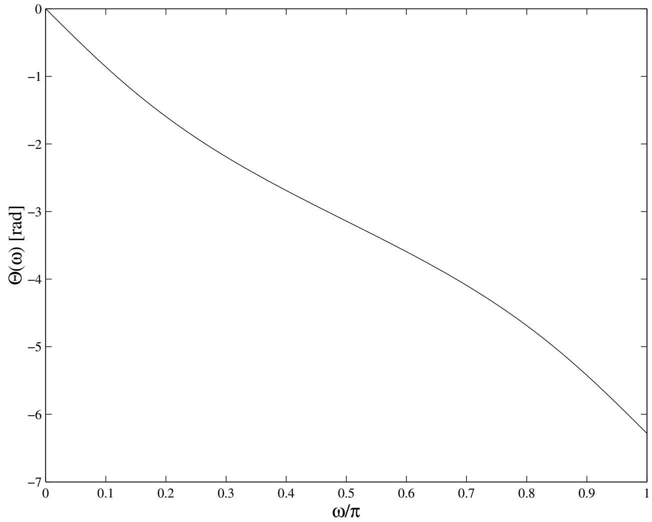

Fig. 4. Designed phase using the BL phase-error.

To employ the GST condition (7) in designing the biquad system, the biquad-system coefficients c_2 and c_1 are transformed to

$$\begin{cases} c_2 = U(x_2)\rho^2\cdot \\ c_1 = U(x_1)(\rho + \rho^{-1}c_2) \end{cases} \tag{8}$$

by using the unity-bounded function

$$U(x) \in [-1, 1].$$

It has been rigorously proved that whatever values the new parameters x_2 and x_1 may take always satisfy the GST condition [9].

To perform the minimax design, the original system coefficients c_2 and c_1 in (6) are first expressed in the forms of (8), and then the variables x_2 and x_1 are optimized via minimizing the peaked error

$$\Delta_\beta = \max\{|e_\beta(\omega)|,\ \omega \in [0, \pi]\}. \tag{9}$$

This optimization is a nonlinear mathematical programming problem, which is accomplished through using the nonlinear optimizer (*fminimax* in MATLAB) [9]. Once the optimum x_2 and x_1 are obtained, the resulting biquad-system performance is evaluated by using the peaked error

$$\Delta_\Theta = \max\{|e_\Theta(\omega)|,\ \omega \in [0, \pi]\}. \tag{10}$$

IV. Biquad-System Design Example

This section gives an illustrative example to show the design accuracy as well as the resultant stability margin of the proposed minimiax design approach that uses the BL phase error to design a biquad phase system.

The ideal phase to be approximated is specified by

$$\Theta_d(\omega) = -3\omega + \pi \sin^2(\omega/2). \tag{11}$$

Fig. 3 plots this given $\Theta_d(\omega)$.

To execute the nonlinear optimizer *fminimax*, the initial values of $(x_1, x_2) = (0, 0)$ are used. Furthermore, the stability-margin requirement is $\rho = 0.9$, and the uniform sampled frequencies $\omega_1, \omega_2, \cdots, \omega_{1001}$ are employed. It is also necessary to mention that the function

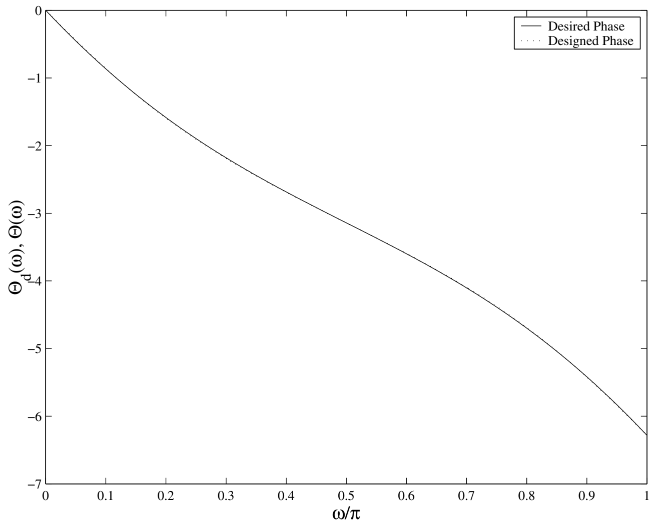

Fig. 5. Designed phase together with the desired one.

$$U(x) = \sin(x)$$

is adopted in transforming the coefficients (x_2, x_1) to the new ones (c_2, c_1) [9]. The execution of the design program obtains the actual phase given in Fig. 4. To compare the ideal $\Theta_d(\omega)$ with the actual $\Theta(\omega)$, the two curves are put together in Fig. 5, where the deviations are indistinguishable (invisible). Fig. 6 shows the detailed deviations with the peak value being -38.2494 dB.

The major objective of this BL-error-based design method is to guarantee that a preset stability margin (requirement) is truly met. The radii of the two poles of the achieved biquad-system $H(z)$ are plotted in Fig. 7. The two poles have the same radius ($\rho = 0.4128$), which meets the preset requirement (the maximum value not larger than $\rho = 0.9$).

V. Conclusion

This paper has applied the GST and the derived BL phase-error formula to the biquad-system design. The designed

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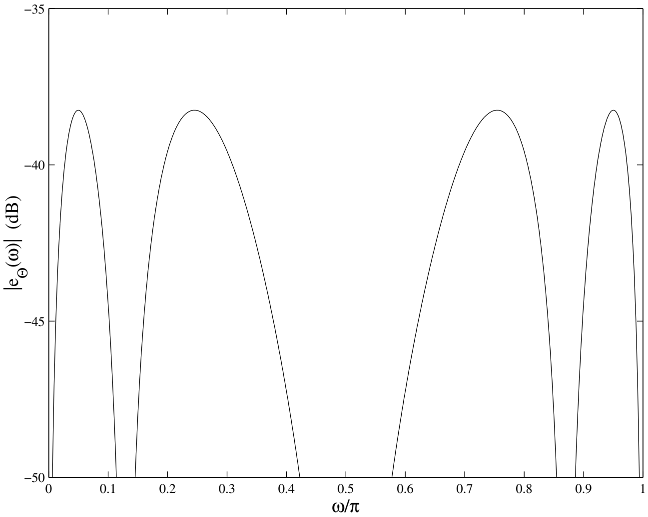

Fig. 6. Phase deviations using the BL phase-error.

biquad phase-compensating system is useful for the phase compensation of a digital system with undesired phase distortions. The GST has the favourable feature that enables the designer to design a biquad system satisfying a preset stability margin. The larger the margin is, the more robust the stability is. A large margin guarantees a strong robustness of the system stability so that the system stability will not suffer from any coefficient-quantization errors, and the system is always stable. A biquad-system design example has illustrated the accuracy of the BL-error-based design algorithm as well as the achieved stability margin.

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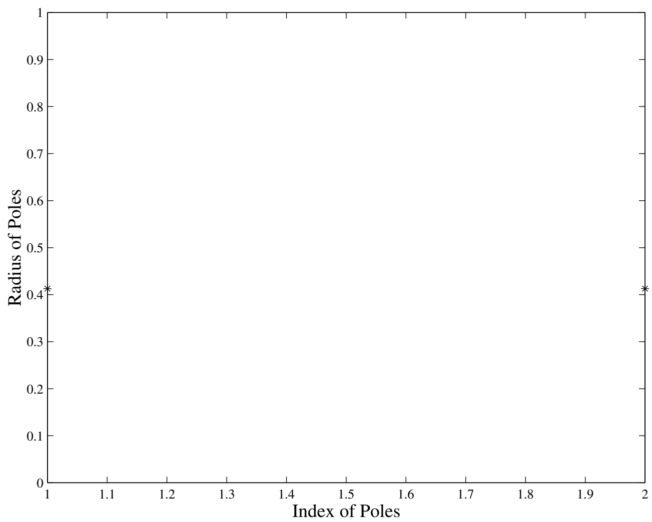

Fig. 7. Radii of the two poles.

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)