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A Shop Location Optimization with Traffic Generators and Lines based on Prediction of Residents' Movement

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Abstract—**This article considers a new shop location model with both traffic generators and lines of residents. In traditional location models, a shop locater almost regards such sites that many residents exist, called traffic generators, or their neighborhood as good locations. However, because such sites are also good for other locaters, locating on them needs many costs and then often results badly for all of them. This article focuses not only residents' sites but their movement, that is, traffic lines. This article proposes a method for prediction of their movement from traffic generators. By representing traffic generators and lines as points and lines on a plain respectively, the proposed location model can be formulated to a location optimization problem in spatial model. Its solution method based upon reduction and decomposition is proposed, and an efficiency of the method is shown by applying it to an example of the location problem.**

Index Terms—**shop, location optimization, traffic lines, spatial model, Brent method.**

I. Introduction

LOCATION optimization is an important area in operations research, and has been studied by many researchers. Researches of location optimization can be classified into two categories according to the type of locating facilities. One is facility location for not obtaining rewards from its users directly, e.g. delivery centers and libraries. An important location problem in the category is "Weber problem", whose recent researches are studied by Hosseininezhad et al.[5], Uno et al.[12], Saleh Farham et al.[8] and so on. The other is facility location for obtaining rewards from its users directly, e.g. shops and supermarkets. This article deals with the later, called shop location optimization (SLO).

In SLO, the objective of shop locater is usually to obtain as many rewards from residents as possible. Mathematical studies on the SLO were originated by Hotteling[6]. He considered the location problem under the conditions that residents are uniformly distributed on a line segment and evaluate shops by the distances to them. As an extension of Hotelling's SLO, Wendell and McKelvey[13] considered residents on a finite number of points, called traffic generators (TGs), and formulated a location problem on a tree network whose vertices are TGs. Drezner extended their location problem to a problem locating on a plain including TGs. This article deals with the location problem on a plain including TGs.

On the above SLO, residents evaluate the located shops only by the distances to them. Huff[7] defined estimation model for shops in a trading area, called Huff location model. In Huff location model, shop's estimation from a resident increases in the proportion of the quality of service it provides, and decrease inversely with the square of the distance from her/his to it. Huff location model is applied to many researches on SLO, whose resent researches are studied by Uno et al.[12] and Blanquero et al[1].

An important issue of Huff location model for SLO is the cost of locating shops. While a site of shop found by Huff location model can expect to obtain a large sales, locating it on such site usually needs many costs and then often results badly. In this article, we focuses not only TGs but their movement, that is, traffic lines (TLs). Hodgson[3] considered a location-allocation model that demands for shops are represented as flows, including TLs. Applications of the Hodgson's model were studied by Hodgson et al.[4] and Riemann[9]. We propose a new SLO model with both TGs and TLs of residents. We construct a method for prediction of their movement from traffic generators. By representing TGs and TLs as points and lines on a plain respectively, the SLO can be formulated to a location problem on the plain. We construct its solution method based upon reduction and decomposition of it, whose efficiency is shown by applying it to an example of the new SLO.

The remaining structure of this article is organized as follows. In the next section, we introduce the new SLO with TGs and TLs based on prediction of residents' movement, and formulate it to a location problem on the plain. We propose a solution method for the formulated location problem based upon reduction and decomposition in Section III. In Section IV, we show an application of the solution method for an example of the new SLO. Finally, conclusions and future studies are summarized in Section V.

II. Formulation of shop location optimization

We consider that a shop locater locates her/his shop on Euclid plain $\mathbf{R}^2$. In $\mathbf{R}^2$, there are n points representing TGs of residents, set of whose indices is denoted by $I = \{1, 2, \ldots, n\}$. We regard TGs as potential demands of locating shop. For i-th TG, $i \in I$, its site and demand are denoted by $\boldsymbol{v}_i \in \mathbf{R}^2$ and $q_i > 0$, respectively.

In the new SLO, we consider TLs of residents on the

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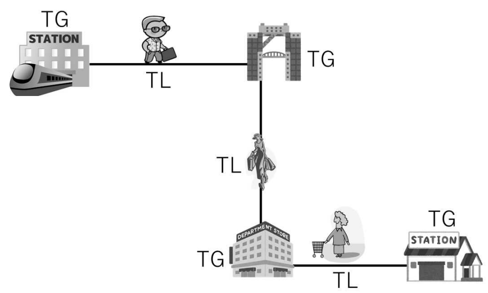

Fig. 1. Illustration of TGs and TLs

location area. TLs can be classified into the following three types:

- TLs both of whose ends are TGs,
- TLs one of whose ends is a TG, and
- TLs being entirely unrelated to any TGs.

In generally, the first TLs are predominantly larger than those of others. Hence we consider the first TLs in the article. Fig. 1 illustrates the relation with TGs and TLs.

Note that if there are three or more TGs being concentrated close, TLs generated by these TGs may be complex shapes. For example, the example of SLO in Section IV deals with the TL being a round route through six TGs.

In the SLO, we give the following assumption for TLs:

- TLs are generated between TGs with **large** demands,
- TLs are be generated between **near** TGs, and
- TLs are shaped as a **shortest path** between TGs.

We construct a mathematical model for generation of TLs according to the above three assumptions. Let d_{ij} be distance between i and j-th TGs, defined by the following Euclidean norm:

$$d_{ij} := ||\boldsymbol{v}_i - \boldsymbol{v}_j|| \tag{1}$$

From the above assumptions, TL between these TGs is generated if the following inequation holds:

$$\frac{q_i q_j}{d_{ij}^2} \geq \sigma, \tag{2}$$

where σ is a threshold for generation of TLs. Let E be set of pairs of TGs generating TLs. For $(i, j) \in E,\ i, j \in I$, its demand is given as $\gamma\sqrt{q_i q_j}$, where γ is a coefficient for demand of TLs, and its shape is given as a shortest path between these TGs, denoted by S_{ij}.

Moreover, we consider the case that there are two or more TLs overlapping. We give demand of sites between i and j-th TGs as sum of demands of TLs overlapping, denoted by w_{ij}, and set of TGs at the ends of all overlapping TLs is denoted by I_{ij}. Fig. 2 illustrates an example of model with four TGs. If TLs are generated between all TGs, four TLs are overlapped between second and third TGs. Then, demand of sites between second and third TGs is given as

$$w_{23} = \gamma \cdot (\sqrt{q_1 q_3} + \sqrt{q_2 q_3} + \sqrt{q_2 q_3} + \sqrt{q_2 q_4})$$

For evaluation of locating shop for TGs, we introduce Huff location model[7]. In the SLO, we assume that the sales of shop not located on any TLs is unaffected by all TLs. Let $\boldsymbol{x} \in \mathbf{R}^2$ be site of location of the shop, and $d_i(\boldsymbol{x})$ be distance

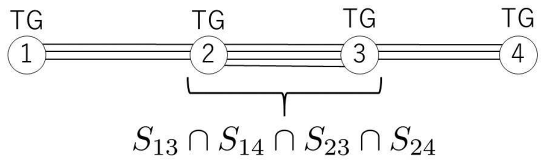

Fig. 2. Overlap of TLs

from i-th TG to the shop, defined by the following Euclidean norm:

$$d_i(\boldsymbol{x}) := ||\boldsymbol{x} - \boldsymbol{v}_i|| \tag{3}$$

In the model, the shop locater expects the sales of her/his shop from i-th TG defined as the following function:

$$h_i(\boldsymbol{x}) := \begin{cases} \alpha \cdot \dfrac{q_i}{d_i^2(\boldsymbol{x})}, & \text{if } d_i(\boldsymbol{x}) > \varepsilon, \\ \alpha \cdot \dfrac{q_i}{\varepsilon^2}, & \text{if } d_i(\boldsymbol{x}) \leq \varepsilon, \end{cases} \tag{4}$$

where α is a positive constant value for TGs, depended upon the kind and quality of shop, and is used in common to all TGs, and where ε is an upper limit of distance that residents can move without any effort.

First we consider the sales of shop not located on any TLs. From (4), the sum of sales obtained from all TGs, without considering TLs, can be represented as follows:

$$f(\boldsymbol{x}) := \sum_{i \in I} h_i(\boldsymbol{x}), \quad \text{if } \boldsymbol{x} \in \mathbf{R}^2 \backslash \mathbf{S} \tag{5}$$

where

$$\mathbf{S} = \bigcup_{(i,j) \in E} S_{ij} \tag{6}$$

On the other hand, for evaluation of locating shop for TLs, we propose a new function for sales of locating shop. Note that for each TL, we can regard that each resident in the TL exists on all sites on the TL. This means that sales obtained from each TL is constant for any site if it is located on the TL. If the shop is located on S_{ij}, we represent the sales from residents on TL between i and j-th TGs as the following function:

$$u_{ij}(\boldsymbol{x}) := \max \left\{ \sum_{k \in I_{ij}} h_k(\boldsymbol{x}),\ \beta w_{ij} \right\}, \text{ if } \boldsymbol{x} \in S_{ij} \tag{7}$$

where β is a positive constant value for TLs, whose value is generally more than one because residents in a TL are nearer by shops on the same TL than those on TGs. It depends upon the kind and quality of shop, and is used in common to all TLs.

From (7), the sum of total sales of shop located on S_{ij} with considering j-th TL can be represented as follows:

$$\begin{aligned} f(\boldsymbol{x}) &:= u_{ij}(\boldsymbol{x}) + \sum_{k \in I \backslash I_{ij}} h_k(\boldsymbol{x}) \\ &= \max \left\{ \sum_{k \in I} h_k(\boldsymbol{x}),\ \beta w_{ij} + \sum_{k \in I \backslash I_{ij}} h_k(\boldsymbol{x}) \right\}, \\ &\qquad \text{if } \boldsymbol{x} \in S_{ij}. \end{aligned} \tag{8}$$

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Next, we consider the cost of locating shop. Huff location model is widely applied to shop location in real world. Then, we assume that the cost of locating shop is represented as follows:

$$c(\boldsymbol{x}) := \sum_{i \in I} h_i(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbf{R}^2. \tag{9}$$

Therefore, we can formulated our SLO as the following reward maximizing problem:

$$\left.\begin{array}{ll} \text{maximize} & r(\boldsymbol{x}) := f(\boldsymbol{x}) - c(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{R}^2 \end{array}\right\} \tag{10}$$

III. Solution method

From (5) and (9), the reward from shop is just zero wherever it is not located on any TLs. From (8) and (9), the reward from shop located on TLs is zero or more. Then, if the reward from shop is also zero wherever located on TLs, it is zero for any location. Hence, (10) can be reduced to the following problem:

$$\left.\begin{array}{ll} \text{maximize} & r(\boldsymbol{x}) = \beta W(\boldsymbol{x}) - \displaystyle\sum_{k \in I_{ij}} h_k(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{S} \end{array}\right\} \tag{11}$$

where

$$W(\boldsymbol{x}) = \begin{cases} w_{ij}, & \boldsymbol{x} \in S_{ij}, \\ 0, & \boldsymbol{x} \notin \mathbf{S} \end{cases} \tag{12}$$

For solving (11), we propose the decomposition of it to subproblems each of whose feasible set is a TL. Then, subproblem for TL between i and j-th TGs, $(i,j) \in E$ is formulated as follows:

$$\left.\begin{array}{ll} \text{maximize} & r(\boldsymbol{x}) = \beta w_{ij} - \displaystyle\sum_{k \in I_{ij}} h_k(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in S_{ij}, (i,j) \in E \end{array}\right\} \tag{13}$$

By solving all subproblems for (11), we can obtain optimal solution of (11), which is the largest objective function value in all solutions of subproblems.

Since the first term of objective function of (13) is constant, (13) is equivalent to the following problem:

$$\left.\begin{array}{ll} \text{minimize} & \displaystyle\sum_{k \in I_{ij}} h_k(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in S_{ij}, (i,j) \in E \end{array}\right\} \tag{14}$$

From (5), the objective function is neither linear nor convex. Unfortunately, there is no efficient solution algorithm for general nonlinear and nonconvex programming problems. However, from the assumption of TLs in the previous section, each TL can be represented as a line segments. Since (14) can be represented as one-dimensional optimization, we can apply Brent method[2] for solving (14).

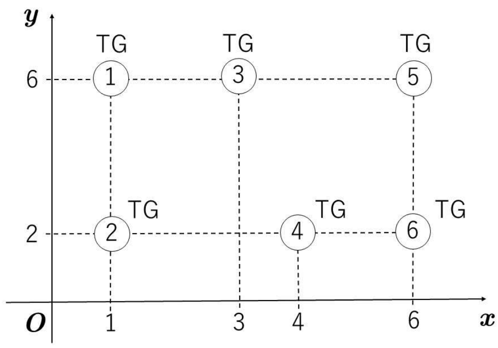

Fig. 3. An Example of SLO

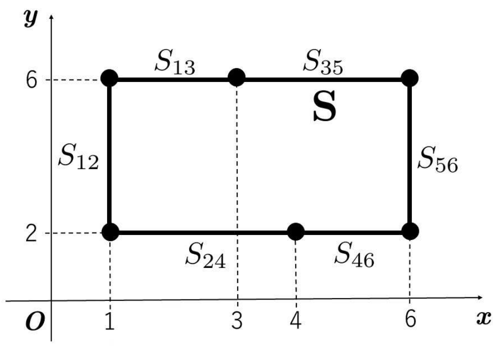

Fig. 4. Generation of TLs

IV. Numerical example of SLO

In this section, we show an efficiency of the proposed solution method by applying it to an example of our SLO. We consider an example of SLO with six TGs, whose sites are shown in Fig. 3. Demands for the TGs are given $q_i = 2$ for all $i \in \{1, 2, \ldots, 6\}$.

For generation of TLs, we give $\sigma = 1$ and $\gamma = 0.5$. Then, TLs are generated in Fig. 4, all of whose demands $w_{ij} = 1$ for all $(i,j) \in E$.

For sales and cost of locating shop, we give $\alpha = 1$ and $\beta = 2$. Then, we can formulate the SLO as the following problem:

$$\left.\begin{array}{ll} \text{maximize} & r(\boldsymbol{x}) = f(\boldsymbol{x}) - \displaystyle\sum_{i=1}^{6} h_i(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{R}^2 \end{array}\right\} \tag{15}$$

For finding an optimal location of the SLO, we reduce (15) to the following subproblems:

$$\left.\begin{array}{ll} \text{maximize} & r(\boldsymbol{x}) = 2 - \displaystyle\sum_{k \in I_{ij}} h_k(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{S} = \displaystyle\bigcup_{(i,j) \in E} S_{ij}, \end{array}\right\} \tag{16}$$

where $E = \{(1,2), (1,3), (2,4), (3,5), (4,6), (5,6)\}$.

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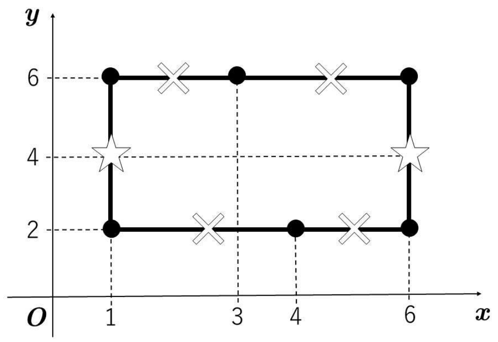

Fig. 5. Optimal Location

We decompose (16) to the following subproblems for $(i, j) \in E$:

$$\left.\begin{array}{ll} \text{minimize} & \displaystyle\sum_{k \in I_{ij}} h_k(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in S_{ij}, (i,j) \in E \end{array}\right\} \quad (17)$$

For solving (17), we code a Brent method by Python with scipy.optimize.brent function[10] in SciPy. Then, we can find the following candidates of optimal solutions of (16)

- If the shop is located on S_{12}, optimal site of (17) is $\boldsymbol{x} = (4.00, 1.00)$, whose objective function value is 1.00.
- If the shop is located on S_{13}, optimal site of (17) is $\boldsymbol{x} = (1.00, 4.00)$, whose objective function value is 4.00.
- If the shop is located on S_{24}, optimal site of (17) is $\boldsymbol{x} = (2.50, 2.00)$, whose objective function value of (13) is 1.78.
- If the shop is located on S_{35}, optimal site of (17) is $\boldsymbol{x} = (3.50, 6.00)$, whose objective function value of (13) is 1.78.
- If the shop is located on S_{46}, optimal site of (17) is $\boldsymbol{x} = (5.00, 2.00)$, whose objective function value of (13) is 4.00.
- If the shop is located on S_{56}, optimal site of (17) is $\boldsymbol{x} = (6.00, 4.00)$, whose objective function value of (13) is 1.00.

These sites are shown in Fig. 5. By comparing these objective function values, we can obtain an optimal solution of (16), and then (15), $\boldsymbol{x} = (1.00, 4.00)$ and $(6.00, 4.00)$. Their objective function values are $r(\boldsymbol{x}) = 2.00 - 1.00 = 1.00$, which means that the shop locater can obtain reward from a shop by these location.

V. Conclusion and Future Studies

In this article, we have considered a new SLO with both TGs and TLs. We proposed a mathematical model for prediction of residents' movement from traffic generators. We defined sales function of shops on TLs and applied Huff location model for representing cost of locating shop. Then, we formulated a new SLO problem with TGs and TLs. For solving the formulated problem, we decompose it to subproblems whose feasible sets are line segments, and then suggest to apply Brent method. An efficiency of solution method is shown by applying an example of SLO.

This article proposes a simple cost function of locating shop. However, cost functions in real world are usually given in a complicated way. Considering SLOs with more complex cost function is an interesting future research. Moreover, this article considers SLO that TLs is given statically. However, TLs often changes their sizes and shapes, and includes uncertainty. Considering SLO with uncertain TLs changing dynamically is also an interesting future study.

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