

# Pricing Strategy for Small Sized Poultry Farmers with Two Different Products

Hidefumi Kawakatsu, Dong Li, and Kosuke Kato

**Abstract**— This study develops an inventory model for a small sized poultry farmer who produces both Broilers (Product 1) and their Branded chickens (Product 2) with different Weibull amelioration rates. Both products are produced in multiple lots. The poultry farmer's total profit per unit of time is formulated as a function of selling prices by using a price-dependent demand function. We establish the existence of a unique optimal selling price for each product, which maximises the total profit. Numerical examples are presented to demonstrate the behaviour of the optimal solution when the ingredients in the poultry feed vary.

**Index Terms**— Amelioration, deterioration, small sized poultry farmers, optimal price

## I. INTRODUCTION

THE commercial chickens are classified into mainly three categories in Japan [1]: (1) *Wakadori* (Broiler), (2) *Meigaradori* (Branded chicken) and (3) *Jidori* (Free-range local traditional pedigree chicken). Broilers are the most popular young and fast-growing chicken, and their production period and commercial weight are about 50 days and from 2.5 kg to 3.0 kg, respectively. As for Branded chickens, there are no regulations on their breed (genotype), the method for rearing them, and the rearing environment. They are characterised by being fed with a low calorie diet, including ingredients such as herbs, tea, enzymes, and seaweeds. The low-calorie poultry diet makes their production period longer than that of broilers ([1], [2]). In contrast, the *Jidori* chickens have been certified governmentally in Japanese Agriculture Standards (JAS) by the Ministry of Agriculture in Japan. JAS 844 defines the meaning of *Jidori*, which refers to its genetic characteristics and longer breeding period as well as lower breeding density [3].

Statistics from Japanese government [4] have shown that the annual amount of production of broiler chickens in Japan has been increasing from 652,429 (thousands birds) in 2014 to 689,280 (thousands birds) in 2018. They also revealed that there has been a decrease in the number of poultry farmers with an annual production capacity less than 200,000 birds whilst the number of larger-scale poultry farmers with the capacity over 200,000 birds has gradually been increasing. Under such circumstances small and medium sized poultry

farmers would introduce alternative category of chickens to secure and increase their profit. Due to the high cost of producing *Jidori* chickens, it is relatively easy for the smaller sized poultry farmers who have limited breeding areas and/or limited budget to introduce their Branded chickens. Moreover, the local governments in 37 of 47 prefectures have been developing the branded chickens in their own experimental stations of livestock as part of an agricultural promotion [5]. There should be enough demand for branded chickens if they are produced in accordance with the instruction from the local governments.

In the field of inventory management, livestock such as chickens, fish, and ducklings are referred to as ameliorating items. Several models have been studied for items with the Weibull amelioration rate, since Hwang ([6], [7]) first developed EOQ models for ameliorating items. Mondal et al. [8] conducted an inventory model for ameliorating items with price dependent demand rate for a prescribed time period. Hwang [9] dealt with a stochastic set-covering location problem for both ameliorating and deteriorating items. Law et al. [10] developed an integrated, production-inventory model for ameliorating and deteriorating, taking into account the time discount. Wee et al. [11] developed an inventory model with consideration of the time value of money in the case where both amelioration and deterioration rates were assumed to follow a Weibull distribution. Vandana and Srivastava [12] proposed an inventory model for ameliorating/deteriorating items with trapezoidal-type demand rate under the condition of inflation and time discounting rate. Chou et al. [13] and Tuan et al. [14] established analytical frameworks to obtain an optimal solution for the EOQ model proposed by Hwang [6]. Our previous study [15] has developed an inventory model for a small sized poultry farmer who produces both Broilers (Product 1) and Branded chickens (Product 2) with different amelioration rates under the assumption that each product is only produced in a single lot.

This study formulates the total profit by relaxing the assumption of a single lot for each product under the following situations. The breeding area is limited and divided into two regions; one for Product 1 and the other for Product 2. Each product has a different breeding period due to the different Weibull amelioration rates. The product with longer breeding period has higher selling prices. We examine the existence of the optimal selling price for each product to maximise the total profit per unit of time.

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The amelioration and the feed conversion rates are considered to vary depending upon the ingredients in feed. Numerical examples are presented to demonstrate the behaviour of the optimal solution when the ingredients in poultry feed are changed.

## II. NOTATION AND ASSUMPTIONS

The main notation used in this paper listed below:

|             |   |
|-------------|---|
| $i$         | index of Product ( $i = 1, 2$ )   |
| $p_i$       | selling price of Product $i$ (decision variable)                                  |
| $Q_i(p_i)$  | order quantity per cycle of Product $i$ , a function of $p_i$                     |
| $d_i(p_i)$  | daily demand quantity of Product $i$ , a function of $p_i$                        |
| $T_i$       | production period for Product $i$   |
| $N_i$       | number of lots for Product $i$  |
| $\bar{D}_i$ | maximum capacity of each lot for Product $i$                                      |
| $\bar{D}$   | maximum capacity of breeding area, which is given by $\sum_{i=1}^2 N_i \bar{D}_i$ |
| $\theta_i$  | deterioration rate for Product $i$  |
| $\xi_i$     | disposal cost of deteriorated birds for Product $i$                               |
| $\eta_i$    | feed conversion rate (FCR) for Product $i$  |
| $\gamma_i$  | unit cost of ameliorating (diet) for Product $i$                                  |
| $h_i$       | unit inventory carrying cost per unit of time for Product $i$                     |
| $c_i$       | unit purchasing price of newly-hatched chick of Product $i$                       |
| $s$         | unit ordering cost  |
| $w_0$       | initial body weight   |
| $w_F$       | final body weight   |

The assumptions in this study are as follows:

- i) The quantity of a product is expressed as its weight and treated as continuous for simplicity.
- ii) No shortages are allowed.
- iii) The breeding area is divided into two areas; one for Product 1 (Broiler chicken) and the other for Product 2 (Branded chicken).
- iv) The poultry farmer has a maximum capacity, denoted by  $\bar{D}$ , of the breeding area.
- v) The inventory level (e.g., total body weight) of Product  $i$  continuously increases due to growth during  $[0, T_i)$ , but at the same time, the inventory level is depleted due to deterioration.
- vi) The amelioration rate of Product  $i$  at time  $t$ , denoted by  $r_i(t)$ , is expressed as the following equation ([6]-[14]):

$$r_i(t) = \frac{f_i(t)}{1-F_i(t)} = \alpha_i \beta_i t^{\beta_i-1} \quad (\alpha_i > 0, \beta_i > 0), \quad (1)$$

where  $f_i(t)$  and  $F_i(t)$  represent the probability density function and the distribution function of Weibull distribution, respectively, i.e.,

$$f_i(t) = \alpha_i \beta_i t^{\beta_i-1} e^{-\alpha_i t^{\beta_i}}, \quad (2)$$

$$F_i(t) = 1 - e^{-\alpha_i t^{\beta_i}}. \quad (3)$$

- vii) The products in each lot are shipped in one batch to the buyer in  $T_i$  ( $i = 1, 2$ ) units of time after the start

of the production of each lot. Figure 1 illustrates an example of the transition of the inventory level in the case of  $N_i = 3$ , where  $\text{Lot}(i, j)$  expresses the  $j$ th lot for Product  $i$  ( $i = 1, 2, j = 1, 2, \dots, N_i$ ).

- viii) The demand rate of Product  $i$ ,  $d_i(p_i)$ , is assumed to be deterministic and decreasing in  $p_i$ . This study focuses on the case where the demand can be expressed as the following constant price elasticity demand function ([16], [17]):

$$d_i(p_i) = a_i p_i^{-b_i}, \quad (4)$$

where  $a_i (> 0)$  denotes a scale parameter and  $b_i (> 1)$  represents an index of price elasticity.

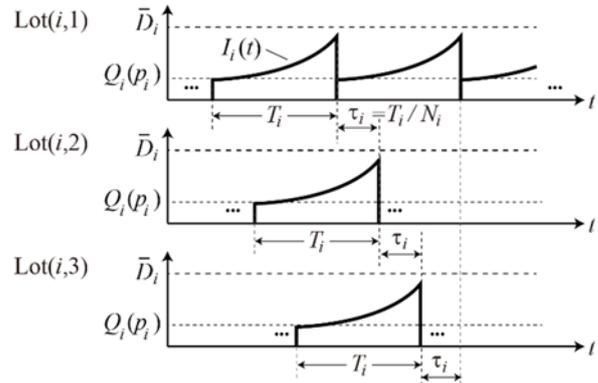


Fig. 1. Transition of inventory level ( $N_i = 3, i = 1, 2$ )

## III. TOTAL PROFIT

By assumptions v) and vi), the inventory level of Product  $i$  ( $i = 1, 2$ ) at time  $t$  can be expressed by

$$dI_i(t)/dt = [r_i(t) - \theta_i]I_i(t) \quad (0 \leq t < T_i). \quad (5)$$

Solving the differential equation in (5) with a boundary condition  $I_i(t) = d_i(p_i)T_i/N_i$  yields

$$I_i(t) = a_i p_i^{-b_i} \frac{T_i}{N_i} e^{-(T_i^{\beta_i} - t^{\beta_i})\alpha_i + (T_i - t)\theta_i}. \quad (6)$$

Equation (6) implies that the order quantity per lot is given by

$$Q_i(p_i) = I_i(0) = a_i p_i^{-b_i} \frac{T_i}{N_i} e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)}. \quad (7)$$

The total amount of ameliorating during  $[0, T_i)$  in each lot is given by

$$\begin{aligned} A_i(T_i) &= \int_0^{T_i} (\alpha_i \beta_i t^{\beta_i-1}) I_i(t) dt \\ &= a_i p_i^{-b_i} \frac{T_i}{N_i} \left[ 1 - e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \right]. \end{aligned} \quad (8)$$

Furthermore, the total amount of inventory level and deteriorated units over cycle in each lot are respectively expressed by

$$\begin{aligned} H_i(T_i) &= \int_0^{T_i} I_i(t) dt \\ &= a_i p_i^{-b_i} \frac{T_i}{N_i} e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \int_0^{T_i} e^{\alpha_i t^{\beta_i} - \theta_i t} dt, \end{aligned} \quad (9)$$

$$\int_0^{T_i} \theta_i I_i^{(j)}(t) = \theta_i H_i^{(j)}(T_i). \quad (10)$$

$$dp_1^2 \psi(p_1) < 0.$$

Therefore, the total profit of Product  $i$  can be represented by

$$\begin{aligned} P_i(p_i) &= (N_i/T_i)[d_i(p_i)T_i/N_i - c_i Q_i(p_i) \\ &\quad - \gamma_i \eta_i A_i(T_i) - (h_i + \xi_i \theta_i) H_i(T_i) - s] \\ &= a_i p_i^{-b_i} \left\{ p_i - c_i e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \right. \\ &\quad \left. - \gamma_i \eta_i \left[ 1 - e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \right] \right. \\ &\quad \left. - (h_i + \xi_i \theta_i) \int_0^{T_i} e^{\alpha_i t^{\beta_i} - \theta_i t} dt \right. \\ &\quad \left. \times e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \right\} - s N_i / T_i. \quad (11) \end{aligned}$$

Let  $C_i$  be the positive multiplies, independent of  $p_i$ , in (11), i.e.,

$$\begin{aligned} C_i &= c_i e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} + \gamma_i \eta_i \left[ 1 - e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \right] \\ &\quad + (h_i + \xi_i \theta_i) e^{-(\alpha_i T_i^{\beta_i} - \theta_i T_i)} \int_0^{T_i} e^{\alpha_i t^{\beta_i} - \theta_i t} dt, \quad (12) \end{aligned}$$

then equation (11) can be rewritten as

$$P_i(p_i) = a_i p_i^{-b_i} (p_i - C_i) - s N_i / T_i. \quad (13)$$

Hence, the poultry farmer's total profit per unit of time, denoted by  $P(p_1, p_2) = P_1(p_1) + P_2(p_2)$ , is given by

$$P(p_1, p_2) = \sum_{i=1}^2 [a_i p_i^{-b_i} (p_i - C_i) - s N_i / T_i]. \quad (14)$$

It is straightforward to see that  $P(p_1, p_2)$  in (14) is increasing in  $a_i$  and decreasing in  $b_i$ ,  $C_i$ ,  $s$ , and  $N_i$  ( $i = 1, 2$ ).

#### IV. FEASIBLE REGION

By assumption iv), a lower bound of selling price  $p_i$  is given by

$$\underline{p}_i = (a_i T_i / \bar{D})^{1/b_i} \quad (i = 1, 2). \quad (15)$$

Solving  $\sum_{i=1}^2 d_i(p_i) T_i \leq \bar{D}$  for  $p_2$ , we have

$$p_2 \geq [a_2 T_2 / (\bar{D} - a_1 p_1^{-b_1} T_1)]^{1/b_2}. \quad (16)$$

Let  $\psi(p_1)$  be the right-hand side of (16), i.e.,

$$\psi(p_1) = [a_2 T_2 / (\bar{D} - a_1 p_1^{-b_1} T_1)]^{1/b_2}. \quad (17)$$

The following property can be shown by the simple differential operation.

#### Property 1.

- (a)  $\lim_{p_1 \rightarrow \underline{p}_1 + 0} \psi(p_1) = +\infty$  and  $\lim_{p_1 \rightarrow +\infty} \psi(p_1) = \underline{p}_2$ .
- (b)  $\psi(p_1)$  is decreasing in  $p_1$  ( $p_1 \geq \underline{p}_1$ ) and  $(d^2 /$

From Property 1, the behaviour of  $\psi(p_1)$  can be characterized as shown in Fig. 2 in our previous study [15]. The feasible region, denoted by  $W$ , is expressed as

$$W = \{(p_1, p_2) | p_2 \geq \psi(p_1)\}. \quad (18)$$

It can be confirmed that  $p_i > \underline{p}_i$  ( $i = 1, 2$ ) holds for  $(p_1, p_2) \in W$ .

#### V. OPTIMAL SELLING PRICES

Under the constraint of  $(p_1, p_2) \in W$ , the problem is formulated as follows:

$$\begin{cases} \text{maximise} & P(p_1, p_2) \\ \text{subject to} & p_2 \geq \psi(p_1) \end{cases} \quad (19)$$

As mentioned in Section IV,  $p_i > \underline{p}_i$  ( $i = 1, 2$ ) holds if  $p_2 \geq \psi(p_1)$  in (19) is satisfied.

By introducing a Lagrange multiplier, denoted by  $\lambda$ , the problem (19) can be expressed as

$$\begin{cases} \frac{\partial}{\partial p_1} L(p_1, p_2, \lambda) = \frac{\partial}{\partial p_2} L(p_1, p_2, \lambda) = 0, \\ g(p_1, p_2) \leq 0, \lambda_k \geq 0, \lambda \cdot g(p_1, p_2) = 0. \end{cases} \quad (20)$$

where

$$L(p_1, p_2, \lambda) = P(p_1, p_2) - \lambda \cdot g(p_1, p_2). \quad (21)$$

$$g(p_1, p_2) = \psi(p_1) - p_2. \quad (22)$$

The conditions in (20) are referred to as the Karush-Kuhn-Tucker (KKT) conditions ([18]).

In the case of  $\tilde{p}_2 \geq \psi(\tilde{p}_1)$  ( $\Leftrightarrow g_1(p_1, p_2) \leq 0$ ), the KKT conditions in (20) indicates that a critical point of  $P(p_1, p_2)$  in (14) becomes a solution to the problem (19). In this case, a unique global solution, denoted by  $(p_1, p_2) = (\tilde{p}_1, \tilde{p}_2)$  ( $\tilde{p}_i > \underline{p}_i$ ), to the problem (19) is given by the following Lemma 1.

#### Lemma 1.

(a) A unique finite positive solution for each product, which maximises  $P_i(p_i)$ , is given by

$$\tilde{p}_i = b_i C_i / (b_i - 1) \quad (i = 1, 2). \quad (23)$$

(b) A unique pair of  $(p_1, p_2) = (\tilde{p}_1, \tilde{p}_2)$  ( $\tilde{p}_i > \underline{p}_i$ ) attains the maximum value of  $P(p_1, p_2)$  in (14), which is a unique finite solution to the problem (19) for  $\tilde{p}_2 \geq \psi(\tilde{p}_1)$ .

In the case of  $\tilde{p}_2 < \psi(\tilde{p}_1)$  ( $\Leftrightarrow g_1(p_1, p_2) > 0$ ), although it cannot be shown that  $P(p_1, p_2)$  is concave for  $p_i > \underline{p}_i$  ( $i = 1, 2$ ), Lemma1(a) implies that  $P_i(p_i)$  in (13) is increasing in  $p_i$  for  $p_i < \tilde{p}_i$ . In contrast, for  $p_i > \tilde{p}_i$ ,  $P_i(p_i)$  is decreasing in

$p_i$  ( $i = 1, 2$ ). Therefore a maximum value of  $P(p_1, p_2)$  is obtained on  $p_2 = \psi(p_1)$  for  $\tilde{p}_2 < \psi(\tilde{p}_1)$ . We can also prove that there exists a unique finite solution  $p_1 = \hat{p}_1$  ( $> \underline{p}_1$ ) that attains the maximum value of  $P(p_1, \psi(p_1))$ . This signifies that the solution to (20) becomes a global solution to (19).

A unique finite solution  $(p_1, p_2) = (\hat{p}_1, \hat{p}_2)$  to (19) for  $\tilde{p}_2 < \psi(\tilde{p}_1)$  can be summarised in the following Lemma 2.

**Lemma 2.** *In the case of  $\tilde{p}_2 < \psi(\tilde{p}_1)$ ,  $\hat{p}_1$  ( $> \tilde{p}_1$ ) is given as a unique solution to*

$$\frac{b_1-1}{b_1T_1} \left( p_1 - \frac{b_1C_1}{b_1-1} \right) - \frac{b_2-1}{b_2T_2} \left[ \psi(p_1) - \frac{b_2C_1}{b_2-1} \right] = 0, \quad (24)$$

and  $\hat{p}_2$  ( $> \tilde{p}_2$ ) is expressed as

$$\hat{p}_2 = \psi(\hat{p}_1). \quad (25)$$

The Theorem 1 below follows immediately Lemma 1(b) and Lemma 2.

**Theorem 1.** *A unique positive finite solution to (19), denoted by  $(p_1, p_2) = (p_1^*, p_2^*)$ , is given by*

$$(p_1^*, p_2^*) = \begin{cases} (\tilde{p}_1, \tilde{p}_2) & (\tilde{p}_i > \underline{p}_i), \\ (\hat{p}_1, \hat{p}_2) & (\tilde{p}_i < \psi(\tilde{p}_1)). \end{cases} \quad (26)$$

## VI. NUMERICAL EXAMPLES

This section considers a case where the poultry diet for branded chickens consists of the ordinary diet (the same as for broiler chickens) and *Plantago lanceolata* (a species of herb that grows in temperate areas and can be used as livestock feed). We assume that the cost of collecting the wild herb can be ignored. The unit cost of amelioration (diet) for Product 2, denoted by  $\gamma_2$ , is expressed as

$$\gamma_2 = (1 - \text{Blending ratio of herb})\gamma_1, \quad (27)$$

where  $\gamma_1$  is the unit cost of amelioration for Product 1.

Nishiwaki et al. [2] have observed the change of body weight of broiler chickens when the ratio of herb (*Plantago lanceolata*) contained in the ordinary diet is 0%, 10%, and 20%. We estimate the parameters of the amelioration rate ( $\alpha_2$  and  $\beta_2$  in (1)) and the length of the production cycle of Branded chicken  $T_2$  by using their data when the initial and final body weight of the birds are  $w_0 = 0.04$  (kg) and  $w_F = 2.7$  (kg), respectively. For 4% of the mortality rate, we estimate the values of  $\theta_2$  since the mortality rate of the broilers is controlled to be less than 4% in Japan.

The estimated values are shown in Table I. The parameters of broiler chickens are set as  $\alpha_1 = \alpha_2$ ,  $\beta_1 = \beta_2$ ,  $T_1 = T_2$ , and  $\theta_1 = \theta_2$  in the case where the blending ratio of herb is 0%.

Regarding to the demand rate, we consider a scenario as follows: (a) the buyer will purchase 1,500kg (1,800kg) of Product 1 if its unit selling price is 300 (270). (b) the buyer will buy 400kg (500kg) of Product 2 if its unit selling price is 500 (400).

In this case, the parameters of the demand rate for each product can be estimated as  $(a_1, b_1) = (567824.727, 1.73)$  and  $(a_2, b_2) = (453699035.445, 3.106)$ .

TABLE I  
ESTIMATED VALUES OF  $\alpha_2, \beta_2, T_2$ , AND  $\theta_2$

| Content ratio of herb | $\alpha_2$ | $\beta_2$ | $T_2$  | $\theta_2$ |
|-----------------------|------------|-----------|--------|------------|
| 0 %                   | 0.909      | 0.390     | 51.100 | 0.000799   |
| 10 %                  | 0.759      | 0.432     | 52.940 | 0.000771   |
| 20 %                  | 0.685      | 0.445     | 59.529 | 0.000686   |

TABLE II  
(A) NUMERICAL EXAMPLE UNDER CONSTANT FCR=1.7

| Subcase | Herb | FCR  | $p_1^*$ | $p_2^*$ | $P^*$ |
|---------|------|------|---------|---------|-------|
| (1)     | 0 %  | 1.70 | 361.8   | 345.1   | 303.0 |
| (2)     | 10 % | 1.70 | 376.2   | 328.0   | 367.7 |
| (3)     | 20 % | 1.70 | 387.0   | 318.2   | 454.6 |

(B) NUMERICAL EXAMPLE UNDER FCR DEPENDING UPON HERB CONTENT RATE

| Subcase | Herb | FCR  | $p_1^*$ | $p_2^*$ | $P^*$ |
|---------|------|------|---------|---------|-------|
| (4)     | 0 %  | 1.70 | 361.8   | 345.1   | 303.0 |
| (5)     | 10 % | 1.82 | 366.6   | 338.7   | 332.7 |
| (6)     | 20 % | 2.26 | 360.5   | 373.6   | 275.9 |

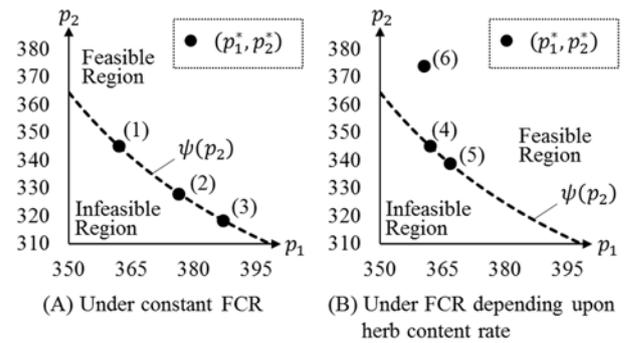


Fig. 2. Numerical Examples (correspond to Table II(A) and (B))

For  $w_0 = 0.04$  and  $w_F = 2.7$ , the feed conversion ratio (FCR) of broiler chickens can be estimated as approximately 1.7 using the data in [19]. The value of FCR is considered to increase with the blending rate of herb due to the decrease in the fat content ratio in the feed. This section estimates the FCR corresponding to each blending rate of herb by assuming that the amount of feed intake for 50 days is the same regardless of the blending ratio of herb. The other parameters are set as

$$(\gamma_1, \bar{D}, N_1, N_2, h_1, h_2, \xi_1, \xi_2, c_1, c_2) = (120, 1500, 3, 3, 1.1, 1.2, 2, 2, 100, 100).$$

Table II(A) shows the numerical examples in reference to  $p_1^*$ ,  $p_2^*$ , and  $P^* = P(p_1^*, p_2^*)$  for the constant FCR=1.7. Table II(B) summarises the results when FCR increases with the blending ratio of herb. Figure 2(A) and (B) correspond to Table II(A) and (B), respectively.

Table II(A) and Fig. 2(A) show that  $p_1^*$  and  $P^*$  increase and  $p_2^*$  decreases as the herb content rate increases under a constant FCR. This is mainly because the total cost of the diet significantly decreases as the herb content ratio increases under a constant FCR, whereas the low-calorie diet containing herb increases the production period of the chickens as shown in Table I.

In contrast, Table II(B) demonstrates that the total profit

per unit of time increases when the herb content rate increases from 0% to 10%, but decreases when it increases from 10% to 20%. In this case, the total cost of amelioration increases with FCR, though the unit cost of the diet becomes small as the mixed ratio of the herb increases. This result implies that there may be cases that there exists an optimal herb content rate, which maximises the total profit, if the cost of diet for branded chickens can be reduced by adding herbs into the ordinary feed for broiler chickens.

## VII. CONCLUSION

This study has proposed an inventory model for two products with different Weibull amelioration rates under the following circumstances. The small sized poultry farmer not only produces Broilers (Product 1) to meet the demand of the dominant buyer, but also introduces their own branded chickens (Product 2) for other customers. The breeding area is limited and divided into two regions; one for Product 1 and the other for Product 2. Each product is produced in more than one lots and has a different breeding period. The product with longer breeding period has higher selling prices.

We formulated the farmer's total profit per unit of time as a function of selling prices and proved that there exists a unique positive finite optimal selling price for each product, which maximises the total profit.

The numerical examples illustrate the behaviour of the optimal solution and the total profit when the content rate of the herb (*Plantago lanceolata*) varies. We have assumed that the cost of collecting the *Plantago lanceolata* can be ignored. Since it is widely accepted that the amelioration rate decreases and the feed conversion rate (FCR) increases with the content ratio of herb, we estimated the parameters of the amelioration rate and the FCRs using the actual data for three different content ratios of herb (0%, 10%, and 20%). The results indicate that there may exist an optimal content ratio of herb if the cost of feed can be reduced by adding herbs into the poultry diet.

In order to show the existence of the optimal content ratio of herb, however, we need to clarify the relationship between the content ratio, the amelioration, and the feed conversion rates more precisely. Furthermore, the feed conversion rate is not constant over the production period but varies with the age of products. These factors will be considered in an extended paper in the future.

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