Multi-stage Inventory Model with Fuzzy Demand in Supply Chain Networks

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ABSTRACT – The pursuit of superior coordination schemes is crucial for contemporary supply chains to survive in a highly competitive environment. In order to allow different order quantities among the selected suppliers to obtain the optimal solutions in this industry, this research addresses both supplier selection and inventory control problems in supply chain management simultaneously by creating a multi-stage inventory model for a serial system. In the perspective of a single-objective method, this research aims to minimize the total ordering costs, holding costs, and purchasing costs, subject to the price, quality, and capacity. Furthermore, the inclusion of fuzzy demand employs the signed distance and a ranking method for fuzzy numbers to find the estimate of the common total cost in the fuzzy sense. Consequently, numerical examples are provided to illustrate the usefulness of proposed models and comparative understanding of various methods.

Index Terms—Fuzzy numbers, supply chain network, multi-stage inventory model, multi-objective programming, sensitivity analysis

I. INTRODUCTION

In today’s global market, the competitive advantage of companies has been the key factor to reduce costs so as to increase the profits. Simultaneously, in order to promote the competition and differentiation advantage in the industry, proper supplier selection is an essential task so as to react to the volatile market.

Previous studies normally build the traditional inventory model in the perfect assumptions. However, in reality, it is necessary to select the right suppliers and the optimum order quantities amongst selected suppliers in order to lower the cost and maximize the profit. Therefore, this study commits to sort out the appropriate order quantities to the suppliers and solve the multi-sourcing problem in supply chain network.

Generally speaking, supplier selection involves single-sourcing and multi-sourcing problems. In a single-sourcing problem, buyers will choose a supplier among potential suppliers according to their competitive advantage and evaluation criteria. This paper addresses the single-objective methods to suit real world applications and solve the previous problems.

Mendoza and Ventura (2010) proposed the supplier selection process is assumed to take place in a serial supply chain system. A set of selected suppliers purchase the raw material for the final production. Capacity, quality, setup cost, and unit price are considered as criteria for supplier selection. This raw material is processed and assembled into a final product as it flows through every stages of the supply chain until it reaches the end customer. M. F. Yang (2013) applied interactive two-phase method for multi-objective linear programming to with fuzzy theory select management decisions in supply chain network.

De Boer et al. (2001) applied the existing models for supplier selection in a framework. This framework exhibited several decision-making steps in the supplier selection process. These models are associated with the allocation of the proper order quantities to the selected suppliers. Narasimhan (2006) developed to a decision model for selecting suppliers and supplier bids given the relative importance of multiple criteria across multiple products over their product life cycle. Hillier and Lieberman (2010) referred to serial supply chain systems often lead to developing partnership relationships with suppliers as well as mutually beneficial supply contracts that enable reducing the total cost of operating a jointly managed multi-echelon inventory system.

Ghodsypour and O’Brien (2001) built a mixed integer nonlinear programming model restricted the model to allocate only one order per cycle to each selected supplier. Mendoza (2007) showed that Ghodsypour and O’Brien’s restriction is unnecessary and adjusted their model by allowing multiple orders per supplier within an order cycle. Kheljani et al. (2009) proposed a multi-supplier single-buyer coordination model to minimize the total cost in supply chain. Gorji et al. (2014) applied a two-level supply chain model to consider both order allocation and supplier selection problems at the same time. Cárdenas-Barrón and Treviño-Garza (2014) referred to a three-level supply chain model to solve the multi-product and multi-period problems.

J.C.-H. Pan (2008) proposed a model included fuzzy annual demand and the production rate. Further, this
method employs the signed distance for fuzzy numbers in order to find the estimate of the common total cost in the fuzzy sense. Subsequently, this model derives the corresponding optimal purchaser’s quantity and the integer number of lots in which the items are delivered from the vendor to the buyer. S. Sarkar and T. Chakrabarti (2013) built the EPQ model in the fuzzy sense, in which the shortage is allowed and the delivery is fully extended. The total cost in the fuzzy model is lower than the original model. Zhang and Xu (2014) studied multiple objective decision making model considered the bi-fuzzy environment and quantity discount policy to solve the uncertainty in supply chain.

C.Y. Chiu (2014) suggested the fuzzy multi-objective integrated logistics model with the transportation cost and demand fuzziness to solve green supply chain problems in the uncertain environment. Vineet Mittal et al. (2015) proposed a joint two-tier inventory model with single supplier and single type of product. This model demonstrated the optimality of inventory decisions under non-fuzzy and fuzzy requirements. M.F. Yang (2015) proposed a three-echelon integrated inventory model for the development of defective products, reprocessing and credit periods. Assuming fuzzy requirements, numerical analysis was applied to observe the impact of fuzzy demand on inventory strategy and total profit.

Fuzzy theory is often utilized in the calculation of demand in order to generate results that fit better to the reality. For instance, H.J. Tu et al. (2011) developed a two-echelon inventory model with mutual beneficial pricing strategy for fuzzy demand in a supply chain. The beneficial pricing strategy can benefit the vendor more than multiple buyers in the integrated system. Bodaghi (2018) built a fuzzy multi-objective model to integrated supplier selection, order quantity allocation and customer order scheduling problem in a make to order manufacturing system. Consequently, this mathematical measure help suppliers contribute to the responsiveness and flexibility of entire supply chain in the face of uncertain customer orders.

The past studies mainly focused on supplier selection and order allocation; this is because the above can efficiently affect the cost and profit. Nevertheless, those studies ignored that different quantity distributed policy may cause a terrible influence to the management and profit. Hence, this paper determines demand based on fuzzy theory because of the uncertain environments.

II. MATERIALS AND METHODS

A. Notations

The following notations are used to establish the propose model.

- $s$: number of available suppliers
- $N$: number of stages
- $d$: demand per time unit
- $H_j$: holding cost per unit and time unit at Stage $j$, for $j = 1,..,N$
- $E_j$: echelon holding cost per unit and time unit at Stage $j$, for $j = 1,..,N$
- $K_{j,i}$: ordering cost for the $i$th player at Stage $j$, for $i = 1,..,s$ and $j = 1,..,N$
- $P_i$: unit price of the $i$th supplier, for $i = 1,..,s$
- $C_i$: capacity of the $i$th supplier per time unit, for $i = 1,..,s$
- $U_i$: quality for the $i$th supplier, for $i = 1,..,s$
- $U_a$: minimum acceptable quality for the manufacturer
- $U_p$: perfect quality
- $J_{j,i}$: number of orders for the $i$th player, at Stage $j$ per order cycle, for $i = 1,..,s$ and $j = 1,..,N$
- $Q_{j,i}$: order quantity for the $i$th player, at Stage $j$, for $i = 1,..,s$ and $j = 1,..,N$
- $n_{j,i}$: lot-size multiplier of the $i$th player at Stage $j$, for $i = 1,..,s$ and $j = 1,..,N$
- $T_{EC}$: total expect costs per month

![Fig.1. A multi-stage inventory system with multi-supplier.](image)

B. Assumptions

1. The demand occurs at Stage $j$ at a constant rate per time unit.
2. Shortages are not allowed.
3. The production rate must be greater than the cumulative demand rate.
4. The purchasing costs are only occurred at Stage 1.
5. The product is transferred internally through the company.
6. Multiple orders of one supplier are allowed within single order cycle.
7. In connection with the quality, the suppliers might produce defective parts.
8. The ideal quality is 100%.

C. A Model with fuzzy demand

In order to more accurately reflect real situations, Chang (2017) refers to the multi-stage inventory model by allowing different order quantities for each selected supplier. A revised concept of the multi-stage is shown in Fig. 2.

Due to the uncertainty of monthly demand, this research presents a new inventory model applied fuzzy theory.
First, an order cycle includes six orders (i.e., \( \sum_{i=1}^{3} J_{1,i} = 6 \)) distributed to three suppliers (i.e., \( J_{1,1} = 2 \), \( J_{1,2} = 3 \), \( J_{1,3} = 1 \)) at Stage 1. Two orders are distributed to Supplier 1, three orders are distributed to Supplier 2, and one order is distributed to Supplier 3. Subsequently, the order cycle includes twelve orders (i.e., \( J_{2,1} = 4 \), \( J_{2,2} = 6 \), \( J_{2,3} = 2 \)) at Stage 2. The same procedure can be adapted to the other stages.

Based on the above notations and assumptions; this model solves the total expected cost (TEC) for the suppliers and this is given by:

The total expected costs = the ordering cost + the holding cost + the purchasing cost

The ordering cost: the ordering cost per order cycle at Stage \( j \) is \( \sum_{i=1}^{N} J_{j,i} K_{j,i} \), and the time of order cycle at Stage \( j \) is \( J_{j+1,i} \). Hence, the total ordering cost can be presented as:

\[
d \cdot \frac{\sum_{i=1}^{N} J_{j,i} P_{i} Q_{1,i}}{\sum_{i=1}^{N} J_{j,i} Q_{1,i}}
\]

The holding cost: the holding cost will increase due to the inventory transferring to the next stage, and the echelon holding cost \( E_{j} \) is adopted in this step. The average inventory quantity for the \( i \)th player, at Stage \( j \) is \( \frac{Q_{j,i}}{2} \). Thus, the total holding cost can be described as:

\[
\frac{1}{2} \cdot \frac{\sum_{j=1}^{N} \sum_{i=1}^{N} J_{j,i} Q_{j,i}^2 E_{j,i}}{\sum_{j=1}^{N} \sum_{i=1}^{N} J_{j,i} Q_{j,i}}
\]

The purchasing cost: this research only considers the purchasing cost incurred at Stage 1. The average unit price in this supply chain network is \( \sum_{i=1}^{3} J_{1,i} P_{i} Q_{1,i} / \sum_{i=1}^{3} J_{1,i} Q_{1,i} \). Therefore, the total purchasing cost can be expressed as:

\[
d \cdot \frac{\sum_{i=1}^{3} J_{1,i} P_{i} Q_{1,i}}{\sum_{i=1}^{3} J_{1,i} Q_{1,i}}
\]

As a result, the total expected costs can be depicted as follow:

\[
TEC = d \cdot \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} J_{j,i} K_{j,i}}{\sum_{j=1}^{N} \sum_{i=1}^{N} J_{j,i} Q_{j,i}} + \frac{1}{2} \cdot \frac{\sum_{j=1}^{N} \sum_{i=1}^{N} J_{j,i} Q_{j,i}^2 E_{j,i}}{\sum_{j=1}^{N} \sum_{i=1}^{N} J_{j,i} Q_{j,i}}
\]

Additionally, based on the above condition, this research defines three kinds of constraints included capacity, quality, and quantity. The TEC is subject to these three types of constraints:

The capacity constraints consist of demand \( d \), the proportion of demand assigned to the \( i \)th supplier \( (J_{1,i}, Q_{1,i}/\sum_{i=1}^{3} J_{1,i} Q_{1,i}) \), and the capacity of the \( i \)th supplier \( (C_{i}) \).

\[
d \cdot \frac{J_{1,i} Q_{1,i}}{\sum_{i=1}^{3} J_{1,i} Q_{1,i}} \leq C_{i}
\]

The quality constraints include the minimum acceptable quality \( U_{a} \), and the average quality manufactured by suppliers \( \sum_{i=1}^{3} J_{1,i} U_{1,i} Q_{1,i}/\sum_{i=1}^{3} J_{1,i} Q_{1,i} \).

\[
\sum_{i=1}^{3} \frac{J_{1,i} U_{1,i} Q_{1,i}}{\sum_{i=1}^{3} J_{1,i} Q_{1,i}} \geq U_{a}
\]

The quantity constraints: The order quantity for the \( i \)th supplier, at Stage \( j \) is \( J_{j,i} \) and the lot-size multiplier for the downstream stage is \( n_{j,i} \).

\[
Q_{j,i} = n_{j,i} J_{j+1,i}, \quad j = 1, \ldots, N - 1
\]

The number of order constraints: The number of order for the \( i \)th supplier, at Stage \( j \) is \( J_{j,i} \) and the lot-size multiplier for the downstream stage is \( n_{j,i} \).

\[
J_{j+1,i} = n_{j,i} J_{j,i}, \quad j = 1, \ldots, N - 1
\]

Definition 1. From Kaufmann and Gupta (1991), Zimmermann (1996), Yao and Wu (2000), for a fuzzy set \( \tilde{D} \in \Omega \) and \( \alpha \)-cut, the \( \alpha \)-cut of the fuzzy set \( \tilde{D} \) is \( D(\alpha) = \{ \xi \in \Omega | \mu_{\tilde{D}}(\xi) \geq \alpha \} = [D_{L}(\alpha), D_{R}(\alpha)] \) , where \( D_{L}(\alpha) = a + \alpha(b - d) \) and \( D_{R}(\alpha) = c - \alpha(c - b) \).

We can obtain the following equation. The signed distance of \( \tilde{D} \) to \( \tilde{0}_{4} \) is defined as:

\[
d(\tilde{D}, \tilde{0}_{4}) = \int_{0}^{d} \frac{1}{2} \int_{0}^{d} D_{L}(\alpha), D_{R}(\alpha) d\alpha = \frac{1}{4} (2b + a + c).
\]

So this equation is:

\[
d(\tilde{D}, \tilde{0}_{4}) = \frac{1}{4} \int_{0}^{d} D_{L}(\alpha), D_{R}(\alpha) d\alpha = \frac{1}{4} (2b + a + c).
\]

Streamlined distance method is used to the defuzzification of TEC \( (Q,n) \).
\[ \bar{D} = d(\bar{D}, \bar{D}_0) = \frac{1}{4}(D - \Delta_1) + 2D + (D + \Delta_2) \]

\[ = D + \frac{1}{4}(\Delta_2 - \Delta_1). \]

The single-objective model with fuzzy demand is depicted as:

\[ \text{Min.} \]

\[ TEC(Q, n) = D + \frac{(\Delta_2 - \Delta_1)}{4} \sum_{j=1}^{s} \sum_{i=1}^{s_j} Q_{j,i} (\Delta_{ji})_{ij} + \frac{1}{2}. \]

Subject to:

\[ \sum_{j=1}^{s} \sum_{i=1}^{s_j} (\Delta_{ji})_{ij} = 1, \text{ integer} \]

\[ Q_{j,i} \geq 0, \text{ integer} \]

\[ \sum_{i=1}^{s_j} Q_{j,i} \geq u_a \]

By substituting fuzzy demand into the original formula, the initial ordering cost and the purchasing cost with fuzzy demand can be obtained. In this way, the total expected can be recalculated. At the same time, the influence of fuzzy theory utilized in this inventory system will be depicted by comparison with single-objective (SO) method in Chang (2017).

III. NUMERICAL RESULTS

This research presents a detailed numerical example to illustrate the results of the proposed models:

| TABLE I | DATA OF EACH STAGE |
|---|---|---|---|
| Stage | Ordering cost | Holding cost | Echelon cost ($) |
| 1 | 1780 | 19 | 15 |
| 2 | 820 | 44 | 25 |
| 3 | 360 | 71 | 27 |
| 4 | 280 | 102 | 31 |

| TABLE II | DATA OF FOUR POTENTIAL SUPPLIERS |
|---|---|---|---|
| Supplier | Ordering cost | Price($) | Quality | Capacity(unit) |
| 1 | 32 | 3200 | 0.96 | 10000 |
| 2 | 46 | 4100 | 0.93 | 14000 |
| 3 | 54 | 2900 | 0.97 | 12000 |
| 4 | 49 | 3000 | 0.95 | 9000 |

(1) When \( \Delta_1 < \Delta_2 \), then \( \bar{D} > D \). While \( (\Delta_2 - \Delta_1) \) decreases, the smaller \( \bar{D} \) is in this fuzzy model, the more similar to the model, and vice versa.

(2) When \( \Delta_1 = \Delta_2 = 15000 \), then \( \bar{D} = D = 30000 \). In this case, this fuzzy model will be exactly the same as the original model.

(3) \( TEC \) normally increases while \( \Delta_2 - \Delta_1 \) increase and \( \sum_{i=1}^{s} J_{1,i} \) decrease.

(4) Figure 3 is the comparison diagram that compares \( TEC \) to \( (\Delta_2 - \Delta_1) \). It shows that four curves almost overlap and the trend of \( TEC \) for different \( \sum_{i=1}^{s} J_{1,i} \) are similar.

(5) According to the numerical result, the minimum of \( TEC \) will occur while \( \sum_{i=1}^{s} J_{1,i} = 100 \) and \( \bar{D} = -7500 \).
IV. CONCLUSION

Both supplier selection and the order quantity often determine the success of supply chain management. Otherwise, lowering the price is not a good strategy for the suppliers. It might decrease profit or increase defective products. Thus, a well-designed supply chain network is crucial. In this study, a novel inventory model is developed by assuming the demand quantity as a triangular fuzzy number. The decision makers can acquire more flexibility due to different upper and lower limits of the demand. This method might allow the theoretical model more close to the real situation. Through the sensitive analysis in this study, it shows that if $\bar{D}$ increases, TEC will increase. Also the smaller $\bar{D}$ is in the fuzzy model; the more similar it gets to the original model. In the future, studies may include multi-objective method, and then different fuzzilized factors in supply chain network can be added into the model to simulate more realistic world applications.

REFERENCES


