Study of Using RFID for Fuzzy Integrated Inventory Model

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Abstract—With the development and progress of science and technology, radio frequency identification (RFID) has drawn more and more attention in various fields. In recent years, more and more research is devoted to the development and introduction of RFID. Enterprises through the proper planning of introduction RFID to reduce costs and even improve investment efficiency. In our study, RFID input will be applied to the integrated inventory model, to explore its application performance, and whether it can effectively improve the integration of inventory models. Whether the cost of RFID input and the risk it faces later can ensure that the investment in the system is valuable and the key to the enterprise's attention. Consequently, this study will also explore the investment benefits and risks of introduction to RFID and its problem might face. In the traditional integrated model and the literature, the scale factor in most models is known, but it is not practical. Therefore, this study establishes a RFID integrated inventory model in a fuzzy environment and applies the triangular fuzzy number often used by general decision makers as the way to adjust the scale coefficient by fuzzy theory, and uses the method of the center of gravity to solve the fuzzy number. The results in this paper proved that can better reflect the real world phenomenon in the supply chain environment.

Keywords—RFID; integrated inventory; investment efficiency; fuzzy theory.

I. INTRODUCTION

In this IT-based era, the accuracy and instantaneity of data transmission have been the key factors of supply management. In order to enhance the competitive strength and have the advantage in the market, upstream and downstream supplier constantly exchange their inventory through the integration of resources so as to react to the volatile market.

RFID utilizes the radio frequency to transfer the data. Thus, the sensor can exchange the data without contacting with the receiver. This wireless data transmission method has several advantages. First, there is no directional limitation. Second, it can be applied to any kinds of product without the restrictions of size and shape while reading data from the RFID tag. In addition, the data presented by the identification system can be overwrote by the computer easily. As a result, we can recycle those reusable labels to approach the purpose of convenience and environmental protection.

In recent years, lots of vendors try to introduce RFID technology system to optimize the processes in the supply chain. With the use of RFID, we can identify the information of products immediately and control the shipping process on time. Especially, the improvement of inventory efficiency is particularly impressive.

The main purpose of integrated inventory model is to manufacture available demand and make the purchase at the same time. In order to execute the JIT productive system, the upstream and downstream supplier need to be closely integrated. Moreover, the buyers and vendors also need to closely cooperate. Goyal (1976) [1] proposed a notion of an integrated inventory model for a single vendor and a single buyer that assumes the order cycle and order quantity as the decision variables.

Banerjee (1986) [2] purposed the concept of economic-lot-size to reduce the inventory cost by extension Goyal’s model. Banerjee also considered the integrated inventory model by the purchasing strategies of buyers, in order to count joint economic-lot-size model. Goyal (1989) [3] extended the assumption of Banerjee’s study by using the assumption that the production lot is the integer multiple of the buyer’s order lot. After finding the appropriate vendor's production lot and the buyer's order lot, Goyal proposed a lower cost integrated inventory model than the model of Banerjee’s study.


In order to avoid the complication of the process, most of the inventory models set the demand (rate) as a known constant. However, in the competitive market, the varied of demand is non-ignorable. Roger (1999) [5] considered the integrated inventory model by the buyer consuming the product at a fixed rate; in other words, this situation would continue to satisfy the external demand. If the demand is always set as a known constant, the building cost model will not achieve the expected objectives. Lo and Yang (2007) [7] set the demand as the price demand function of the most
traditional form, where the price elasticity is negative number. Otherwise, J.G.Szmerekovsky (2011) [8] assumed that retailers could satisfy the demand by setting product prices and allocating shelf space. Different from Lo and Yang (2007) [7], J.G.Szmerekovsky’s study set the upper and lower limits of shelf space elasticity. Price elasticity is not the same as shelf space elasticity. Price elasticity is the elasticity of price demand, mostly negative number; shelf space elasticity is any number between 0 and 1, which is considered a definition of scoping.

The key decision variables in investment in RFID are investment costs. Therefore, understanding the cost structure will help decision makers evaluate the introduction of RFID technology into existing processes. Based on the traditional economic order quantity (EOQ). In Lee (2010) [9] considered three factors that affect investment in RFID: ordering efficiency, Just-in-time (JIT) efficiency, operational efficiency. By their assumption, they proposed “Supply chain RFID Investment Evaluation Mode” to minimize the total cost and led to the investment level when introduced the RFID technology in supply chain. Y.C. Tsao et al.(2016)[10] first proposed the design of a closed supply chain network with RFID technology, which proved that CA model can get the distribution centers (DCs) and the remanufacturing centers (RCs)numbers, locations and investment levels. On the other hand, they evaluated the level that investment of RFID affects forward flows and reverse flows by the three efficiency proposed by Lee’s study[9] and the Return rate proposed by Nativi and Lee (2012) [11].

RFID is an automatic identification technology. It is used to tag on items as a label with using radio frequency as signal transmission and data exchange. The key of supply chain and logistics management is information accuracy and instantaneous. In order to transmit the information of sales, inventory or cost immediately and save labors or mistakes which appear during shipping process such as product misplacement, damage and shipping errors, Li(2006) [12] applied RFID equipment into supply chain system. This application really reduced communication costs and replenishment time.

A.Sarac (2010) [13] summarized some literatures that related to RFID technology and supply chain. He realized that the primary areas that RFID application can deal with are inaccurate inventory, bullwhip effect and choosing the best replenishment policy. There are four ways to analyze the influence of RFID on supply chain system: analysis methods, simulation, case studies and experiments. These ways are usually related to return on investment (ROI), so that can quantify the impact of RFID application in the supply chain.

A. Ustundag (2008) [14] first investigates product value, lead time and demand uncertainty of the products, and then used the simulation model to calculate the expected profit of the three-echelon supply chain by improving the efficiency, accuracy, visibility and security level. Park (2010) [15] proposed a three-level model for supply chain network design with the aim of using two-way heuristic solution which based on Lagrange relaxation method to establish the optimal number and location of vendors and distribution centers and minimize the total cost of the supply chain network.

In order to determine the influence of RFID on the area of regional distribution centers and the number of orders, Y.C.Tsao (2017) [16] applied RFID into the design of supply chain network and simplified the problem through a two-stage approximation.

Due to the effectiveness of the system will also affect the cost, the model of our research takes the purchase efficiency and JIT efficiency into consideration. But most of the related literature assumed that the scale factor is fixed constants. However, it is usually difficult for the practitioners to set the scale factor as crisp value in reality. Therefore, more and more researchers start to make effort on applying fuzzy sets theory and techniques to develop and solve inventory problems. In order to minimize the joint total expected annual cost, this study introduces RFID to the integrated inventory model in fuzzy environment.

II. FUZZY SETS THEORY

Professor Zadeh of Berkeley University [17], [18]proposed a fuzzy set in 1965. Converting a semantic or colloquial narrative into a fuzzy set and transforming it into usable information through a series of systematic fuzzy operations.

Fuzzy number is a special fuzzy subset. Its membership function can make the degree membership continuously correspond from the real number R to the universe of discourse between the intervals [0, 1], if the fuzzy set A is a fuzzy number, it has the following characteristics:

1. \( U_A(x) = 0, \text{when } x \in (-\infty, 0) \cup [\delta, \infty] \)
2. \( U_A(x) \) is strictly increasing in \([\alpha, \beta]\) and strictly decreasing in \([\gamma, \delta]\)
3. \( U_A(x) = 1, \text{when } x \in [\beta, \gamma] \)
4. when \( U_A(x) \) is a straight line segment in \([\alpha, \beta]\) and \([\gamma, \delta]\), its membership function \( U_A(x) : R \rightarrow [0,1] \) is:

\[
U_A(x) = \begin{cases} 
\frac{x - \alpha}{\beta - \alpha} & \alpha \leq x \leq \beta \\
1 & \beta \leq x \leq \gamma \\
\frac{\delta - x}{\delta - \gamma} & \gamma \leq x \leq \delta \\
0 & \text{otherwise}
\end{cases}
\]

In the above formula, \( \alpha \leq \beta \leq \gamma \leq \delta \), there are three forms of this membership function:

1. when \( \alpha = \beta \) and \( \gamma = \delta \), the membership function is a rectangular fuzzy number.
2. when \( \beta = \gamma \), the membership function is a triangle fuzzy number.
3. In general case, the membership function is a trapezoidal fuzzy number.

A linguistic variable is a variable in a human natural language that describes a statement, a phrase, or a word for a certain degree, and uses a membership function to represent its degree membership.

The linguistic variables can be expressed in the form of linguistic terms, and a linguistic term can be represented by a fuzzy value, usually expressed as a singleton, for example: \( A = U_A(x) \), and multiple linguistic terms have multiple degree membership to represent this fuzzy set can represent the input data in more detail.(Wang and Mendel[19])

In general, a linguistic variable contains four pieces of information: name, class, range, and degree.
Applying fuzzy theory to measure the subjective judgment process can be divided into two steps. Firstly, the linguistic meaning word used in the linguistic variable is converted into a fuzzy number, and then convert the fuzzy number into a crisp value.

Due to the current changing environment, the traditional inventory model cannot meet the demand, so many scholars began to apply fuzzy theory to solve the problem of inventory and production. 

For example, Park [20] considers the fuzzy inventory cost to use the extension to solve its minimum total cost. Chen and Wang[21] consider the case of allowing for back order by using triangular fuzzy numbers to blur demand, ordering cost, inventory cost, and back order cost into the economic order quantity model. Roy and Maiti [22] construct a fuzzy economic order quantity model and the demand depends on the unit cost in a limited storage capacity. Lee and Yao [23] blur the number of demand and the number of production per day in the economic production quantity model. Yao et al. [24] assume that an economic order quantity model in this model is blurred into a triangular fuzzy number of orders and total demand. Chang [25] applies the fuzzy set theory to the quality inspection project and the annual demand in the economic order quantity model. Pan and Yang [26] uses the triangular fuzzy number to blur the annual demand for the mixed inventory model under controllable lead time. Chang and Ouyang [27] and Yao et al. [28] use fuzzy total inventory cost through the center-of-gravity method and the single distance method. The result is that the method of using a single distance to defuzzify is better than center-of-gravity, so this paper uses a single distance method to defuzzify the total cost under fuzzy linguistic.

### III. NOTATIONS AND ASSUMPTIONS

This study is based on the cost allocation of the integrated inventory model (Yang Jinshan, Fang Zhengzhong, Zhang Yuteng, 2014) [29]. First develop a model with the order quantity and setup cost as the decision variables. Then consider the purpose of this research, change the model in the background of Lee’s study [9] and propose a fuzzy adjustment scale factor with the order quantity and price as the decision variable model, in order to have the best performance and output under the appropriate RFID construction cost. To establish the proposed model, the following notations and assumptions are used.

The decision variable is:

- \( Q \) : Economic order Quantity
- \( p \) : Price (price elasticity = -1)

The remaining parameters are:

- \( A \) : Buyer’s order cost per time
- \( B \) : Annual production
- \( S \) : Vendor’s set-up cost
- \( H_S \) : Holding Cost per unit each year for the vendor
- \( H_B \) : Holding Cost per unit each year for the buyer
- \( F \) : Fixed facility cost for RFID
- \( C_G \) : Unit Cost for each RFID tag attached into the product
- \( E_1 \) : Ordering efficiency
- \( E_2 \) : JIT efficiency
- \( \alpha \) : Scale factor

The Integrated inventory model is developed on the basis of the following assumptions:

We assume that the cost of applying RFID is only fixed facility cost and RFID labeling cost (variable cost). The remaining costs (such as maintenance, human error, etc.) are not discussed in this study.

1. Both buyers and sellers are trading in complete information
2. Do not allow out of stock
3. This study only considers a single product
4. The period of replenishment of the retail industry is short, in other words, the lead time is approaching zero
5. Products are allowed to be sent downstream from upstream

6. the joint total cost is \( \frac{B(p)}{Q} \left( \frac{S}{n} + A \right) \), the vendor’s total order cost is \( \frac{B(p)}{Q} (S) \), and the buyer’s total order cost is \( \frac{B(p)}{Q} (A) \)

7. Holding cost for annual joint total cost = buyer’s annual holding cost + vendor’s annual holding cost \( \cdot \) which is shown as \( \alpha \frac{Q}{B} H_S + H_B \)

8. The total cost of RFID is \( F + (C_G) (Q) \)

9. We assume that the scale factor \( \alpha \) is a triangular fuzzy number because the triangular fuzzy number is easily defined by the general decision maker.

### IV. MODEL FORMULATION

Based on the above notations and assumptions, after the application of RFID technology that the joint total expected annual cost is the following:

\[
\text{JTEC}(Q, p) = E_1 \left[ \frac{B(p)}{Q} (S + A) \right] + E_2 \frac{Q}{B} \left[ \frac{B(p)}{Q} H_S + H_B \right] + \left[ F + (C_G)(Q) \right] \]

(1)

This study assumes that the scale factor \( \alpha \) is a triangular fuzzy number, where \( \alpha = (\alpha - \Delta_1, \alpha, \alpha + \Delta_2) \), \( 0 < \Delta_1 < \alpha \), \( 0 < \Delta_2 \), and \( \Delta_1 \cdot \Delta_2 \) are both determined by decision-makers. In this case, the joint total expected annual cost is a fuzzy function and can be expressed as:

\[
\bar{w}(Q, p) = E_1 \left[ \alpha \frac{B(p)}{Q} (S + A) \right] + E_2 \frac{Q}{B} \left[ \alpha \frac{B(p)}{Q} H_S + H_B \right] + \left[ F + (C_G)(Q) \right] \]

(2)

Theorem 1. From George and Yuan[30] for a fuzzy set \( \bar{B} = \Omega \) and \( \alpha = [0,1] \), the \( \bar{a} - \text{cut of this set} \) \( \bar{B} = B(\alpha) = \{ x \in \Omega | \mu_B \geq \alpha \} = \{ B_1(\alpha), B_0(\alpha) \} \) , where \( B_i(\alpha) = a + (b - a)\alpha \) and \( B_0(\alpha) = c - (c - b)\alpha \) use this method to defuzzify, we call the center-of-gravity method of \( \mu_B(x) \):

\[
\bar{B} = \int_{\Omega} x \mu_B(x) dx \int_{\Omega} \mu_B(x) dx = \frac{1}{3} (a + b + c) \]

(3)

\( \bar{a} \) is obtained by theorem 1:

\[
\bar{a}(\bar{a}, \bar{b}) = 1 \left[ \left( \alpha - \Delta_1 \right) + \alpha + (\alpha + \Delta_2) \right] = \alpha + \frac{1}{3}(\Delta_2 - \Delta_1)
\]

(4)

Substituting the result of (4) into (2), we have

\[
\bar{w}(Q, p) = E_1 \left[ \alpha + \frac{1}{3}(\Delta_2 - \Delta_1)p^{-1} (S + A) \right]
\]
results are summarized in Table 1.

The decision-makers due to the uncertainty of the problem. Using classical optimization, we take the first and second derivatives of $\bar{W}(Q, p)$ with respect to $Q$, and obtain

$$
\begin{align*}
\frac{\partial \bar{W}(Q, p)}{\partial Q} &= -E_1\bar{a}\bar{p}^{-1}(S + A), \\
\frac{\partial^2 \bar{W}(Q, p)}{\partial Q^2} &= -\frac{2E_1\bar{a}\bar{p}^{-1}(S + A)}{Q^2} + \frac{E_2}{2}\bar{a}\bar{p}^{-1}H_S + H_B + C_t, \\
\text{and} \\
\frac{\partial^2 \bar{W}(Q, p)}{\partial Q \partial p} &= -\frac{2E_1\bar{a}\bar{p}^{-1}(S + A)}{Q^3},
\end{align*}
$$

Since $(\partial^2 \bar{W}(Q, p))/(\partial Q^2) > 0$, $W(Q, p)$ is convex in $Q$, and the minimum value of $W(Q, p)$ will occur at the point that satisfies $(\partial^2 \bar{W}(Q, p))/(\partial Q^2) = 0$. Setting (6) equal to zero and solving for $Q$, then the optimal order quantity of the buyer is shown as:

$$
Q = \sqrt{\frac{2E_1\bar{a}\bar{p}^{-1}(S + A)}{E_2\bar{a}\bar{p}^{-1}H_S + H_B}}
$$

In the above formula $\bar{a} = \frac{1}{2}(\Delta_2 - \Delta_1)$. The derivation of equation (8) is shown in appendix 2.

In order to find the optimal $p$, we use the following formula:

$$
p^* = \frac{E_1\bar{a}\bar{p}^{-1}(S + A)}{2E_2\bar{a}\bar{p}^{-1}H_S + H_B} + \sqrt{\left(\frac{E_1\bar{a}\bar{p}^{-1}(S + A)}{2E_2\bar{a}\bar{p}^{-1}H_S + H_B}\right)^2 + 1}
$$

The equation (9) is proved in appendix 2.

We use the following program steps so that we can find the optimal values of $Q$ and $p$.

Step 1: Obtain $\Delta_1$ and $\Delta_2$ from the decision-maker.

Step 2: Find the optimal solution $p^*$ value from by equation (9).

Step 3: Substitute the $p^*$ value to find $Q^*$ value through the formula (6).

Step 4: The $W(Q^*, p^*)$ is the optimal joint total expected annual cost.

V. NUMERICAL EXAMPLES

To illustrate the results of the proposed models, consider an inventory situation with the following parametric values partially adopted in [9], P.C. Yang[26], [31], and Tom Watson[34]. buyer’s ordering cost $A = $25/order, vendor’s set-up cost $S = $400/set-up, annual production $B = $800/unit, vendor’s holding cost $H_S = $25/unit, buyer’s holding cost $H_B = $5/unit, fixed facility cost for RFID $F = $50000, cost of RFID tag attached into the product $C_t = $5/unit, Ordering efficiency $E_1 = 0.9$, JIT efficiency $E_2 = 0.9$, scale factor $\alpha = 1500$.

Based on the above data and the model constructed in this article, solve for the optimal order quantity of buyer and find the optimal joint total expected annual cost $W(Q^*, p^*)$ in the fuzzy sense for various given sets of $\Delta_1$ and $\Delta_2$. Note that in practical situations, $\Delta_1$ and $\Delta_2$ are determined by the decision-makers due to the uncertainty of the problem. The results are summarized in Table 1.

| $\Delta_1$ | $\Delta_2$ | $\bar{a}$ | $p^*$ | $Q^*$ | $W(Q^*, p^*)$ | $V_Q(\%)$ | $V_W(\%)$
|---|---|---|---|---|---|---|---|
| 25 | 50 | $1500$ | $2.17$ | $1500$ | $153.18$ | $2.03$ | $100\%$
| 50 | 100 | $1500$ | $2.10$ | $1500$ | $156.13$ | $2.04$ | $100\%$
| 75 | 150 | $1500$ | $2.03$ | $1500$ | $159.01$ | $2.03$ | $100\%$
| 100 | 200 | $1500$ | $2.00$ | $1500$ | $164.58$ | $2.00$ | $100\%$
| 125 | 250 | $1500$ | $1.97$ | $1500$ | $165.12$ | $2.04$ | $100\%$
| 150 | 300 | $1500$ | $1.93$ | $1500$ | $165.66$ | $2.05$ | $100\%$
| 175 | 350 | $1500$ | $1.92$ | $1500$ | $168.33$ | $2.07$ | $100\%$
| 200 | 400 | $1500$ | $1.90$ | $1500$ | $170.94$ | $2.08$ | $100\%$

Table 1 also lists which cases have the same calculation results as the crisp mode when in the context of this model. From the classical optimization techniques used by Pan and Yang (2004)[31], we can get the optimal order quantity of buyer $Q_c^*$, and the optimal joint total expected annual cost $W(Q_c^*)$. Consequently, we have $Q_c^* = 165.12$ units and $W(Q_c^*) = 53319.15$. At the same time, in order to compare the variation of the optimal order quantity of buyer and the optimal joint total expected annual cost between this model and the crisp model, we use $V_Q = \frac{Q - Q_c^*}{Q_c^*} \times 100\%$ and $V_W = \frac{W(Q, p^*) - W(Q_c^*)}{W(Q_c^*)} \times 100\%$ to measure. The comparison results are shown in the last two columns of Table 1.

The traditional integrated inventory model focuses on the single buyer and the single vendor’s business transactions. With the close cooperation between upstream and downstream of timely production, the RFID technology importance in the technical cooperation is increasing. Therefore, based on the EOQ model, we use the economic order quantity and price as the decision variable, and import the influence of the cost and the demand rate of RFID in order to seek the cooperative productivity with maximum efficiency under the optimal cost. However, in the part of the demand, most literatures discuss constants. But in this study, the demand is defined as a function of price elasticity and scale factor, which makes the model closer to the actual surface.

Using RFID in supply chain system, can use its characteristics of accurate record of the transaction process, to save labor and shipping due to product misuse, shipping
errors, human loss caused by unnecessary loss, and to reduce the integration costs and the time of immediate replenishment of the upstream and downstream, buyers and vendors by a certain extent. Such as: Wal-Mart used RFID technology to accurately know the supply situation of goods on each shelf from thousands of supermarkets throughout the United States, so the headquarters can immediately place an order to the vendor and provide full sales information to the vendor. In order to achieve the best replenishment efficiency, thus created Wal-Mart’s famous continuous supply model Weinstein (2005) [34]. Therefore, we put RFID into cost consideration and study the relationship between the order quantity and price under the optimum RFID cost.

However, in the traditional integration model and the literature, the scale factor in most models is known, but in the current changing and competitive supply chain environment, the product life cycle is shortened, so the production variables are also many, such as to use the traditional integration model may not achieve the best results. Therefore, this study establishes an integrated inventory model using RFID in fuzzy environment, and applies the triangular fuzzy number often used by general decision makers as the way to adjust the scale coefficient by fuzzy, and uses the method of the center of gravity to solve the fuzzy number.

In addition, the results show that in some cases, the proposed fuzzy model can be simplified to a clear problem, and the buyer’s optimal order cost in the fuzzy sense can be reduced to the traditional integrated inventory model. Although we can’t be sure that the solution obtained from the fuzzy model is better than the clear model, the advantage of the fuzzy method is that it is more flexible and the decision makers deal with the problem according to the profession. Therefore, the model proposed in this paper can better reflect the real world phenomenon in the supply chain environment.

APPENDIX

Appendix 1: Derivation of equation (8)
Assume (6) = 0
\[-E_s\tilde{\alpha}p^{-1}(S+A)\cdot Q^2 + \frac{E_s}{2}\left[\tilde{\alpha}p^{-1}\cdot H_s + H_B\right] + C_t = 0\]
\[E_sH_s\tilde{\alpha}p^{-1} + E_sH_B + 2BC_t = \frac{E_s(S + A)\tilde{\alpha}p^{-1}}{Q^2}\]
\[Q^2 = \frac{2E_s\tilde{\alpha}p^{-1}(S + A) + B}{2E_s\tilde{\alpha}p^{-1}(H_s + H_B) + 2C_t}\]
\[Q^* = \frac{2E_s\tilde{\alpha}p^{-1}(S + A)}{\sqrt{2E_s\tilde{\alpha}p^{-1}(H_s + H_B) + 2C_t}}\]

Appendix 2: The calculation for (9)
Substitute (8) into
\[\text{JTEC} = E_t\left[\sqrt{\frac{\tilde{\alpha}p^{-1}}{Q}(S + A)}\right] + \frac{E_sQ}{2}\left[\frac{\tilde{\alpha}p^{-1}}{B}H_s + H_B\right] + F + C_tQ\]

obtain

\[\text{JTEC} = 2\sqrt{\frac{E_t(S + A)\tilde{\alpha}p^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)}{2B}} + F\]

if \(y = f(x) = \left[\frac{2E_t(S + A)\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t}{B}\right]^{\frac{1}{2}}\)
\[u = \frac{2E_t(S + A)\tilde{\alpha}p^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)}{B}\]
\[y = (u)^{\frac{1}{2}}\]
\[\frac{du}{dp} = \frac{2}{B}E_t(S + A)\tilde{\alpha}[\frac{-2E_sH_s\tilde{\alpha}p^{-3} - BE_sH_Bp^{-2}}{2BC_t(p^{-2})}]\]
\[\frac{dy}{du} = \frac{1}{2u^{-\frac{1}{2}}}\]
\[\frac{dy}{dp} = \frac{dy}{du} \times \frac{du}{dp} = \frac{1}{2} \frac{E_t(S + A)\tilde{\alpha}[\frac{-2E_sH_s\tilde{\alpha}p^{-3} - BE_sH_Bp^{-2}}{2BC_t(p^{-2})}]}{B} = 0\]

\[\frac{E_t(S + A)\tilde{\alpha}}{\sqrt{2Bp^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)}} \times \frac{E_t(S + A)\tilde{\alpha}(E_sH_s\tilde{\alpha}p^{-3})^2}{2Bp^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)} \times \frac{E_t(S + A)\tilde{\alpha}}{2Bp^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)} \times (-E_sH_s\tilde{\alpha}p^{-3})\]

\[\frac{E_t(S + A)\tilde{\alpha}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)}{2Bp^{-1}} \times p^{-2} - \frac{E_t(S + A)\tilde{\alpha}(E_sH_s\tilde{\alpha}p^{-3})^2}{2Bp^{-1}(E_sH_s\tilde{\alpha}p^{-1} + BE_sH_B + 2BC_t)} \times p^{-4}\]

\[= E_sH_s\tilde{\alpha}(p^{-2} - 1) = -p^{-1}(BE_sH_B + 2BC_t)\]

\[\frac{p - 1}{p} = \frac{BE_sH_B + 2BC_t}{E_sH_s\tilde{\alpha}}\]

\[p^2 - p \times \frac{BE_sH_B + 2BC_t}{2E_sH_s\tilde{\alpha}} - 1 = 0\]

\[p^* = \frac{BE_sH_B + 2BC_t}{2E_sH_s\tilde{\alpha}} + \sqrt{\left(\frac{BE_sH_B + 2BC_t}{2E_sH_s\tilde{\alpha}}\right)^2 + 1}\]
REFERENCES


