Integrated Inventory Model for Returnable Transport Items considered Container Return Time and Fuzzy Demand

Hsin-Hsiung Fang, Chien-Min Su, Sheng-Long Kao, Lu-An Chen

Abstract - This study considers single supplier supplying single buyer with finished products packed in returnable transport items (RTIs) to facilitate safe transport to the customer. Empty RTIs are collected at the buyer and returned to the supplier. The return time of RTIs is considered to be stochastic in this study because of unforeseen events, such as damages of RTIs or labor shortage to empty RTIs. We formulate the problem of integrating the flow of both the finished product and RTIs and minimizing the supply chain expected total costs as a non-linear program. Secondly, it presents the results of this study in which the behavior of the model is analyzed. In addition, we add fuzzy to this study in order to acquire a more realistic result because of the uncertain environments.

Index Terms—returnable transport item, stochastic return time, expected total costs, fuzzy, supplier

I. INTRODUCTION

In recent years, an increasing environmental awareness and social responsibility of industrial companies have motivated many companies to address sustainable resources. Including reverse operations in the management of supply chains enables companies to use reusable packaging material, which then back into the forward supply chain and lower material usage if the packaging material is managed appropriately. So-called reusable packaging materials, referred to as returnable transport items (RTIs), such as containers, boxes or pallets are widely used in industry today. Management activities related to the use of RTIs include the initial procurement, the replacement of damaged or the collection and return of used RTIs. Integrating the use of RTIs and finished goods is getting more critical in supply chain and helps to lower the cost of the cost of procuring new RTIs.

Manuscript received November 29, 2018; revised December 6, 2018. This work was supported in part by the Department of Transportation Science, National Taiwan Ocean University. Paper title is Integrated Inventory Model for Returnable Transport Items considered Container Return Time and Fuzzy Demand. (paper number: ICOR_10) L. A. Chen Author is with the Department of Transportation Science, National Taiwan Ocean University, No. 2, Beining Rd., Keelung City 202, Taiwan (R.O.C). (corresponding author to provide phone: 0939-939-487; e-mail: k841185366@gmail.com).

H. H. Fang Author is with Taipei University of Marine Technology, Department of Air & Sea Logistics and Marketing, email: fhh@mail.tcmnt.edu.tw

C. M. Su Author is with National Taiwan Ocean University, Department of Transportation Science; Center of Excellence for Ocean Engineering, email: cmsu@mail.ntou.edu.tw

S. L. Kao Author is with National Taiwan Ocean University, Department of Transportation Science; Center of Excellence for Ocean Engineering, email: slkao@mail.ntou.edu.tw

In many practical cases, damages of RTIs that need to be repaired exist or the fact that the customer does not have enough labor resources for handling both RTIs and products at the same time, which may cause delays in returning empty RTIs at the buyer's end and thus we assume that the return time of RTIs is stochastic. However, delays in the return of empty RTIs at the supplier's side may lead to stock out, which make it difficult to achieve the next shipment of finished products on time. Consequently, this paper develops an integrated supply chain inventory model by assuming that the products produced at the supplier are stored in RTIs during transportation, and that if a stock out of RTIs occurs at the supplier, deterioration of products would not happen. Also, we consider the uncertainty of buyer’s demand and therefore fuzzy is added to this study. The intention of this paper studies how delays in the return of RTIs impact the performance of expected total cost considered fuzzy demand.

We must determine the optimal RTI lot size (n) to find the minimum expected total cost, so taking first and second-order partial derivative of the expected total cost function with respect to n and showing the function is consequently convex. In this case, an optimal solution for n can be found by increasing n stepwise from 1 until ETC increases for the first time, and then calculating the minimum total costs.

II. LITERATURE REVIEW

The use of RTIs has increasingly been the object of research in recent years. A related research is the one of Kim, Glock, and Kwon (2014), who studied the case of a single supplier transporting a deteriorating product to a single buyer. RTI return lead times were considered to be stochastic in this study and that delay in return RTIs may lead to shortages at the supplier’s side. In case the forward shipment is delayed, the finished products stored in the container would start to deteriorate. Glock and Kim (2015) researched a single-supplier single-retailer integrated supply chain system where RTIs are used to deliver the finished product. The authors considered that a reliable fraction of RTIs needs to be substituted from cycle to cycle and researched the impact of the downstream companies and upstream industry transportation frequencies on the operation of the supply chain. Buchanan and Abad (1989) researched the problem of inventory control system about containers and assumed the returns in a given time to be a stochastic programming model of the quantity of container in the field. The authors obtained the optimum inventory control policy of the system by using dynamic programming. One of Chew et al. (2002) in this line of study programmed performance evaluation to monitor and supervise the disposal of containers. Toktay et al. (2000)
studied the problem of purchasing new products in supply chains where reusable products are returned to the supplier for reusing, and programmed an optimum ordering policy for the system.

Under stochastic lead times, return in RTIs may arrive early, on time or late at the supply chain system. Consequently, unessential inventory carrying charge may occur in this case. Works that researched inventory supplement strategies with stochastic lead times comprise the ones of Liberatore (1979), Sphicas (1982), and Friedman (1984). Sajadieh and Jokar (2009) researched the problem of a supply chain comprising a single vendor and a single retailer and that the lead time of the vendor is considered to be stochastic. The authors discriminated between two different situations, one where the reorder point surpasses the lead time demand and one where the contrary situation occurs, also programmed and computed total cost functions for two different scenarios. Sajadieh and Jokar (2009) programmed a JELS model consisting of a single supplier and a single buyer and that the supplier’s transportation lead time is considered to be stochastic and constantly distributed. Sajadieh, Jokar, and Modarres (2009) published a closely associated model compared with the one of Sajadieh and Jokar (2009) primarily assuming that lead times are considered to be distributed exponentially.

In today’s changeable environment, traditional inventory model cannot meet our needs and thus many scholars decided to apply fuzzy theory in solving the problem of inventory and production, for example, Chen and Wang (1996) applied triangular fuzzy number to the EOQ model and used it to fuzzy demand, ordering cost, inventory carrying cost and backorder cost with allowance of back order. Pan and Yang (2006) used the conception of fuzzy to combine the mixture inventory model and programmed an optimal solution process to obtain the optimum order quantity and lead time. The authors evaluated the annual demand by using the signed distance. M.F. Yang and Y. Lin (2012) programmed an integrated inventory model consisting of single-supplier multiple-buyer to minimize the total related annual cost resulted by the supplier and the buyers while the lead time demand is normally distributed, and the selection of investing in procedure quality improvement is contained. M. F. Yang and Y. Lin (2013) programmed an efficient two-phase method that can assist to solve the problems of fuzzy multi-objective decision in project management.

III. MATERIALS AND METHODS

To establish the proposed model, the following nomenclature will be used throughout this study.

3.1 Notations

- \( A \): buyer’s ordering cost ($/order)
- \( S \): supplier’s set-up cost ($/setup)
- \( D \): demand rate for finished products at the buyer (kg/year)
- \( P \): production rate for finished products at the supplier (kg/year)
- \( g_b \): inventory carrying charge for finished products at the buyer’s site ($/unit/time)
- \( g_r \): inventory carrying charge for RTIs at the supplier’s site ($/unit/time)
- \( h_a \): inventory carrying charge for finished products at the supplier’s site ($/unit/time)
- \( \pi \): shortage cost factor for finished products at the buyer’s site ($/unit/time)
- \( L_0 \): expected RTI return time with \( L_0 > 0 \)
- \( Q \): lot size of finished products (kg)
- \( \alpha \): transport capacity of a single RTI (kg)
- \( n \): RTI lot size to ship Q units of the finished product (an integer variable)
- \( \sigma \): standard deviation of RTI return time
- \( t \): real RTI return time with \( t > 0 \)
- \( \bar{D} \): Triangular fuzzy number, \( \bar{D} = (D - \Delta_d, D, D + \Delta_d) \), \( 0 < \Delta_d < D, 0 < \Delta_d \)
- \( f(.) \): probability density function of RTI return time
- \( f(t) = \lambda e^{-\lambda t} \) with \( \lambda = 1/L_0 \)

The model is developed under the following assumptions:

3.2 Assumption

- This paper studies a supply chain consisting of a single supplier and a single buyer.
- We used an exponential distribution with mean \( L_0 \), which is a common assumption in models with stochastic return time.
- The time for loading, transporting and unloading RTIs is negligible.
- Shortages are allowed and assumed to result in shortage cost.
- Deterioration of the finished products can be neglected during the production time and stock out at the supplier.

3.3 Model development

If the return lead time of RTIs is stochastic with mean \( L_0 \) and standard deviation \( \sigma \), three different cases may occur depending on the real RTIs return time:

**Case 1:** \( 0 < t \leq L_0 \)

RTIs are returned earlier before the lot size of finished products has been completed, which makes it necessary to store the return RTIs and causes inventory carrying cost for RTIs at the supplier. Given that the returned RTIs arrive at the supplier between times 0 and \( L_0 \). The expected total cost per cycle of case 1 is given by:

\[
TC_{case1}(Q) = \int_0^{L_0} \left( S + (L_0 - t)ng_s + \frac{h_QQ^2}{2P} \right) f(t)dt
+ \int_0^{L_0} \left( A + \frac{h_QQ^2}{2\bar{D}} + R_1g_b \right) f(t)dt
\]

where

\[
R_1 = \left( 1 \times \frac{\alpha}{\bar{D}} + 2 \times \frac{\alpha}{\bar{D}} + \ldots + (n - 1) \times \frac{\alpha}{\bar{D}} \right) = \frac{an(n - 1)}{2\bar{D}}
\]

**Case 2:** \( L_0 < t \leq L_0 + Q/\bar{D} \)

Late return shipments cause inventory carrying cost for finished products at the supplier but RTI inventory has not to be stored at the supplier as returning RTIs are instantly loaded and delivered to the buyer. Once a shipment of finished products arrives at the buyer, we’ll meet first eventual backorders which are lower than the lot size Q. Given that the returned RTIs arrive at the supplier between
times $L_0$ and $L_0 + Q/\bar{D}$. The expected total cost per cycle of case 1 is given by:

$$\begin{align*}
TC_{case1}(Q) &= \int_{L_0}^{L_0 + Q/\bar{D}} \left( S + \frac{Q^2}{2p} + (t - L_0)Q \right) h_s f(t) dt \\
&+ \int_{L_0}^{L_0 + Q/\bar{D}} \left( A + \frac{t - L_0}{2} \bar{D} \pi + h_b (t - L_0)^2 \right) f(t) dt \\
&+ R_2 g_b f(t) dt
\end{align*}$$

where

$$R_2 = \frac{w + (n(n-1) - w(w+1))}{2\bar{D}} \alpha$$

with $w = (t - L_0)/\alpha$.

Case 3: $L_0 + Q/\bar{D} < t < \infty$

Late return shipments, where the backorder quantity equals the lot size $Q$. Given that the returned RTIs arrive at the supplier with $L_0 + Q/D < t < \infty$. The expected total cost per cycle of case 1 is given by:

$$\begin{align*}
TC_{case3}(Q) &= \int_{L_0 + Q/\bar{D}}^{\infty} \left( S + A \right) f(t) dt \\
&+ \int_{L_0 + Q/\bar{D}}^{\infty} \left( \frac{Q^2}{2p} h_s \frac{\pi}{\bar{D}} - L_0 \bar{Q} (h_s + \pi) \right) f(t) dt \\
&+ \int_{L_0 + Q/\bar{D}}^{\infty} (h_s + \pi) Q t f(t) dt
\end{align*}$$

After summing up case 1 to case 3, the expected total cost per unit of time can be expressed by:

$$\begin{align*}
ETC(Q) &= \frac{(S + A)D}{Q} + \int_0^{L_0} \left( k_1(t) - (t - L_0) \frac{g_b}{\alpha} \right) f(t) dt \\
&+ \int_{L_0}^{L_0 + Q/\bar{D}} \left( D(b - h_s(t - L_0) + k_1(Q) \\
&+ L_0(L_0 - 1) \frac{D^2}{\alpha} g_b \right) f(t) dt \\
&+ \frac{\bar{D}}{2Q} \left( \pi + h_b \frac{g_b}{\pi} \int_{L_0 + Q/\bar{D}}^{\infty} (t - L_0)^2 f(t) dt \\
&+ \frac{g_b}{\pi} \int_{L_0}^{L_0 + Q/\bar{D}} (t - L_0) f(t) dt \\
&+ \bar{D} \int_{L_0 + Q/\bar{D}}^{\infty} \left( h_s + \pi \right) \left( t - L_0 \right) + \frac{Q h_s}{2p} \frac{\pi}{\bar{D}} \right) f(t) dt
\end{align*}$$

where

$$k_1(Q) = \frac{QD}{2p} h_s \frac{\pi}{\bar{D}} + \frac{Q - \alpha}{2\alpha} g_b$$

By substituting $na$ for $Q$, the expected total cost of per unit of time can be rearranged by:

$$\begin{align*}
ETC(n) &= \frac{(S + A)D}{na} + \int_0^{L_0} \left( k_2(n) - (t - L_0) \frac{g_b}{\alpha} \right) f(t) dt \\
&+ \int_{L_0}^{L_0 + na/\bar{D}} \left( \bar{D} (h_s - h_b) (t - L_0) + k_2(n) \\
&+ L_0(L_0 - 1) \frac{D^2}{\alpha} g_b \right) f(t) dt
\end{align*}$$

3.4 Solving Procedure

The second order partial derivative of the expected total cost function with respect to $n$ is positive and the function is consequently convex which proves that there is a minimum to the solution.

$$\frac{\partial ETC(n)}{\partial n^2} = 2(S + A)\frac{D}{n^3} + ((n + 1)\lambda - 1) \frac{g_b}{nD} e^{-\frac{na}{D}} > 0$$
\[ \int f(t) \, dt + g = -\alpha(\pi + h) + g + h_{\frac{\pi}{2}} - \pi \int e^{-(na/\beta t)} \]

For the case of an exponentially distributed lead time, i.e., 
\[ f(t) = e^{-at} \]  
with \( a = 1/L_0 \), an optimal solution for \( n \) can be reformulated by:

\[ n^* \]

\[ \int f(t) \, dt + g = -\alpha(\pi + h) + g + h_{\frac{\pi}{2}} - \pi \int e^{-(na/\beta t)} \]

where 
\[ \bar{D} = \frac{1}{4}[(D - \Delta_1) + 2D + (D - \Delta_2)] = D + \frac{1}{4}(\Delta_2 - \Delta_1) \]

Subsequently, an optimal solution for \( n \) can be found by increasing \( n \) stepwise from 1 until ETC increases for the first time. An optimal solution for \( n^* \) has to satisfy the following condition:

\[ \text{ETC}(n^* - 1) \geq \text{ETC}(n^*) \leq \text{ETC}(n^* + 1) \]

IV. NUMERICAL EXAMPLE

We adopt the data of Kim, Glock, and Kwon (2014) to illustrate the results of our proposed models. For the case study considered here:

The return time of RTIs takes, on average, 0.0023 years (\( L_0 = 0.0023 \) years). The RTI capacity, \( a \), is 500 kg. Inventory holding costs for finished products and RTIs at the supplier’s and buyer’s side are \( h_s = 0.0045/\text{kg} \), \( h_b = 0.005/\text{kg} \), \( g_s = g_b = $13.6 \) per RTI. The other model parameters are as follows: \( D = 3,400,000 \) kg per year, \( P = 4,533,000 \) kg per year, \( S = $2970 \) per setup, \( A = $1562 \) per order and \( \pi = $2.5 \) per unit short. In addition, we need multiple \( \Delta_1, \Delta_2 \) to obtain the optimal solution, and that \( \Delta_1, \Delta_2 \) were determined by the decision maker to solve problems under the uncertain environments. Table III illustrates all of the results.

<table>
<thead>
<tr>
<th>( \Delta_1 )</th>
<th>( \Delta_2 )</th>
<th>( \bar{D} )</th>
<th>( n )</th>
<th>ETC(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250000</td>
<td>500000</td>
<td>3462500</td>
<td>1854</td>
<td>33294.8</td>
</tr>
<tr>
<td>500000</td>
<td>1000000</td>
<td>3522500</td>
<td>1854</td>
<td>33622.2</td>
</tr>
<tr>
<td>750000</td>
<td>1500000</td>
<td>3587500</td>
<td>1854</td>
<td>33950.1</td>
</tr>
<tr>
<td>1000000</td>
<td>2000000</td>
<td>3650000</td>
<td>1854</td>
<td>34276.3</td>
</tr>
<tr>
<td>1250000</td>
<td>2500000</td>
<td>3712500</td>
<td>1854</td>
<td>34603.9</td>
</tr>
<tr>
<td>1500000</td>
<td>3000000</td>
<td>3775000</td>
<td>1854</td>
<td>34758.7</td>
</tr>
<tr>
<td>2000000</td>
<td>1000000</td>
<td>3400000</td>
<td>1854</td>
<td>32966.6</td>
</tr>
<tr>
<td>2500000</td>
<td>1250000</td>
<td>3087500</td>
<td>1854</td>
<td>31325.8</td>
</tr>
<tr>
<td>3000000</td>
<td>1000000</td>
<td>3150000</td>
<td>1854</td>
<td>31654.1</td>
</tr>
<tr>
<td>1500000</td>
<td>750000</td>
<td>3212500</td>
<td>1854</td>
<td>31983.2</td>
</tr>
<tr>
<td>1000000</td>
<td>500000</td>
<td>3275000</td>
<td>1854</td>
<td>32311.1</td>
</tr>
<tr>
<td>500000</td>
<td>2500000</td>
<td>3337500</td>
<td>1854</td>
<td>32639.6</td>
</tr>
</tbody>
</table>

**Fig.1.** The distribution relationship diagram of ETC and \( \Delta_2 - \Delta_1 \)

(1) When \( \Delta_1 < \Delta_2 \), then \( d(\bar{D}, \bar{D}_1) > D \); If the variation between \( \Delta_1 \) and \( \Delta_2 \) is smaller in this fuzzy model, the correlated variation between the fuzzy model and the traditional model will also be smaller.

(2) When \( \Delta_1 = \Delta_2 = 30000 \), then \( d(\bar{D}, \bar{D}_1) = D = 3400000 \). In this case, the optimal solution of this fuzzy model will be the same as the solution of the
traditional model.

(3) According to the Fig.1, the minimum of expected total cost equals 31325.8$ which can be obtained while $\Delta_1 = 2500000$, $\Delta_2 = 1250000$.

V. CONCLUSION

In traditional integrated inventory model, we assumed that demand rate for finished products at the buyer can be obtained by historical data. However, product life time is getting shorter in today’s competitive supply chain environment and products are substituted by another one within a very short time. Thus, we programmed a fuzzy demand integrated inventory model which is commonly used by general decision maker and used streamlined distance method to defuzzify expected total cost.

In addition, we assumed a stochastic return lead time in our inventory model, but did not assume that the return quantity of RTIs is stochastic in practical terms. For instance, RTIs may be confronted with lose or damage that need to be repaired during shipping and not high-quality anymore so the supplier may consider the way of renting to avoid shortage of RTIs. Moreover, the production procedure of the supplier may be restricted to some stochastic factors such as deterioration situation of agricultural products or stochastic demand for the customer. Considering such scenarios and including the situation mentioned above in our model would lead to an even more realistic result of supply chain. These extensions are worth studying for future research.

REFERENCE