Shilnikov's Homoclinic Loops in Attitude Dynamics of CubeSAT-3U Nanosatellites with One Movable Unit

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***Abstract*—In the paper the nonlinear dynamics of the attitude motion of the CubeSAT-3U nanosatellites with one movable unit. In this dynamics the well-known dynamical objects as Shilnikov's loops are identified. It is quite interesting nonlinear phenomenon, which can be applied to solving task of chaotic reorientation of the spacecraft.**

***Index Terms*—Spacecraft, satellite, attitude, homoclinic loop**

I. Introduction

THE CubeSAT-3U is one of the most common classes of nanosatellites. The research of the attitude dynamics of such class of satellites and design of attitude control systems are important engineering and scientific tasks. Especially, the complexity of these tasks solution is connected with the requirement of the fundamental simplicity of the design and control system structure. It means that actuators must have simple principle of functionality and simple ways to the implementation of the control laws.

In this research, we consider the scheme of the attitude control, using the displacements one of three units to change the geometrical position of the mass center of the satellite, and the constant jet-engine force, which does not change its direction and magnitude. So, the jet-engine is placed into the main two units of the satellite along the longitudinal direction, and the movable position of the mass center creates the torque, which rotates the satellite.

The displacements of movable unit are created with the help of flexible cables (fig.1) to change the relative position of the movable unit. These flexible cables slide out and in the main central unit such way, that the point *O* remains its immovable position relative the main central unit (i.e. two corresponding opposite pairs of cables slide in and out on

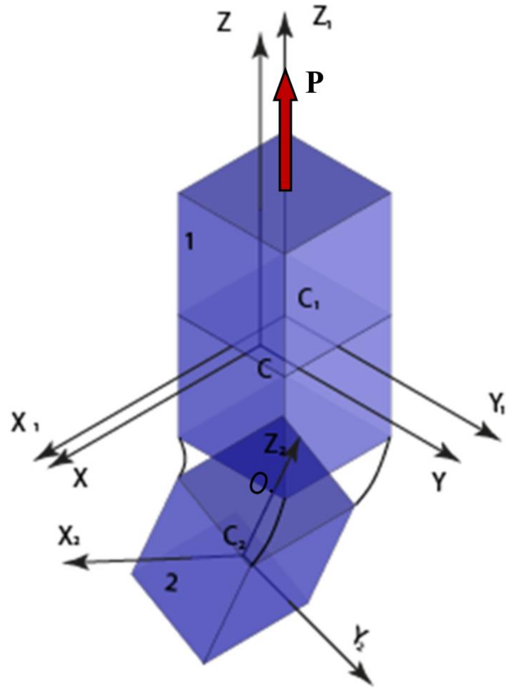

Fig.1. Structure of the CubeSAT-3U with one movable unit on the sliding flexible cables

the same distance, and rotating the movable unit around the point *O*). Such rotations of the movable unit around the point *O* moves the center of mass of the unit C_2, and, therefore, geometrically displaces the mass center of complete satellite C relative the main longitudinal axis, that creates the torque around the C from the constant jet-engine force **P**. In this case, the angular displacement of the movable unit produces the control torque (the arm of the jet-force), and, that is clear, we can control the torque by controlling the angular displacement of the unit.

The satellite in the considered case represents the structure compound from three units: the central cube-units with the main equipment and with the control system, the jet-engine-unit rigidly connected to the central unit and the third movable unit with some functional outfits (e.g., antennas, optical telescopes, radiometers, etc.). This third movable unit is, per se, the actuator of the motion control system, creating the appropriate arm of the jet-force for the control torque.

The motion of mechanical systems with movable parts/masses is important modern problem of the mechanics and space flight dynamics [1-7]. Detecting nonlinear phenomena in the dynamics, including strange attractors,

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homoclinic loops and heteroclinic chaos, in turn is the significant aspect of such research [3-7]. Such phenomena not only have interesting dynamical behavior, but also can be used to solve practical tasks of the space flight dynamics, e.g., they can even be applied to spatial reorientation of spacecraft [6, 7].

In the next sections, we construct the mathematical model of attitude dynamics of the satellite, and fulfill the motion modeling with detecting Shilnikov's homoclinic loops [3-5] in the phase space of the system.

II. Mathematical Model

First of all, let us involve the following coordinates frames (fig.1):

CXYZ – the coordinates system with the origin in the mass center of the complete satellites (all three units) with axes parallel to the axes of inertia of the main part of the satellite (two connected and fixed units);

$C_1X_1Y_1Z_1$ – the coordinates system with the origin in the mass center of the main part of the satellite (two rigidly connected units) with axes coinciding with principal axes of the inertia of the main part;

$C_2X_2Y_2Z_2$ – the coordinates system with the origin in the mass center of the movable unit with axes coinciding with its principal inertia axes;

C'X'Y'Z' – the "immovable" inertial frame.

The mathematical model of the motion can be constructed based on the angular momentum law:

$$\frac{d\mathbf{K}}{dt} = \frac{d\tilde{\mathbf{K}}}{dt} + \boldsymbol{\omega}_1 \times \mathbf{K} = \mathbf{M}\ , \qquad (1)$$

where $\mathbf{K}$ – is the angular momentum of the complete satellite, $\boldsymbol{\omega}_1$ – the vector of the angular velocity of the main part of the satellite, $\mathbf{M}$ – is the torque from the jet-engine and from other external forces.

The angular momentum can be find in the following form:

$$\mathbf{K} = \mathbf{K}_1 + (\boldsymbol{\sigma}_1 \mathbf{K}_2)\ , \qquad (2)$$

where $\mathbf{K_1}$ – the angular momentum of the main part of the satellite, $\mathbf{K_2}$ – the angular momentum of the movable unit, $\boldsymbol{\sigma}_1$ – is the transition matrix from the frame $C_2X_2Y_2Z_2$ into the frame $C_1X_1Y_1Z_1$.

The complete angular momentum of the main part of the satellite is:

$$\mathbf{K}_1 = \mathbf{I}_1 \boldsymbol{\omega}_1 + \mathbf{CC}_1 \times M_1 \mathbf{V}_1\ , \qquad (3)$$

where $\mathbf{I}_1$ – the inertia tensor of the main part jf the satellite, $\mathbf{CC}_1$ – the radius-vector of the mass center of the main part of the satellite in the frame $C_1X_1Y_1Z_1$, M_1 – the mass of the main part of the satellite, $\mathbf{V}_1$ – the linear velocity of the mass center of the main part relative the complete mass center C (calculated in the frame $C_1X_1Y_1Z_1$).

The angular momentum of the movable unit is:

$$\mathbf{K}_2 = \mathbf{I}_2 \boldsymbol{\omega}_2 + \mathbf{CC}_2 \times M_2 \mathbf{V}_2\ , \qquad (4)$$

where $\mathbf{I}_2$ – the inertia tensor of the movable unit, $\boldsymbol{\omega}_2$ – the angular velocity of the movable unit, $\mathbf{CC}_2$ – the radius-vector of the mass-center of the movable unit (calculated in the frame $C_2X_2Y_2Z_2$), M_2 – the mass of the movable unit, $\mathbf{V}_2$ – the linear velocity of the movable unit mass center C_2 relative the mass center of the complete system C (calculated in the frame $C_2X_2Y_2Z_2$).

The indicated linear velocities are:

$$\mathbf{V}_1 = \frac{d\mathbf{CC}_1}{dt} + \boldsymbol{\omega}_1 \times \mathbf{CC}_1\ . \qquad (5)$$

$$\mathbf{V}_2 = \frac{d\mathbf{CC}_2}{dt} + \boldsymbol{\omega}_2 \times \mathbf{CC}_2\ . \qquad (6)$$

The radius-vector of the mass center of complete system (calculated in the frame $C_1X_1Y_1Z_1$):

$$\mathbf{R}_c = \frac{M_1}{(M_1 + M_2)} \mathbf{OC}_1 + \frac{M_2}{(M_1 + M_2)} (\boldsymbol{\sigma}_1 \mathbf{OC}_2) \qquad (7)$$

The radius-vector of the mass center of the main part of the satellite relative the mass center C is (in $C_1X_1Y_1Z_1$):

$$\mathbf{CC}_1 = \mathbf{OC}_1 - \mathbf{R}_c\ . \qquad (8)$$

The radius-vector of te movable unit relative the mass center C is (in $_2X_2Y_2Z_2$):

$$\mathbf{CC}_2 = \mathbf{OC}_2 - (\boldsymbol{\sigma}_2 \mathbf{R}_c) \qquad (9)$$

The angular velocity of the main part of the satellite in projections on the axes $C_1X_1Y_1Z_1$ has the form:

$$\boldsymbol{\omega}_1 = \begin{bmatrix} p_1 \\ q_1 \\ r_1 \end{bmatrix} \qquad (10)$$

The vector of the absolute angular velocity of the movable unit will be compound from the angular velocity of the main part of the satellite and the relative angular velocity:

$$\boldsymbol{\omega}_2 = \begin{bmatrix} p_2 \\ q_2 \\ r_2 \end{bmatrix} = (\boldsymbol{\sigma}_2 \boldsymbol{\omega}_1) + \begin{bmatrix} \dot{\alpha}\cos\gamma\cos\beta + \dot{\beta}\sin\gamma \\ -\dot{\alpha}\cos\beta\sin\gamma + \dot{\beta}\cos\gamma \\ \dot{\alpha}\sin\beta + \dot{\gamma} \end{bmatrix} \qquad (11)$$

where p_2, q_2, r_2 – the components of the absolute angular velocity of the movable unit calculated in the frame $C_2X_2Y_2Z_2$, and α, β, γ – are the angles of rotation of the movable unit relative the main part of the satellite.

The inertia tensor of the main part of the satellite in the frame $C_1X_1Y_1Z_1$ has the form:

$$\mathbf{I}_1 = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & B_1 & 0 \\ 0 & 0 & C_1 \end{bmatrix}, \qquad (12)$$

where A_1, B_1, C_1 – are the main central inertia moments of the main part of the satellite. The inertia tensor of the movable unit in the frame $C_1X_1Y_1Z_1$ has the form:

$$\mathbf{I}_2 = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix}, \qquad (13)$$

where A_2, B_2, C_2 – are the main central inertia moments of the movable unit.

Then fulfilling the differentiations and substituting results into (1), it is possible to write the scalar form of the dynamical equations – the final expressions will be cumbersome and, therefore, do not presented here.

III. Modeling Results

Now we can implement the mathematical modeling of the motion. Let us choice he following control laws for the relative angles of the movable unit:

$$\begin{cases} \alpha = \alpha(p_1, q_1, r_1) = a_1 p_1 + a_2 q_1 + a_3 r_1 \\ \beta = \beta(p_1, q_1, r_1) = b_1 p_1 + b_2 q_1 + b_3 r_1 \\ \gamma \equiv 0 \end{cases} \qquad (14)$$

In case with the control (14) the satellite will realize the interesting attitude dynamics. The phase space of this dynamics can be described by the 3D-space $\{p_1, q_1, r_1\}$ in which the phase trajectories will produce the complex forms of the curves, including the homoclinic loops corresponded to the Shilnikov's attractors [3-6]. All of integration were fulfilled with the following parameters: A_1=0.013, B_1=0.01, C_1=0.005, A_2=0.0025, C_2=0.0025 [kg*m^2]; M_1=3, M_2=2 [kg].

So, let us present the corresponded results in the graphical shape (fig. 2-4).

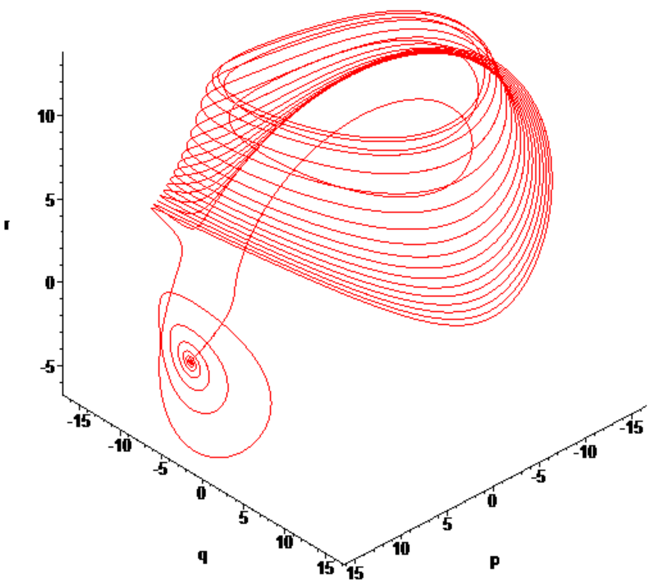

Fig.2. The Shilnikov's homoclinic loop:
a_1=0.008, a_2=-0.011, a_3=0.01;
b_1=0.01, b_2=0.005, b_3=-0.01;
p(0)=11.559 q(0)=-3.77, r(0)=-0.637

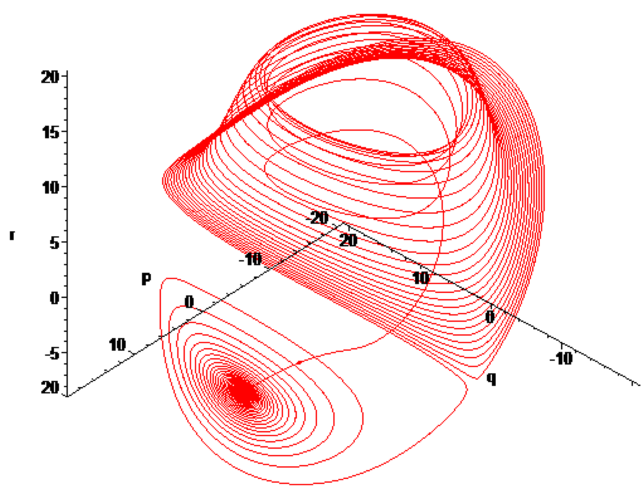

Fig.3. The Shilnikov's homoclinic loop with complex form:
a_1=0.008, a_2=0, a_3=0.01;
b_1=0.009, b_2=0, b_3=-0.01;
p(0)=11.559 q(0)=-3.77, r(0)=-0.637

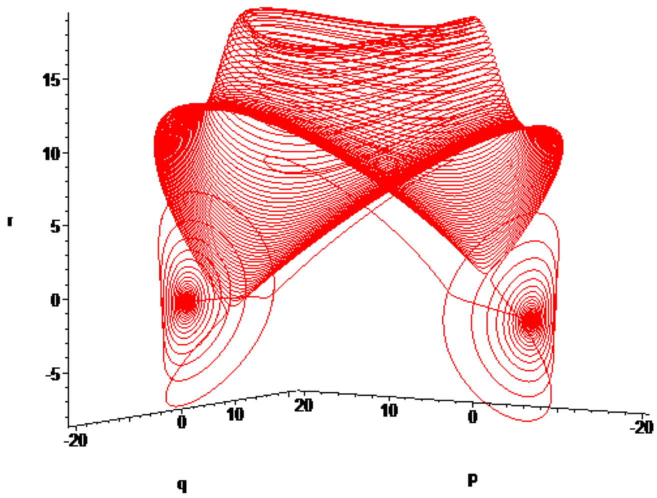

Fig.4. The Shilnikov's homoclinic loop with duplex form:
a_1=0.008, a_2=0, a_3=0;
b_1=0.009, b_2=0, b_3=0;
p(0)=-14.831 q(0)=2.857, r(0)=-0.803

So, the complex homoclinic loops were detected in the phase space of the attitude dynamics of the CubeSAT-3U nanosatellite with one movable unit. It is the important scientific result, which must be considered in details in next works.

IV. Conclusion

The obtained results can be tuck as the new base for the next investigations in the framework of nonlinear attitude dynamics of the compound spacecraft and satellites.

Presented complex loops can be used for the tasks of attitude dynamics, including possible chaotic reorientations of spacecraft, that is the quite interesting method of attitude control.

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