EWMA-NP Control Chart for Time Truncated Pareto Distribution of Second Kind

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***Abstract*—This paper proposes the exponentially weighted moving average (EWMA) control chart for monitoring the number of nonconforming with the truncated lifetime of a product when lifetimes of products are Pareto distribution of the second kind. The performance of the control chart is measure with the behavior of average run length (ARL). The simulation shows that the proposed control chart is much better performs to monitoring shifts in scale and shape parameters.**

***Index Terms*— EWMA, attribute, control chart, average run length, Pareto distribution.**

I. Introduction

Nowaday, quality control in industrial processes is a prominent tool for increasing productivity. That can reduce the amount of waste and various deficiencies that occurred in the production process. Statistical Process Control (SPC) is applied to help in collecting data and fixing those imperfections. The popular tool for control the production process is the control chart. It is a prominent tool for detecting and controlling the production process to ensure high-quality products. These tools also control the specification of production before quality inspections. So the control chart features have been developed to reduce non-conforming products and alternately reduce the cost of the production process. The control charts are divide into two main categories as follows variable and attribute control charts. Variable control charts are used for continuous data such as Shewhart $\bar{X}$ and R control charts. On other hand, Shewhart attribute control charts are using for discrete data, for example, P chart use for monitoring a fraction of defective items, NP control chart used for monitoring the number of non-conforming, C control chart used for counting the number of nonconforming and U control chart used for counting nonconforming per unit [1]. Usually, Shewhart charts are more powerful for detecting large shifts ($\geq 1.5\sigma$) in the process [2]. These charts are not fast enough to detect small changes in the production process. The exponentially weighted moving average (EWMA) control chart is an optimal control chart for detecting small shifts in the manufacturing process, it is developed by Roberts [3]. The EWMA control chart is used in statistical process control to monitor the output of the manufacturing process by tracking the moving average of performance over the lifetime of the manufacturing process. It has been used in various industries especially the chemical industry. Some researchers have studied these control charts, for example, Chen and Cheng [4] designed EWMA control chart for detecting mean and variance in the one control chart, Khan et al. [5] utilized EWMA control chart on fuzzy data, Petcharat [6] proposed EWMA control chart for monitoring process variation. Moreover, the EWMA control chart was utilized for monitoring attribute data such as Eleftheriou and Farmakis [7] proposed a one-sided EWMA control chart for monitoring the fraction of nonconforming products (p) in high yield processes. Sukparungsee and Mititelu [8] proposed EWMA control chart for monitoring the number of nonconforming.

Presently, consumers are demanding high-quality products. Therefore, product life testing often takes a long time to test. Therefore, many researchers have conducted product lifetime truncation studies and developed control charts to detect nonconforming under such a situation. Aslam and Jun [9] proposed an attribute control chart (NP control chart) when the lifetime of a product follows as Weibull distribution. They truncated lifetime based on the number of nonconforming products and can be effective applied to real-life data. Next, Aslam et al. [10] proposed the NP control chart when the product's lifetime distributed Pareto of the second kind under lifetime truncation. They found that the NP control chart can be effectively detecting of nonconforming items in the industrial process. In addition, Arif et al. [11] designed the EWMA control chart for monitoring the number of nonconforming or failure items, which they called the EWMA-NP control chart. They found that the EWMA-NP control chart is better to perform than the NP control chart for monitoring mean changes under lifetime truncation when the product's lifetime follows Weibull distribution. Moreover, Alghamdi et al. [12] developed a moving average control chart for monitoring the number of nonconforming or failure items of products under lifetime truncated Weibull distribution.

The criteria used to compare the performance were average run length (ARL). The ARL is an average number of observations taken from an in-control process until the control chart falsely signals out-of-control is denoted by ARL_0. An ARL_0 can be regarded as acceptable if it is large enough to keep in the level of false alarms at the acceptance level. The expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control denote in ARL_1. By reviewing the

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literature, the researcher is interested in developing the exponentially weighted moving average control chart to monitor nonconforming products for lifetime truncated Pareto distribution of the second kind.

This paper aims to develop the exponentially weighted moving average (EWMA) control chart for lifetime truncation of product follow as Pareto distribution of the second kind. In addition, the proposed control chart be compared with the NP control chart refer to [10]. In Section 2, the present distribution of lifetime follows as the Pareto distribution of the second kind. In section 3 presents the NP control chart. Section 4, designing EWMA-NP control chart of lifetime truncation when product follows Pareto distribution of the second kind. Section 5 discusses the performance of the proposed control chart in the behavior of the average run length (ARL) values when the scale and the shape parameters are shift with different sample sizes. Finally, Section 6 provides a conclusion.

II. Pareto distribution of the second kind

Pareto distribution is a heavy-tailed distribution with many applications in the real world such as survival analysis income and biomedical sciences. The distribution is applied in acceptance sampling plans [10]. The cumulative distribution function (CDF) of Pareto distribution of the second kind is

$$F(t;\alpha,\beta)=1-\left(1+\frac{x}{\alpha}\right)^{-\beta} \quad , x>0, \tag{1}$$

and probability density function (pdf) is

$$f(t;\alpha,\beta)=\frac{\beta}{\alpha}\left(1+\frac{x}{\alpha}\right)^{-(\beta+1)} \quad , x>0, \tag{2}$$

where β is a shape parameter ($\beta > 0$) and α is a scale parameter ($\alpha > 0$).

The mean value of life for the distribution is

$$\mu=\frac{\alpha}{\beta\text{-}1} \ , \beta>1. \tag{3}$$

III. The Np Control chart

Aslam et al. (2016) proposed the np control chart (np chart) for time truncated lift test. The upper control limits (UCL) and lower control limit (LCL) of np control chart as follows:

$$\mathrm{UCL}_{np}=np_0+K\sqrt{np_0(1-p_0)}\ , \tag{4}$$

$$\mathrm{LCL}_{np}=\max[0,np_0-K\sqrt{np_0(1-p_0)}], \tag{5}$$

where p_0 is the fraction of nonconforming before experiment time t_0 when process is in control, n is size of subgroup, K is the width of control limit of control chart.

In real situation, p_0 is usually unknown, therefore p_0 can approximate as follow:

$$\bar{D}=\sum_{i=1}^{n} D_i / n\ ,$$

where D_i is number of defect items

n is number of subgroups

$\bar{D}$ is the average number of defects items over subgroups. Then the control limit of np control chart defined as follow:

$$\mathrm{UCL}_{np}=\bar{D}+K\sqrt{\bar{D}(1-\bar{D}/n)}\ , \tag{6}$$

$$\mathrm{LCL}_{np}=\max\left[0,\bar{D}-k\sqrt{\bar{D}(1-\bar{D}/n)}\right], \tag{7}$$

IV. Design of EWMA-NP Control chart

This paper assumes that the product's lifetime follows the Pareto distribution CDF and pdf defined according to (1) and (2). The proposed EWMA-NP control chart under time truncated lift test for Pareto distribution of the second kind is explained as follow;

First

Random a sample of size n from the production process at each subgroup and put them on time-truncated life test. Count number of defect item (D_i) at subgroup i by time t_0.

The EWMA statistic is

$$EWMA_{D_i}=\lambda D_i+(1-\lambda)EWMA_{D_{i-1}}, \tag{8}$$

where λ is an exponential smoothing parameter ($0<\lambda<1$).

Second

The process is out of control when $EWMA_{D_i}$ > UCL or $EWMA_{D_i}$ < LCL . The process is in control when LCL ≤ $EWMA_{D_i}$ ≤ UCL .

Since p_0 is the fraction of nonconforming before experiment time t_0 when process is in control, such that from (1) p_0 is became

$$p_0=F(t_0,\alpha_0,\beta_0)=1-(1+t_0/\alpha)^{-\beta_0}\ , \tag{9}$$

where $t_0=a\mu_0$, t_0 is the truncation time and multiple of the in-control mean for a specified constant a. Form (3) and $t_0=a\mu_0$ in (5), such that

$$p_0=1-(1+a/(\beta_0-1))^{-\beta_0}\ . \tag{10}$$

Since D_i is binomial distribution with n subgroup and p_0. For the large of i, the process is in control state then the EWMA statistic performs on the normal distribution with mean $=np_0$ and variance $=(\lambda/(2-\lambda))np_0(1-p_0)$ can be rewrite as:

$$EWMA_{D_i} : N\left(np_0,\frac{\lambda}{(2-\lambda)}np_0(1-p_0)\right) \tag{11}$$

The Upper Control Limit of EWMA-NP control chart is defined by the following recursion:

$$UCL_{EWMA-np} = np_0 + L\sqrt{\frac{\lambda}{(2-\lambda)}np_0(1-p_0)}. \quad (12)$$

The Lower Control Limit of EWMA-NP control chart is defined by the following recursion:

$$LCL_{EWMA-np} = \max\left[0, np_0 - L\sqrt{\frac{\lambda}{(2-\lambda)}np_0(1-p_0)}\right]. \quad (13)$$

where λ is an exponential smoothing parameter, $0<\lambda<1$
L is the width of control limit of EWMA-NP control chart.

V. Comparison of Performance of Control Chart

In this section, we provide a comparative discussion of the results obtained in Sections 3 and 4. Moreover, the efficiency of the EWMA-NP control chart is also compared with the NP control chart. First, consider the case in which observations are Pareto distribution of the second kind with lifetime truncated. In a situation the process is in-control state, scale (α) and shape (β) parameters are $\alpha = \alpha_0$ and $\beta = \beta_0$, respectively. The process is out of control state, which means that the process shifts to a new shape or new scale parameters.

In case scale is shifted, the new scale parameter is $\alpha_1 = \delta\alpha_0$, where δ is scale shift size. For example, if $\delta = 1$ then $\alpha_1 = \alpha_0$, it implied that the process is in-control state. Let p_1 be the fraction of nonconforming before experiment time t_0, corresponding to (9), p_1 defined as

$$p_1 = F(t_0, \alpha_1, \beta_0) = 1-(1+t_0/\delta\alpha)^{-\beta_0}$$

$$= 1-(1+a/(\delta(\beta_0 - 1))^{-\beta_0} \quad (14)$$

In case shape is shifted, the new scale parameter is $\beta_1 = \tau\beta_0$, where τ is shape shift size. For example, if $\tau = 1$ then $\beta_1 = \beta_0$, it implied that the process is in control state. Let p_2 be the fraction of nonconforming before experiment time t_0, corresponding to (9), p_2 defined as

$$p_2 = F(t_0, \alpha_0, \beta_1) = 1-(1+t_0/\alpha)^{-\tau\beta_0}$$

$$= 1-(1+a/(\beta_0 - 1)^{-\tau\beta_0} \quad (15)$$

Usually, the specified value of in-control average run length (ARL_0) is equal to 370, Parameters of the control chart are determined based on the Monte Carlo simulations technique. The average run length was estimated by averaging the number of a simulation study (M). The ARL_1 values are discovered by different shifts size.

Consider width of control limit K and L from (4), (5) and (12), (13) will be set on the target values ARL_0=370. The algorithm is used to determine K and L as follow:

1. Setting values of sample size (n), scale (α), shape (β) and time truncate (a).

2. Determine K and L of each control chart until ARL_0 get nearly target value.

3. Define shift size δ or τ of scale and shape to obtain ARL_1 by using K and L from step 2.

The Run Length is given by

$$RL_l = \begin{cases} 1\ ;\ LCL_l \le C \le UCL_l \\ 0\ ;\ Otherwise. \end{cases}, \quad (16)$$

where l = 1, 2, … , M.

Therefore, the Average Run Length = $\dfrac{\sum_{l=1}^{M} RL_l}{M}$,

where LCL_l is the Lower Control Limit of the chart

UCL_l is the Upper Control Limit of the chart

M is the number of the simulation study.

We consider the situation of the sample sizes (n) are equal 10 and 20. The summarized information of these control charts are provided in Table I to Table III.

TABLE I

Comparison of ARLs values for Pareto(2,1) between EWMA-NP and NP control chart with a = 0.3 when parameter scale shifted

n	shift (δ)	EWMA-NP λ=0.05 L=2.98885	EWMA-NP λ=0.10 L=2.9407	NP K=3.158
10	1.00	370.1239	368.701	369.5259
	0.9	**214.2585**	222.4118	238.2674
	0.8	**88.08352**	78.26667	141.5887
	0.7	**53.95056**	38.44897	77.18100
	0.6	**38.70696**	25.06463	36.74613
	0.5	20.60327	18.50321	**17.91073**
	0.4	17.04185	14.46062	**6.666664**
	0.3	20.60327	11.63359	**2.493118**
	0.2	17.04185	9.55113	**0.448538**
	0.1	14.5009	8.07172	**0.016756**
n	shift (δ)	λ=0.05 L=2.8209	λ=0.10 L=2.8999	K=3.025
20	1.00	369.2237	368.7595	369.3604
	0.9	**150.0820**	156.6615	210.1415
	0.8	**65.06128**	49.28056	93.94336
	0.7	45.66077	**28.61538**	31.38582
	0.6	35.1023	**20.40135**	13.05147
	0.5	28.02276	15.79773	**4.451588**
	0.4	22.6057	12.69912	**1.555152**
	0.3	18.83135	10.41363	**0.208284**
	0.2	15.80874	8.6632	**0.00803**
	0.1	13.61081	7.8329	**2×10^{-6}**

From Table 1, the EWMA-NP control chart is better than NP control chart for detecting change in scale parameter $0.6 \le \delta \le 0.9$. On the other hand, the NP control chart is better than the EWMA-NP control chart for detecting scale parameter $0.1 \le \delta \le 0.5$.

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TABLE II
COMPARISON OF ARLS VALUES FOR PARETO (3, 1) BETWEEN EWMA-NP AND NP CONTROL CHART WITH $a = 0.6$ WHEN PARAMETER SCALE SHIFTED

n	shift (δ)	EWMA-NP		NP
		λ=0.05 L=2.701	λ=0.10 L=2.9302	K=2.8901
10	1.00	371.9335	369.5507	371.2848
	0.9	**183.9437**	206.968	218.0323
	0.8	79.8306	**68.73215**	125.8108
	0.7	53.14312	**36.32338**	60.07719
	0.6	40.85675	**25.21784**	32.92172
	0.5	33.2834	19.55624	**14.56436**
	0.4	28.00572	16.01008	**6.067725**
	0.3	24.11381	13.56164	**2.039305**
	0.2	21.28785	11.85287	**0.440269**
	0.1	19.47676	11.0133	**0.024664**
n	shift (δ)	λ=0.05 L=2.650	λ=0.10 L=2.961	K=3.1885
20	1.00	371.4185	370.9613	371.55158
	0.9	**128.1798**	137.1258	274.6423
	0.8	64.09054	**45.09038**	162.0041
	0.7	46.25628	**28.04507**	67.572
	0.6	36.80586	**21.08466**	23.9959
	0.5	30.5633	17.02422	**8.18214**
	0.4	26.02386	14.27276	**2.22316**
	0.3	22.59149	12.27919	**0.40216**
	0.2	20.05951	10.87695	**0.01966**
	0.1	18.36778	10.00282	**2×10^{-5}**

From Table II, the EWMA-NP control chart performs better than the NP control chart for detecting change in scale parameter $0.6 \le \delta \le 0.9$. On the other hand, the NP control chart is better perform than the EWMA-NP control chart for detecting change in scale parameter $0.1 \le \delta \le 0.5$.

TABLE III

COMPARISON OF ARLS VALUES FOR PARETO (3, 2) BETWEEN EWMA-NP AND NP CONTROL CHART WITH $a = 0.3$ WHEN PARAMETER SHAPE SHIFTED

n	shift (τ)	EWMA-NP		NP
		λ=0.05 L=2.888	λ=0.10 L=3.183	K=3.113
10	1.00	368.1285	370.9935	368.4833
	0.9	**196.3727**	239.1479	335.3097
	0.8	79.13775	**71.32761**	272.1299
	0.7	47.64854	**34.11735**	186.8683
	0.6	33.63487	**22.08365**	80.97324
	0.5	25.98492	**15.59376**	31.8709
	0.4	20.30933	**11.90421**	9.88373
	0.3	15.9099	9.19771	**2.245404**
	0.2	12.84015	7.37167	**0.190234**
	0.1	11.01811	6.10925	**9.2e-05**
n	Shift (δ)	λ=0.05 L=2.802	λ=0.10 L=3.253	K=3.372
20	1.00	372.1886	370.2295	372.6863
	0.9	**137.0195**	189.2941	346.0238
	0.8	59.20698	**45.51509**	275.6027
	0.7	41.03132	**25.57432**	140.5696
	0.6	29.64567	**18.18731**	42.88544
	0.5	23.18072	13.39078	**12.28446**
	0.4	18.36923	10.46704	**2.46309**
	0.3	14.70774	8.32533	**0.21028**
	0.2	11.98564	6.77928	**0.000856**
	0.1	10.07567	6.0000	**0**

From Table III, it found that for n=10, EWMA-NP control charts performs better than the NP control chart for detecting change in shape parameter $0.4 \le \tau \le 0.9$ and the NP control chart is better than EWMA-NP control chart for detecting change in scale parameter $0.1 \le \tau \le 0.3$. For n=20, the EWMA-NP control chart is better than the NP control chart for detecting change in shape parameter $0.6 \le \tau \le 0.9$ and NP control charts is better than the EWMA-NP control chart for detecting change in scale parameter $0.1 \le \tau \le 0.5$.

VI. CONCLUSION

This paper proposed the EWMA-NP control chart based on the lifetime truncated when the product's lifetime follows Pareto distribution of the second kind. The proposed control chart using to monitor scale or shape parameters of the distribution is shifting. If the scale or shape parameters of the distribution are shift, it can imply that the mean of the distribution is changing as decreasing shift size behavior. To summarize that, the performance of the proposed control chart indicates that the EWMA-NP control chart is better performs to detect small changes in shift size of scale and shape parameters than NP control charts. On the other hand, the NP control chart is superior for detecting large shift sizes of scale and shape parameters.

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