Redistributions of Relative Angular Momentum in Multi-rotor Systems of Multi-spin Spacecraft

Anton V. Doroshin

***Abstract*— Attitude dynamics of multi-spin spacecraft is considered in cases of redistributions of angular momentum between a main body and axial rotors of this multi-rotor mechanical system. Direct connections of rotors are described, which make it possible to instantly ensure the rotation of opposite axial rotors in different directions with equal angular velocities. This mechanical scheme of direct connection of opposite rotors uses auxiliary gears and provides a nonholonomic mechanical constraints. The indicated schemes of direct axial connection of rotors can be used for instant redistribution of the relative angular momentum between the bodies of spacecraft, and can also be used for instant change of the motion mode, for example, for instant exit from the arisen chaotic regimes of attitude dynamics.**

***Index Terms*—Multi-rotor system, multi-spin spacecraft, chaotic regimes, relative angular momentum, nonholonomic mechanics**

I. INTRODUCTION

THIS work deals with a problem of redistribution of relative angular momentum of multi-spin spacecraft between its internal rotors. The mechanical schemes of multi-spin spacecraft were indicated in classical works, e.g. [1]. An importance of such multi-rotors scheme can be defined by the practical tasks of attitude dynamics of spacecraft, when it is needed to immediately change the parameters of the angular motion [2, 3] and to instantly switch to another dynamical mode. Here we can indicate the problem the problem of leaving the arisen chaotic regime [4] of the angular motion of spacecraft.

To partially solve this problem in this paper the mechanical scheme of the direct connections of rotors of multi-spin spacecraft is suggested. The considered here direct connections can be implemented with the help of auxiliary gears, that provides a nonholonomic mechanical constraints arising.

So, the considered problem is quite important from the scientific point of view in aspects of investigation of nonholonomic angular motion of multi-rotor rigid bodies systems around the center mass. In addition, the results of the research can be applied into practice of space flight dynamics in the framework of multi-spin spacecraft attitude dynamics and control.

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II. MECHANICAL AND MATHEMATICAL MODELS

Let us consider the angular motion of the multi-rotor rigid bodies system depicted at the figure 1. This mechanical scheme correspond to the structure of the multi-spin spacecraft and it contains axial pairs of opposite rotors ({1 and 2}, {3 and 4}, {5 and 6}).

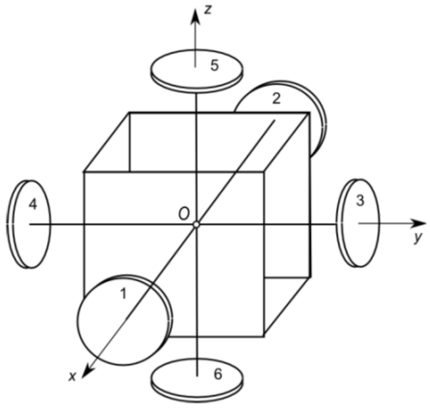

Fig.1. The mechanical model of the multi-spin spacecraft.

Each rotor can have its own independent dynamics. In addition, in this work the possibility of instantaneous direct connection of opposite rotors is simulated, when their subsequent relative rotations are fulfilled in opposite directions with equal values of relative angular velocities and relative angular momentums. This direct connection can be implemented by small auxiliary intermediate gears, as it depicted at the figure 2.

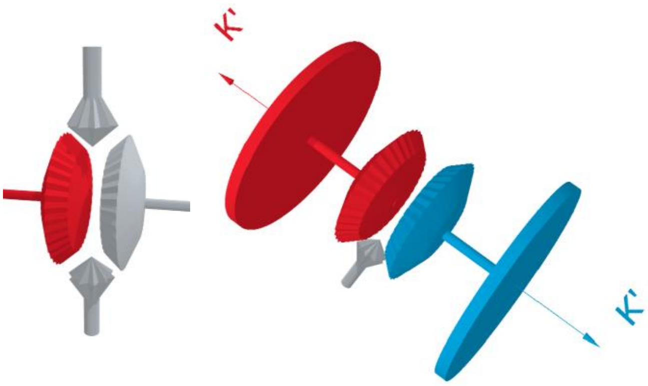

Fig.2. The scheme of the direct connection of axial opposite rotors with the help of small auxiliary intermediate gears.

This system has six general rotors on the main axial directions along the axes x, y, z (fig.1), which coincide with the principle axes of the spacecraft main body. Let us consider the case of dynamics with equal rotors parameters. The angular momentum of the multi-rotor mechanical system written in the frame Oxyz has the form:

$$\mathbf{K} = \mathbf{K}_m + \mathbf{K}_r;$$
$$\mathbf{K}_m = \begin{bmatrix} Ap \\ Bq \\ Cr \end{bmatrix}; \quad \mathbf{K}_r = I \begin{bmatrix} \sigma_1 + \sigma_2 \\ \sigma_3 + \sigma_4 \\ \sigma_5 + \sigma_6 \end{bmatrix}; \tag{1}$$

The vector $\mathbf{K}_m$ corresponds to the angular momentum of the mechanical system with all rotors fixed relative the main body; $\mathbf{K}_r$ - is the relative angular momentum of rotors; p, q, r – are components of the angular velocity of the main body σ_k - are relative angular velocity of rotors relatively the main body; $A = \tilde{A} + 4J + 2I$, $B = \tilde{B} + 4J + 2I$, $C = \tilde{C} + 4J + 2I$; $\{\tilde{A}, \tilde{B}, \tilde{C}\}$ - the general inertia moments of the spacecraft main body; I and J - the longitudinal and the equatorial inertia moments of the rotor calculated relatively the point O (and we will consider the case when I and J are equal for all six rotors). The equations of the angular motion of the system motion in the connected frame Oxyz (O is the center of mass of the complete system) is:

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}^e \tag{2}$$

The scalar form of the equation (2) is:

$$\begin{cases} A\dot{p} + I_l\left\{\dot{\sigma}_1 + \dot{\sigma}_2\right\} + (C - B)qr + \\ \qquad + I\left[\left\{\sigma_5 + \sigma_6\right\}q - \left\{\sigma_3 + \sigma_4\right\}r\right] = M_x^e; \\ B\dot{q} + I\left\{\dot{\sigma}_3 + \dot{\sigma}_4\right\} + (A - C)pr + \\ \qquad + I\left[\left\{\sigma_1 + \sigma_2\right\}r - \left\{\sigma_5 + \sigma_6\right\}p\right] = M_y^e; \\ C\dot{r} + I\left\{\dot{\sigma}_{5l} + \dot{\sigma}_{6l}\right\} + (B - A)qp + \\ \qquad + I\left[\left\{\sigma_3 + \sigma_4\right\}p - \left\{\sigma_1 + \sigma_2\right\}q\right] = M_z^e \end{cases} \tag{3}$$

The equations of relative rotations of rotors are also can be build from the law of the angular momentum changing:

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_1\right) = M_1^i + M_{1x}^e; \\ I\left(\dot{q} + \dot{\sigma}_3\right) = M_3^i + M_{3y}^e; \\ I\left(\dot{r} + \dot{\sigma}_5\right) = M_5^i + M_{5z}^e; \end{cases} \tag{4}$$

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_2\right) = M_2^i + M_{2x}^e; \\ I\left(\dot{q} + \dot{\sigma}_4\right) = M_4^i + M_{4y}^e; \\ I\left(\dot{r} + \dot{\sigma}_6\right) = M_6^i + M_{6z}^e \end{cases} \tag{5}$$

here M_j^i are internal torques acting on the rotor #j, and $M_{j\{x,y,z\}}^e$ - are the external torques. Systems Equations (3)-(5) describe the attitude dynamics of the multi-spin spacecraft, when each rotor is independent and has its own degree of freedom without any constraints.

Let us consider the possibility of the direct connection of opposite rotors ## 1 and 2. In this case, the following kinematical constraint will be actual:

$$\sigma_2 = -\sigma_1 \tag{6}$$

As we can see, at the constraint (6) fulfilling, the sum of relative angular velocities and accelerations of rotors 1 and 2 are equal to zero:

$$\left(\sigma_1 + \sigma_2\right) \to 0; \qquad \left(\dot{\sigma}_1 + \dot{\sigma}_2\right) \to 0\,. \tag{7}$$

The conditions (7) represent the nonholonomic constraints. And there is important to give short explanations about nonholonomic dynamics.

III. The Nonholonomic Dynamics

It is possible to briefly give the main approach of the nonholonomic system modeling [5-8]. Let the following nonholonomic constraints are actual for the mechanical system:

$$\begin{gathered} \sum_{j=1}^{m} b_{\beta j}\left(q_1, \dots, q_m, t\right)\dot{q}_j + b_\beta\left(q_1, \dots, q_m, t\right) = 0, \\ \beta = 1, 2, \dots, s \end{gathered} \tag{8}$$

where q_j – are the generalized coordinates of our system (m – is quantity of degrees of freedom). The equations of the system can be written on the base of the Lagrange formalism:

$$\sum_{j=1}^{m}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j\right)\delta q_j = 0 \tag{9}$$

The coordinates' variations δq_j due to (8) will be connected by the s independent expressions:

$$\sum_{j=1}^{m} b_{\beta j}\left(q_1, \dots, q_m, t\right)\delta q_j = 0 \tag{10}$$

We can subtract from (9) corresponding expressions (10). which are multiplied by multipliers λ_β (so called "indefinite Lagrangian multipliers"):

$$\sum_{j=1}^{m}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j - \sum_{\beta=1}^{s}\lambda_\beta b_{\beta j}\right)\delta q_j = 0 \tag{11}$$

Then due to independences of variations $\delta q_1, \dots, \delta q_n$, we can to write the equations:

$$\begin{gathered} \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j - \sum_{\beta=1}^{s}\lambda_\beta b_{\beta j}\left(q_1, \dots, q_m, t\right); \\ j = 1, \dots, m \end{gathered} \tag{12}$$

Equations (12) with constraints (8) allow us to completely investigate the nonholonomic system dynamics and find all of the multipliers λ_β.

In our case with one constraint (6) we can write the expression (8) in the form:

$$b_{11}\sigma_1 + b_{12}\sigma_2 = 0, \quad b_{11} = b_{12} = 1 \tag{13}$$

After differentiation we have:

$$b_{11}\dot{\sigma}_1 + b_{12}\dot{\sigma}_2 = 0 \tag{14}$$

Nonholonomic dynamical equations (4) and (5) for relative rotations of rotors at the constraints (13) take the shape:

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_1\right) = M_1^i + M_{1x}^e + b_{11}\lambda; \\ I\left(\dot{q} + \dot{\sigma}_3\right) = M_3^i + M_{3y}^e; \\ I\left(\dot{r} + \dot{\sigma}_5\right) = M_5^i + M_{5z}^e; \end{cases} \tag{15}$$

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_2\right) = M_2^i + M_{2x}^e + b_{12}\lambda; \\ I\left(\dot{q} + \dot{\sigma}_4\right) = M_4^i + M_{4y}^e; \\ I\left(\dot{r} + \dot{\sigma}_6\right) = M_6^i + M_{6z}^e; \end{cases} \tag{16}$$

From systems (15), (16) and (14) we can find the Lagrangian multipliers:

$$\lambda = \frac{I\dot{p}\left(b_{11}+b_{12}\right)-b_{11}\left[M_1^i+M_{1x}^e\right]-b_{12}\left[M_2^i+M_{2x}^e\right]}{b_{11}^2+b_{12}^2} \quad (17)$$

So, now we can to investigate the nonholonomic dynamics of the multi-rotor system with the direct connection of two rotors (##1 and 2).

IV. The Numerical Modeling

In this section we try to give some illustrations of step-wise dynamics of the angular motion of our multi-spin spacecraft though the tree intervals: (I) starting without constraint (13), (II) moving to the internal active constraint (13), and (III) again go to the interval with disabled constraint.

Let us to consider the following conditions for rotors dynamics of the indicated intervals I-III:

I). The internal torques correspond to free rotations of rotors ##1 and 3, to the small harmonically perturbed rotor #5, and to fixed relative the main body rotors ## 2, 4, 6 (when the we formally use the torque of a liquid friction with large value of a medium resistance ν), at all zero values of the external torques:

$$\begin{cases} t\in\left(-\infty,0\right): \\ M_{j\{x,y,z\}}^e=0; \\ M_1^i=0;\;\; M_3^i=0;\;\; M_5^i=\varepsilon\sin\left(\eta t\right); \\ M_2^i=-\nu\sigma_2;\;\; M_4^i=-\nu\sigma_4;\;\; M_6^i=-\nu\sigma_4; \end{cases}$$

II). The rotors ##1 and 2 are directly connected with constraint (13) without any other torques, but at the small harmonically perturbed rotor #5:

$$\begin{cases} t\in\left[0,T\right]: \\ M_{j\{x,y,z\}}^e=0; \\ M_1^i=0;\;\; M_3^i=0;\;\; M_5^i=\varepsilon\sin\left(\eta t\right); \\ M_2^i=0;\;\; M_4^i=0;\;\; M_6^i=0; \end{cases}$$

III). The rotors ##1-4 and 6 are free, the constraint (13) is disabled, still at the small harmonically perturbed rotor #5:

$$\begin{cases} t\in\left(T,\infty\right): \\ M_{j\{x,y,z\}}^e=0; \\ M_1^i=0;\;\; M_3^i=0;\;\; M_5^i=\varepsilon\sin\left(\eta t\right); \\ M_2^i=0;\;\; M_4^i=0;\;\; M_6^i=0; \end{cases}$$

In other words, the system goes through successive stages without the constraint, with the active constraint, and again without the constraint.

The following parameters for numerical modeling are used: A=5, B=6, C=7, I=0.03 [kg*m^2]; ε=0.05 [N*m]; η=0.1 [1/s]; ν=1000 [N*m*s], T=200 [s].

The "initial" conditions correspond to time t=0 [s], but here we need take into account the some impact phenomena, when the constraint arising: the conditions before and after enabling constraint must match each other by the angular momentum and kinetic energy value. Therefore, the initial condition can be defined at the time t=0-δ (δ is infinitesimal), and after enabling constraint (t=0+δ) these "initial" conditions are recalculated from the conservation of axial relative angular momentum and corresponded kinetic energy due to some impact at the initiation of constraint. This aspect defines the dynamics "in the past" and "in the future" by the time axis. So, in our research the following conditions for modeling:

1) before activation of constraint (13): p(0-)=-0.13, q(0-)=0.25, r(0-)=0.353, σ_1(0-)=18.44, σ_2(0-)=0, σ_3(0-)=1, σ_4(0-)=0, σ_5(0-)=6, σ_6(0-)=0 [1/s]:

2) after activation of constraint (13): p(0+)=0.02, q(0+)=0.25, r(0+)=0.353, σ_1(0+)=- σ_2(0+)=13, σ_3(0+)=1, σ_4(0+)=0, σ_5(0+)=6, σ_6(0+)=0 [1/s].

Moreover, in our research the indicated above initial conditions were chosen in a special way to show the implementation of chaotic dynamics before the constraint (13) was turned on, as well as to demonstrate the fact of getting out of chaos when the constraint was turned off and always further. So, in the considered case point of time t=0 defines the border of switching on constraint (13) and exiting from dynamical chaos.

The dynamical chaos, which we see before t=0, corresponds to heteroclinic type of chaos – it can arise when the phase point of the system came in the neighborhood of the separatrix region in the phase space, and when a small perturbation (in our case it is the internal torque $M_5^i=\varepsilon\sin\left(\eta t\right)$) is active [4]. Thus, we not only demonstrate the dynamics in the process of switching on the limitation, but also show the possibility of getting out of chaos in this way.

The modeling results are depicted at the figures 3-5.

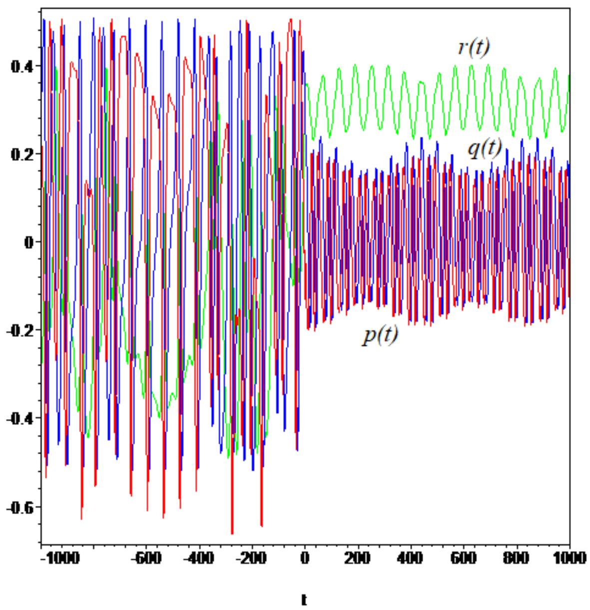

Fig.3. The time-history of the angular velocity components (in electronic version: p – red, q – blue, r – green)

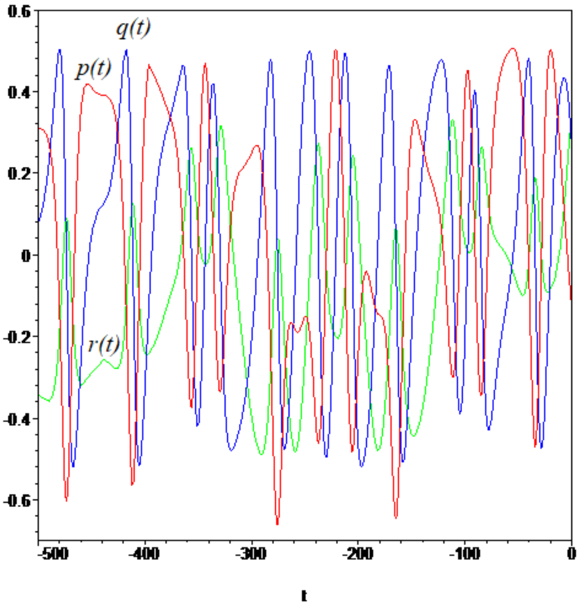

Fig.4. The time-history of the angular velocity components at the realization of the heteroclinic chaos
(in electronic version: p – red, q – blue, r – green)

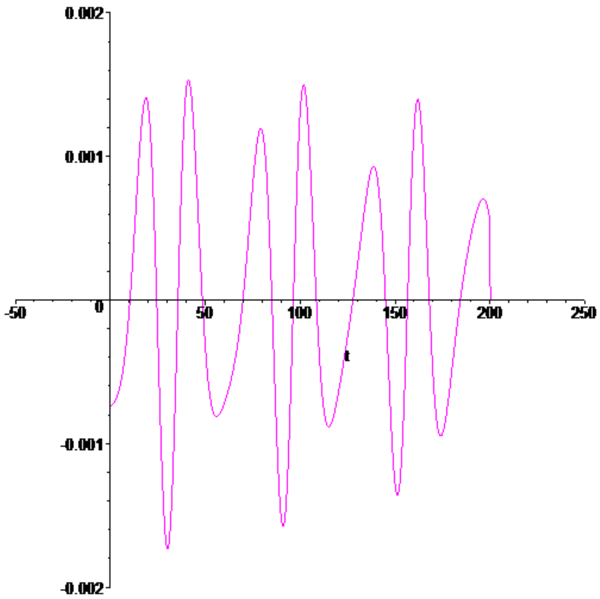

Fig.5. The time-history of the Lagrangian multiplier λ(t)

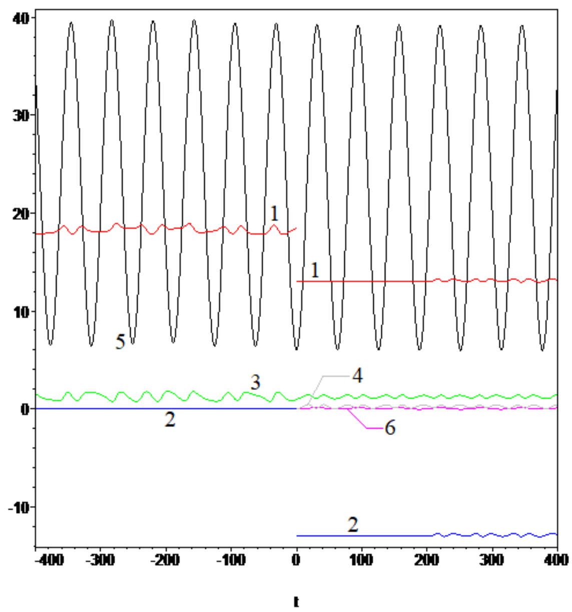

Fig.5. The time-history of the relative angular velocities of rotors: $\sigma_1(t)$ – red (1), $\sigma_2(t)$ – blue (2), $\sigma_3(t)$ – green (3), $\sigma_4(t)$ – gray (4), $\sigma_5(t)$ – black (5), $\sigma_6(t)$ – magenta (6).

V. Conclusion

The attitude dynamics of multi-spin spacecraft was considered in cases of redistributions of angular momentum between the main body and axial rotors of this multi-rotor mechanical system. The direct connection of rotors was described, which make it possible to instantly ensure the rotation of opposite axial rotors in different directions with equal angular velocities. This mechanical scheme corresponds to the arising nonholonomic mechanical constraints. The indicated schemes of direct axial connection of rotors can be used for instant redistribution of the relative angular momentum. Also it can be applied to change of the current motion mode, e.g., to instant exit from the chaos.

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