Aspects of Chaotic Regimes of a Nanosatellite With Movable Unit

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***Abstract*— The motion of a nanosatellite with one movable unit is considered from the point of view of chaotic regimes initiations at the action of small internal perturbations. The Poincaré sections are plotted, and these sections illustrate the chaotic regimes, which arise in the framework of attitude dynamics of nanosatellites.**

***Index Terms*—Nanosatellite, attitude dynamics, chaotic regimes, Poincaré sections**

I. Introduction

THIS work deals with a problem of chaotic dynamical regimes arises in the attitude dynamics of spacecraft and nanosatellites with movable parts.

Here we present short illustration of chaotic regimes initiation in the nanosatellite, which consists from two parts - a carrying body and a movable unit, attached to carrier body by means of flexible rods [1-3]. The mechanical structure of the nanosatellite is depicted at the fig.1.

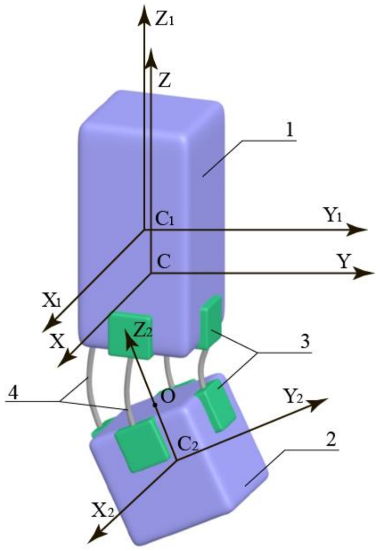

Fig.1. The mechanical model: 1 – carrier body, 2 – movable module, 3 – control systems of flexible rods extraction, 4 – flexible roads.

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II. The Mathematical Model

The nanosatellite consists of two parts - a carrying body and a movable unit, attached to carrier body by means of flexible rods.

The movable unit can perform angular motion relative to the carrier body (the main body), that is carried out by changing the lengths of the flexible rods (the lengths of the rods change by different values depending on the selected axis of rotation). Assume here, that the point O is the center of rotation of the movable unit that is not moving relative to the main body during the unit maneuvers on the rods with changing length.

Let us use the following coordinates frames:

- $CXYZ$ is coordinate frame with the origin in the mass center, which axes are parallel to the main axes of the main body. The point C is the center of mass of the complete nanosatellite;
- $C_1X_1Y_1Z_1$ is the frame with the origin in mass center of the main body, which axes are parallel to the main axes of the main body. The point C_1 is the mass center only of the main body;
- $C_2X_2Y_2Z_2$ is the main connected frame of the movable unit. The point C_2 is the mass center only of the movable unit.

Let us write an expression for the angular momentum of NS, which is the sum of the angular momentums of its parts. In the frame $CXYZ$ the vector of the angular momentum has the form:

$$\mathbf{K} = \mathbf{K_1} + \boldsymbol{\delta}_1 \mathbf{K_2} \tag{1}$$

where $\boldsymbol{\delta}_1$ transition matrix from the $C_2X_2Y_2Z_2$ coordinate system to the $C_1X_1Y_1Z_1$ coordinate system, $\mathbf{K_1}$ – the angular momentum of the main body in the frame $C_1X_1Y_1Z_1$, $\mathbf{K_2}$ – the angular momentum of movable unit in projections on axys $C_2X_2Y_2Z_2$.

In this research let us assume that the angular displacements of the movable unit can be possible only about the direction C2X2 on the small angle $\alpha << 1$. Then the following linearized matrix takes place (we neglect the terms of order α^2 or less):

$$\boldsymbol{\delta}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix} \tag{2}$$

The angular momentums of NS parts in CXYZ are:

$$\mathbf{K_1} = \mathbf{I_1}\boldsymbol{\omega}_1 \tag{3}$$

$$\mathbf{K}_2 = \mathbf{I}_2\boldsymbol{\omega}_2 \tag{4}$$

where $\mathbf{I_1}, \mathbf{I}_2$ – are the inertia tensors of the main body and the movable units:

$$\mathbf{I_1} = \begin{bmatrix} I_{1x} & 0 & 0 \\ 0 & I_{1y} & \alpha k_1 \\ 0 & \alpha k_1 & I_{1z} \end{bmatrix}, \tag{5}$$

$$\mathbf{I}_2 = \begin{bmatrix} I_{2x} & 0 & 0 \\ 0 & I_{2y} & \alpha k_2 \\ 0 & \alpha k_2 & I_{2z} \end{bmatrix}, \tag{6}$$

where m_1 is the mass of the carrier body, m_2 – mass of the movable unit, $k_1 = -m_1 l_2 (l_1 - z_1 - l_2)$, $k_2 = -m_2 l_1 (l_1 + z_2 - l_2)$, $l_1 = m_1 z_1 / (m_2 + m_2)$, $l_2 = m_2 z_2 / (m_2 + m_2)$, z_1 – the distance the distance between points C_1 and O, z_2 – the distance between points C_2 and O.

The angular velocity of the main body in projections on the axes of the $C_1X_1Y_1Z_1$ coordinate system is:

$$\boldsymbol{\omega_1} = [p,\, q,\, r]^T \tag{7}$$

Taking into account the relative rotation of the movable unit, the angular velocity of the mobile module in projections on the axes of the $C_2X_2Y_2Z_2$ is:

$$\boldsymbol{\omega_2} = \boldsymbol{\delta_2} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p + \dot{\alpha} \\ q + \alpha r \\ r - \alpha q \end{bmatrix} \tag{8}$$

where $\boldsymbol{\delta_2}$ – linearized transition matrix from the $C_1X_1Y_1Z_1$ coordinate system to the $C_2X_2Y_2Z_2$ coordinate system:

$$\boldsymbol{\delta_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & -\alpha & 1 \end{bmatrix} \tag{9}$$

To analyze the attitude dynamics of the nanosatellite, it is appropriate to write dynamical equations using the well-known canonical Andoyer-Deprit variables $\{l, g, h, L, G, H\}$ (fig.2).

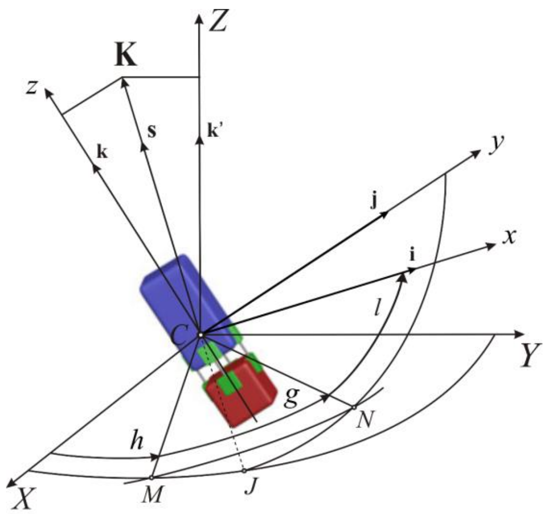

Fig.2. The canonical Andoyer-Deprit coordinates and impulses

The angular momentum components in Andoyer-Deprit variables take the form:

$$\begin{cases} K_x = \sqrt{G^2 - L^2} \sin l; \\ K_y = \sqrt{G^2 - L^2} \cos l; \\ K_z = L \end{cases} \tag{10}$$

Also we must add the canonical impulse A for the relative rotation of the movable unit:

$$\mathrm{A} = \frac{\partial T}{\partial \dot{\alpha}} = I_{2x} (p + \dot{\alpha}) \tag{11}$$

where T is the kinetic energy of the nanosatellite:

$$T = \frac{1}{2}(\mathbf{K_1} \cdot \boldsymbol{\omega_1} + \mathbf{K_2} \cdot \boldsymbol{\omega_2}) \tag{12}$$

Using (7)-(12) we can explicitly express $p, q, r, \dot{\alpha}$ in terms of Andoyer-Deprit variables:

$$\begin{cases} \widehat{p} = \dfrac{\mathrm{A} - \sqrt{G^2 - L^2} \sin l}{I_{2x} - I_{cx}} \\ \widehat{q} = \dfrac{I_{cz} \sqrt{G^2 - L^2} \cos l - \alpha k L}{I_{cy} I_{cz}} \\ \widehat{r} = \dfrac{I_{cy} L - \alpha k \sqrt{G^2 - L^2} \cos l}{I_{cy} I_{cz}} \\ \widehat{\dot{\alpha}} = \dfrac{I_{2x} \sqrt{G^2 - L^2} \sin l - I_{cx} \mathrm{A}}{I_{2x} (I_{2x} - I_{cx})} \end{cases} \tag{13}$$

where $I_{cx} = I_{1x} + I_{2x}$, $I_{cy} = I_{1y} + I_{2y}$, $I_{cz} = I_{1z} + I_{2z}$, $k = I_{2y} + k_2 + k_1 - I_{2z}$. The symbol "$\frown$" above the variable indicates the explicit expressions for angular velocity components throw the Andoyer-Deprit canonical variables – exactly these expressions will be substituted into the expression of the kinetic energy to write the explicit expression of the Hamiltonian. On the base of equations (12) and (13) we can write the explicit expression for the Hamiltonian of the mechanical system and fulfill the investigation of the dynamics at presence of any external and internal potential fields:

$$\begin{aligned} &\mathcal{H}(\widehat{p}, \widehat{q}, \widehat{r}, \widehat{\dot{\alpha}}, \alpha) = \\ &\quad = T(l, g, h, \alpha, L, G, H, \mathrm{A}) + P(l, g, h, \alpha) \end{aligned} \tag{14}$$

So, for complete investigation of the dynamics of the nanosatellite with movable unit we can use the Hamiltonian expression (14), where it is needed only to write the concrete form of the potential energy *P*, corresponding to the concrete potential forces and torques.

III. The Chaotic Regimes Investigation at Small Slow Internal Oscillations by the α-Angle

In this work we will consider the torque-free dynamics of our system in "monobody" format without any external and internal forces. This will be the general unperturbed case, when we investigate the torque-free motion of the nanosatellite as the mechanical system consisting from two rigid bodies glued into a single rigid form (at small and slow relative angular displacement of the movable unit by the angle α). Due to the smallness of the angle of the relative position of the movable unit, the described mechanical

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system can be considered as a rigid body, but at the presence of small change of its inertia-geometrical parameters that depend on the α value. Then we can consider the angle α as the geometrical parameter (it will define a geometrical/kinematical constrain $\alpha = \alpha(t)$).

It is possible to divide the expressions (14) on parts, proportional to a small parameters powers: the "generating part" $\mathcal{H}_0$ and the part caused by the formal parametrical perturbations $\mu\mathcal{H}_1$, $\mu^2\mathcal{H}_2$:

$$\mathcal{H} = \mathcal{H}_0 + \mu\mathcal{H}_1 + \mu^2\mathcal{H}_2 ... \tag{15}$$

where

$$\mathcal{H}_0 = \frac{1}{2}\left[\left(G^2 - L^2\right)\left(\frac{\sin^2 l}{I_{cx}^2} + \frac{\cos^2 l}{I_{cy}^2}\right) + \frac{L^2}{I_{cz}^2}\right] \tag{16}$$

$$\mu\mathcal{H}_1 = -\mu\tilde{\alpha}k\frac{L}{I_{cy}I_{cz}}\sqrt{G^2 - L^2}\cos l \tag{17}$$

Here μ is the formal small dimensionless parameter $\left(\mu << 1\right)$ denoting the small deviation of the relative angular motion of the movable unit $\alpha = \mu\tilde{\alpha}$. So, the Hamiltonian (15) corresponds to the usual Hamiltonian of the unperturbed rigid body $\mathcal{H}_0$ with small perturbations formally defined by the Hamiltonian's parts proportional to small parameters powers $(\mu\mathcal{H}_1, ...)$, which corresponds to the variability of the "geometrical" parameter α. Here we neglect the terms $\mu^2\mathcal{H}_2$. The corresponding system of Hamilton's equations in the Andoyer-Deprit canonical coordinates is:

$$\begin{aligned} \dot{L} &= -\frac{\partial\mathcal{H}}{\partial l}; & \dot{l} &= \frac{\partial\mathcal{H}}{\partial L}; \\ \dot{H} &= -\frac{\partial\mathcal{H}}{\partial h}; & \dot{h} &= \frac{\partial\mathcal{H}}{\partial H}; \\ \dot{G} &= -\frac{\partial\mathcal{H}}{\partial g}; & \dot{g} &= \frac{\partial\mathcal{H}}{\partial G} \end{aligned} \tag{18}$$

As we can see from the Hamiltonian (15)-(17), the mechanical system is completely described by the pare of coordinates {l, L}, since

$$\dot{H} = -\frac{\partial\mathcal{H}}{\partial h} = 0, \quad \dot{h} = \frac{\partial\mathcal{H}}{\partial H} = 0, \quad \dot{G} = -\frac{\partial\mathcal{H}}{\partial g} = 0$$

Coordinate $g(t)$ can be integrated separately after the integrations for $\{l, L\}$, and, therefore, coordinate $g(t)$ do not influence on the main dynamical properties.

For convenience, we write the system of differential equations (18) in the following form:

$$\dot{L} = f_L + \mu g_L; \quad \dot{l} = f_l + \mu g_l \tag{19}$$

where

$$\begin{aligned} f_L &= -\frac{\partial\mathcal{H}_0}{\partial l}; & g_L &= -\frac{\partial\mathcal{H}_1}{\partial l}; \\ f_l &= \frac{\partial\mathcal{H}_0}{\partial L}; & g_l &= \frac{\partial\mathcal{H}_1}{\partial L} \end{aligned} \tag{20}$$

So, the system (19) completely describe the dynamics of the rigid body at the presence of the small perturbations in its inertia parameters due to small variability of its shape, that describe in linear approximation the dynamics of the nanosatellite with small inclinations of movable unit. In this consideration, the fourth canonical coordinate and corresponding canonical impulse $\{\alpha, \mathrm{A}\}$ lose their independent meaning, but they are still important to consideration of the perturbed dynamics.

It is known, that the rigid body at the action of external perturbations or at the presence of internal asymmetric rotator can have the chaotic regimes of the angular motion dynamics [4-6]. What can be realized in the case of torque-free motion of the rigid body in cases of small internal changes of its inertial/geometrical parameters from the chaotic motion point of view – it is the question, which is solving in this section of the paper.

The chaotic dynamics is associated with the presence in the phase space of the system such objects like homo/heteroclinic nets and/or strange chaotic attractors. The possible presence of the homo/heteroclinic nets, first of all, can be detected by the Melnikov's method [7] and its developments, including Wiggins [5] or Holmes-Marsden [4] multidimensional methods. Also the heteroclinic nets can be detected using direct integration of the dynamical equations and Poincare sections plotting.

To show chaotic regimes arising in the dynamics of the rigid body with the small variability of inertia parameters (the nanosatellite with small inclinations of movable units), we will use in this work the classical Melnikov's method.

The Melnikov's method is based on the Melnikov function evaluation. The Melnikov function expression for the perturbed system (19) has the form:

$$M(t_0) = \int_{-\infty}^{+\infty} \left[f_L g_l - f_l g_L \right]\Big|_{\left(\bar{L}(t), \bar{l}(t), t+t_0\right)} dt \tag{21}$$

where $\bar{L}(t), \bar{l}(t)$ - are the explicit exact solutions corresponding to the heteroclinic orbit, which can be expressed throw the well-known [4] heteroclinic solutions $\left\{\bar{p}_0(t), \bar{q}_0(t), \bar{r}_0(t)\right\}$ for the torque-free dynamics of a rigid body:

$$\begin{cases} \bar{q}_0(t) = \sqrt{\dfrac{T_0}{I_{cy}}}\,\mathrm{th}(at) \\ \bar{r}_0(t) = -\sqrt{\dfrac{T_0\left(I_{cx} - I_{cy}\right)}{I_{cz}\left(I_{cx} - I_{cz}\right)}}\dfrac{1}{\mathrm{ch}(at)} \\ \bar{p}_0(t) = \sqrt{\dfrac{T_0\left(I_{cy} - I_{cz}\right)}{I_{cx}\left(I_{cx} - I_{cz}\right)}}\dfrac{1}{\mathrm{ch}(at)} \end{cases} \tag{22}$$

where coefficient $a = \sqrt{T_0\left(I_{cx} - I_{cy}\right)\left(I_{cy} - I_{cz}\right) / \left(I_{cx}I_{cy}I_{cz}\right)}$, T_0 - the constant of the kinetic energy of a rigid body at the absence of any external forces and torques:

$$T_0 = \frac{1}{2}\left(I_{cx}p_0^2 + I_{cy}q_0^2 + I_{cz}r_0^2\right) = \mathrm{const} \tag{23}$$

Now we need to write the f_L, f_l, g_L, g_l functions. After differentiation fulfilling (20) and after substituting the heteroclinic solutions (22), we can obtain:

$$f_l\left(\bar{L}(t),\bar{l}(t)\right)=\bar{r}_0-I_{cz}\bar{r}_0(t)\frac{\bar{p}_0^2(t)I_{cx}+\bar{q}_0^2(t)I_{cy}}{G^2-\bar{r}_0^2(t)I_{cz}^2};$$
$$f_L\left(\bar{L}(t),\bar{l}(t)\right)=\left(I_{cx}-I_{cy}\right)\bar{p}_0(t)\bar{q}_0(t);$$
$$\begin{aligned}g_l\left(\bar{L}(t),\bar{l}(t),t+t_0\right)&=\\&=k\left(\frac{\bar{q}_0(t)\bar{r}_0^2(t)I_{cz}}{G^2-\bar{r}_0^2I_{cz}^2}-\frac{\bar{q}_0(t)}{I_{cz}}\right)\tilde{\alpha}(t+t_0);\end{aligned}\tag{24}$$
$$g_L\left(\bar{L}(t),\bar{l}(t),t+t_0\right)=-\frac{kI_{cx}}{I_{cy}}\bar{p}_0(t)\bar{r}_0(t)\tilde{\alpha}(t+t_0)$$

Then the integrand of (21) has the following form:

$$\begin{aligned}&\left[f_Lg_l-f_lg_L\right]\Big|_{(\bar{L}(t),\bar{l}(t),t+t_0)}=\\&=\tilde{\alpha}(t+t_0)\,f\left(\bar{p}_0(t),\bar{q}_0(t),\bar{r}_0(t)\right);\end{aligned}\tag{25}$$

$$\begin{aligned}&f\left(\bar{p}_0(t),\bar{q}_0(t),\bar{r}_0(t)\right)=\\&=k\bar{p}_0\left(\bar{q}_0^2\frac{I_{cx}-I_{cy}}{I_{cz}}\left(\frac{\bar{r}_0^2I_{cz}^2}{G^2-\bar{r}_0^2I_{cz}^2}-1\right)+\right.\\&\left.+\bar{r}_0^2\frac{I_{cx}}{I_{cy}}\left(1-I_{cz}\frac{\bar{p}_0^2I_{cx}+\bar{q}_0^2I_{cy}}{G^2-\bar{r}_0^2I_{cz}^2}\right)\right)\end{aligned}\tag{26}$$

Taking into account the fact, that the component $\bar{p}_0(t)$ is even function (this follows from (22)), and squared values $\bar{p}_0^2,\ \bar{q}_0^2,\ \bar{r}_0^2$ also are even functions, we can sure, that the function (26) is even function.

Now to make a judgment about presence of chaotic regimes using the Melnikov's function (21) we only need to know the concrete functional form of dependence $\alpha=\alpha(t+t_0)$, because this dependence defines the result of improper integration of the integrand (25).

So, the first simplest but very important case of dynamical analysis, we take the time-dependence of the angle $\alpha=\alpha(t)$, which corresponds to a simplest harmonic form:

$$\begin{aligned}&\alpha=\mu\tilde{\alpha}=\mu\cos\left(\Omega t+\Omega t_0\right);\\&\Omega=\text{const};\qquad \mu<<1\end{aligned}\tag{27}$$

The form (27) can simulate simplest flexible oscillations of the movable unit about a flexible elastic joint in the point O, or small harmonic self-oscillations of the movable unit due to a backlash in the blocks of the flexible rods, or other dynamical aspects. Substituting the expression (27) into (25) gives after simplifications the following Melnikov's function:

$$\begin{aligned}M(t_0)&=\cos(\Omega t_0)\int_{-\infty}^{+\infty}\cos(\Omega t)f(t)dt-\\&-\sin(\Omega t_0)\int_{-\infty}^{+\infty}\sin(\Omega t)f(t)dt\end{aligned}\tag{28}$$

Since $f(t)$ is an even function, and $\sin(t)$ is the odd function, the improper integral of the odd function $f(t)\sin(t)$ will be equal to zero, and, therefore:

$$M(t_0)=\cos(\Omega t_0)\int_{-\infty}^{+\infty}\cos(\Omega t)f(t)\,dt\tag{29}$$

The integrand in (29) is even function, and then the value of the improper integral can be calculated by the following way:

$$\begin{aligned}M(t_0)&=2\cos(\Omega t_0)\int_0^{+\infty}\cos(\Omega t)f(t)\,dt=\\&=\lambda_1\cos(\Omega t_0),\end{aligned}\tag{30}$$

where λ_1 is constant depended on the inertial-mass parameters of the mechanical system and the initial conditions:

$$\lambda_1=2\int_0^{\infty}f(t)\cos(\Omega t)dt=\text{const}\neq 0\tag{31}$$

As we can see, the Melnikov's function in the considered simplest case is the usual harmonic function (30) with nonzero amplitude (31), and it has infinity quantity of simple roots. This fact analytically proves the presence of infinity quantity of intersections of split manifolds of the heteroclinic points – it implies the heteroclinic net and chaos arising.

To illustrate the heteroclinic chaos at the perturbation (27) we can plot the Poincaré sections of the phase portrait of the system in coordinates $\{l, L/G\}$ for zero and nonzero values of the small parameter μ. Here (fig.3 and 4) it is not important numerical values for the system motion parameters; here only quality properties are important.

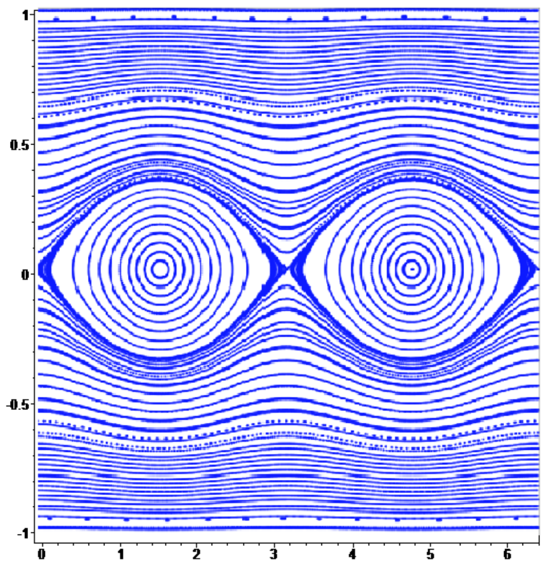

Fig.3. The Poincaré section of the unperturbed system $(\mu=0)$

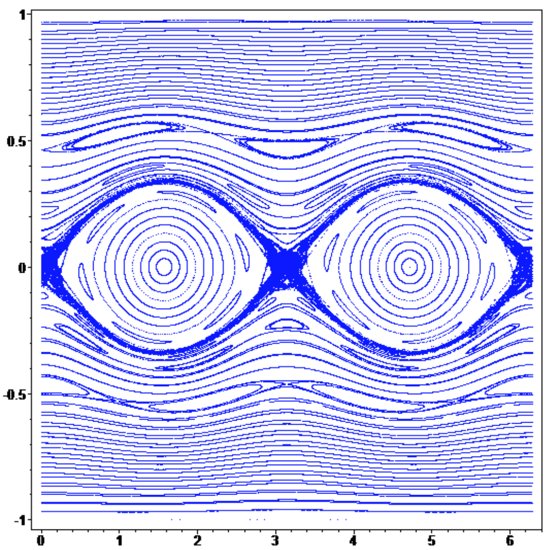

Fig.4. The Poincaré section of the perturbed system $(\mu=0.1,\quad \Omega=1)$

We see at the fig.4 the the heteroclinic net in the separatrix-region of the phase space: the points of the Poincaré section cover the separatrix region with a “dense fog”, and the secondary heteroclinic bundles and new heteroclinic trajectories are born. This is the main sign of the realization of chaotic regimes near the heteroclinic region of the phase space.

IV. Conclusion

As can we see from the analytical and numerical modeling, the chaotification of the dynamical regimes near the separatrix region of the phase space takes place at the action of any simplest harmonical perturbation (27). This is the first and more important conclusion of the work. It is worth to indicate the case of research of chaotic dynamics of the nanosatellite with flexible panels, that is quit close case by the natural properties of the motion [8, 9].

But if will consider the perturbed dynamics at the complete modeling of the system and oscilations relative the angle α, then we can take into our account more aspects (such the vibration stiffness, damping and friction in the structure of the nanosatellite), then the chaotic regimes can be suppressed. The possibility of the suppression of the chaos is the quite important task for further research.

For example, if we try to consider the dynamics of the angle α with the help of the Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right)-\frac{\partial T}{\partial \alpha}=M_{\alpha} \tag{32}$$

at the action of the internal torque

$$M_{\alpha}=-c_1\dot{\alpha}-c_2\alpha+\mu\sin\left(\Omega t\right) \tag{33}$$

with damping and natural vibration stiffness, then the factors of the chaotic dynamics and the factor of the energy dissipation (due to internal damping) will define the resulting dynamics of the system – it can turn out to be both chaotic and regular, and this is the direct task of our research.

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