

# On the Statistical Distribution of Stationary Segment Lengths of Road Vehicles Vibrations

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**Abstract**— This paper presents an important outcome of a research programme which focuses on the development of a method for synthesizing, under controlled conditions in the laboratory, the non-stationary random vibrations generated by road transport vehicles. It addresses an important limitation of current methods used for synthesising random vehicle vibrations which assume that vibrations produced by wheeled vehicles can be approximated by a zero-mean, normally-distributed (Gaussian) random process and, therefore, fails to accurately reproduce the fluctuations in vibration levels that occur naturally during road transportation realizations [1]. The paper builds upon the observation that non-stationary random vehicle vibrations are composed of a sequence of zero-mean random Gaussian processes of varying standard deviations [2]. It discusses the important parameters that need to be addressed when dealing with the synthesis of random sequences. The paper presents the development of a change-point detection algorithm that was used to determine the length of stationary segments within a large number of typical non-stationary random vibration records. These include measured vibration records as well as numerically-generated records based on measured pavement profiles. The algorithm, based on the cum-sum / bootstrapping techniques, was developed and applied to the instantaneous magnitude of sample vibration records. The statistical distribution of segment lengths for each vibration record was computed with the aim of identifying similarities and trends for the development of an overall statistical model for segment lengths to be used for synthesis purposes. One outcome of note was that the shape of the segment length distributions computed from a wide range of vibration records are generally comparable and exhibit an asymptotic-like decrease in probability of occurrence as the segment length increases. This behaviour was found to be adequately modelled with a hyperbolic trigonometric function which was found to be useful for characterising the general statistical behaviour of segment length for non-stationary random vibrations produced by road vehicles. Finally, the significance and relevance of this outcome with respect to the synthesis of non-stationary vibrations for package evaluation and validation purposes is highlighted.

**Index Terms**—Non-stationary vibrations, random vibrations, vehicle vibrations, change-points, segment lengths.

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## I. INTRODUCTION

It is self-evident that vehicle vibrations are caused, in the main, by uneven pavement surfaces. These have been found to be random in nature which, in turn, causes the vehicle vibrations to be random. Variations in pavement roughness and fluctuations in the vehicle's speed within a particular journey combine to produce variations in the overall energy levels of the vehicle vibrations. These variations make the process highly non-stationary [2, 3] and introduce a level of complexity that cannot be adequately dealt with using conventional methods that are used for Gaussian and stationary processes. The most commonly-used technique for laboratory simulation of transport vibrations has been in place for some years. The method assumes that the vibrations produced by wheeled vehicles can be approximated by a zero-mean, normally-distributed (Gaussian) random process. In addition, the overall root-mean-square (rms) level of the process is often assumed to be constant thus implying stationarity. It has been shown that vibration synthesis at a constant rms level fail to accurately reproduce the fluctuations in vibration levels that occur naturally during road transportation realizations [1]. Most laboratories make use of a random vibration controller (RVC) which uses a feedback loop to synthesize and control random signals according to a PSD function. When non-stationarity is taken into account, vibrations are synthesized at various rms levels for pre-determined durations. However, the length and sequence of each constant rms synthesized segment is not known. This is the outstanding issue addressed herein.

One important aspect pertaining to the synthesis of non-stationary random vibrations is the issue of signal segmentation. If the hypothesis that the process can be modelled statistically as a sequence of segments, each belonging to a family of Gaussian process with varying standard deviations, is valid [2 ] then there must exist identifiable boundaries (change-points) at which the transition from one segment to another occurs. The detection of such change-points should make it possible to determine the segment lengths and their relationship with the segment standard deviation as well as their statistical characteristics. Segmentation is critical when considered in the context of synthesis. This will determine the statistical parameters upon which the length and sequence of each

Gaussian segment will be synthesized.

## II. CHANGE POINT DETECTION

Change-point detection is a process that emerged out of a need to identify real changes in random processes such as economic indicators and the monitoring and control of quality in manufacturing processes. Techniques range from cumulative sum (cum-sum) schemes first proposed by Page [4, 5, 6] and further developed by Hinkley [7] and Pettitt [8] to Singular-Spectrum Analysis [9]. The cumulative sum techniques are primarily geared toward detecting significant deviations in the mean of random processes. The procedure is greatly enhanced by including a bootstrapping algorithm which is used to provide an estimate in the significance of the change-point. Bootstrapping effectively rearranges the sequence or order of the sample in a random fashion a number of times while re-evaluating the sample (in this case using the cum-sum algorithm). This gives a basis for determining whether the change is truly significant or merely apparent. In this study, a change-point detection algorithm, based on the cum-sum / bootstrapping techniques was developed and applied to the instantaneous magnitude of the sample vibration records. The argument for using the instantaneous magnitude are based on evidence which shows that level type non-stationarities in random signals are well manifested through changes in the instantaneous magnitude and are not reliant on subjective parameters such as the window width required to compute the moving RMS [10].

The instantaneous magnitude accounts for the deficiencies with the moving RMS method in detecting short duration fluctuations in the magnitude of vehicle vibrations which are typically induced when the vehicle encounters sudden changes in the roughness of pavements and when severe and localised pavement surface defects are present [11]. The method presented here makes use of the Hilbert transform to compute the instantaneous magnitude of the record. In its original form, the Hilbert Transform is used to produce the imaginary component,  $\tilde{a}(t)$ , of a measured, real signal,  $a(t)$ , thus enabling the creation of an analytical signal  $\hat{a}(t)$  [12]:

$$\hat{a}(t) = a(t) + i \tilde{a}(t) \quad (1)$$

where the Hilbert transform is defined as:

$$\begin{aligned} \tilde{a}(t) = H\{a(t)\} &= \frac{1}{\pi} \int_{-\infty}^{\infty} a(\tau) \left( \frac{1}{t-\tau} \right) d\tau \\ &= \frac{1}{\pi} a(t) * \left( \frac{1}{t} \right) \end{aligned} \quad (2)$$

Executing the convolution in equation (2), it can be shown that the Hilbert transform is equivalent to a  $90^\circ$  phase shift of

the real signal. This is significant in that it allows the computation of the Hilbert transform to be performed in the frequency domain via the Fourier transform as follows:

$$\begin{aligned} \mathcal{F}\{\tilde{a}(t)\} &= A \perp (f) \\ &= A(f) \cdot [-j \operatorname{sgn}(f)] \end{aligned} \quad (3)$$

where  $A(f)$  is the spectrum of  $a(t)$ . This operation where the positive and negative frequency components of the signal are given a phase shift of  $-90^\circ$  (multiplied by  $-j$ ) and  $+90^\circ$  (multiplied by  $j$ ) respectively. Once the analytical signal is created, the instantaneous magnitude of the vibration,  $M[a(t)]$ , signal is easily computed:

$$M[a(t)] = \sqrt{a(t)^2 + \tilde{a}(t)^2} \quad (4)$$

The instantaneous magnitude of the signal is manifested as the envelope of the signal as illustrated in Fig. 1 which shows a typical vibration record together with its instantaneous magnitude computed from the Hilbert Transform. It demonstrates how the instantaneous magnitude is useful to describe short-term and rapid variations in the amplitude of the signal. The main benefit is that there is no averaging process employed and the true magnitude-based non-stationary character of the signal is revealed.

The change-point detection algorithm was developed specifically to deal with the amplitude type non-stationarities that are prevalent in road vehicle vibrations. In order to test and validate the algorithm, a number of vehicle vibration samples were collected from a variety of vehicle types, routes and payload conditions as shown in Table 1. In addition, the vertical acceleration responses of various linear quarter-car numerical models, made available in the literature, were computed for a range of pavement profiles (Table 2). These vibration records were generated to supplement and complement the collection of measured vibration records with data simulated from different vehicle types and pavement surfaces. Although it is acknowledged that the rudimentary nature of the simulation model (only linear elements were modelled) produces vibration estimates that are not necessarily accurate, the simulation is sufficiently realistic to reproduce the random non-stationarities that occur in reality and are, therefore, deemed adequate to the purpose of this study. The simulation was carried out with a purpose-designed program coded in Matlab® and Simulink®. The boundary conditions were accounted for by introducing a vehicle velocity ramp at a constant forward acceleration until the target cruise speed was reached. The vertical vibrations of the quarter-car model were then computed at constant vehicle velocity for the entire pavement profile.

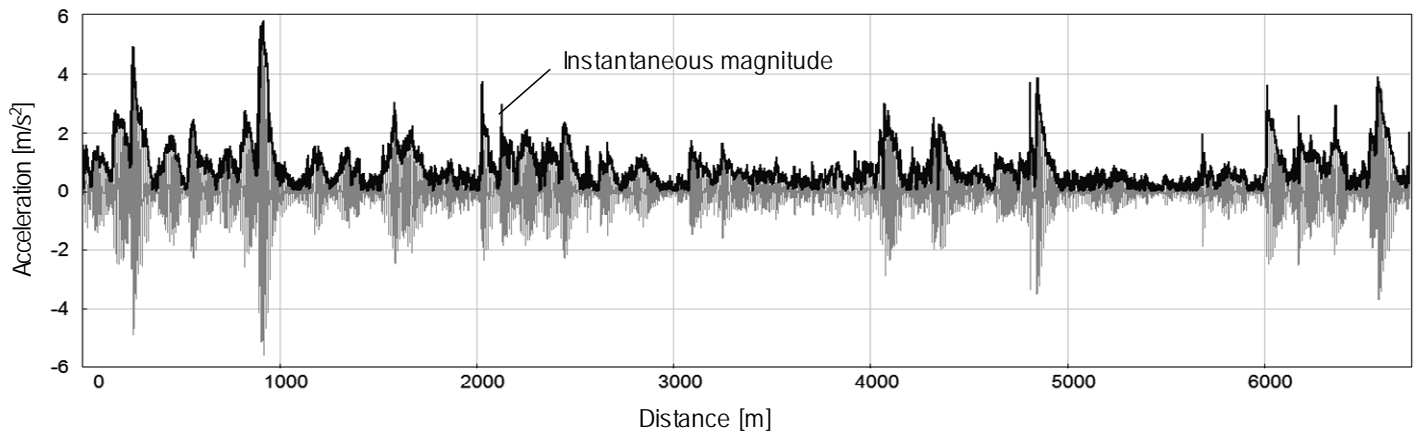


Figure 1. Example of the manifestation of a vibration signal's instantaneous magnitude (dark line).

Table I. Summary of measured vibration record parameters.

ID	Vehicle type & load	Route Type
MA	Utility vehicle (1 Tonne capacity). Load: < 5% cap.	S'urban streets
MB	Prime mover + Semi trailer (Air ride susp.). Load: 90% cap.	Country roads
MC	Transport van (700 kg capacity). Load: 60% cap.	Suburban streets
MD	Transport van (700 kg capacity). Load: 60% cap.	Suburban hwy.
ME	Transport van (700 kg capacity). Load: 60% cap.	Motorway
MF	Prime mover + Semi trailer (leaf spring susp.). Load: < 5% cap.	Country roads
MG	Tipper truck (16 Tonnes capacity, Air ride susp.). Load: 25% capacity.	Country roads
MH	Small flat bet truck (1 Tonne capacity, leaf spring susp.). Load <5% cap.	Suburban streets
MJ	Flat bed truck (5 Tonnes capacity, leaf spring susp.). Load >95% cap.	Country roads
MK	Sedan car. Load: 1 passenger	Suburban

Table II. Summary of routes used for numerically-generated vibration records.

ID	Route (Victoria, Australia)
SA	Murray Valley Highway (Major county road)
SB	Bendigo – Maryborough road (Major county road)
SC	Princess Highway (Freeway)
SD	Timboon Road, Victoria, Australia (Major county road)
SE	Road sequence: Timboon Road – Princess Hwy and Murray Valley Hwy, Victoria, Aust.

The change-point algorithm developed for this study and associated statistical analysis is described hereunder.

- The instantaneous magnitude (envelope) of vibration is computed using the Hilbert Transform
- Compute the cumulative sum (cum-sum) of the instantaneous magnitude vector normalised with respect to the mean magnitude.

- Apply the bootstrap algorithm sequence whereby the entire instantaneous vibration vector is randomly re-samples (shuffled) a number of times and the cum-sum re-computed for each re-sampled vector
- The maximum and minimum envelopes from the bootstrap (shuffled) samples are computed
- The largest extremum of the original record is detected and identified as a change-point. Its value is compared with that of the bootstrap sample as illustrated in Fig. 2.
- The change point is identified as significant or valid if the ratio of the largest extremum to the bootstrap extremum exceeds a pre-determined value. In all cases studied, the ratio threshold of 5.5 was identified as adequate. The ratio threshold effectively controls the sensitivity of the algorithm. Large ratio threshold values results in low sensitivity whereby small changes in instantaneous magnitude will not be detected. Small ratio threshold values will enable the algorithm to detect small changes in instantaneous magnitude. Given the stochastic nature of the processes under consideration coupled with the relatively broad range of frequencies present, too small a ratio threshold is problematic in that the algorithm will identify a large number of fictitious change-points within the natural fluctuations in a stationary segment.
- If the change point is identified as real or significant, the instantaneous magnitude record is bisected about the change-point and the resulting two segments are subjected to the same cum-sum / bootstrap algorithm until no more change-points are identified or the segment length reaches a pre-determined limit known here as the “Minimum Segment Length”. In the case of road vehicle vibration, a compromise needs to be arrived at when determining a suitable Minimum Segment Length. For synthesis purposes, a Minimum Segment Length of 0.5 seconds was deemed sufficiently small to include the vast majority

stationary segments within the process. Fig.3 shows an example of the outcome of the algorithm for a typical vibration record.

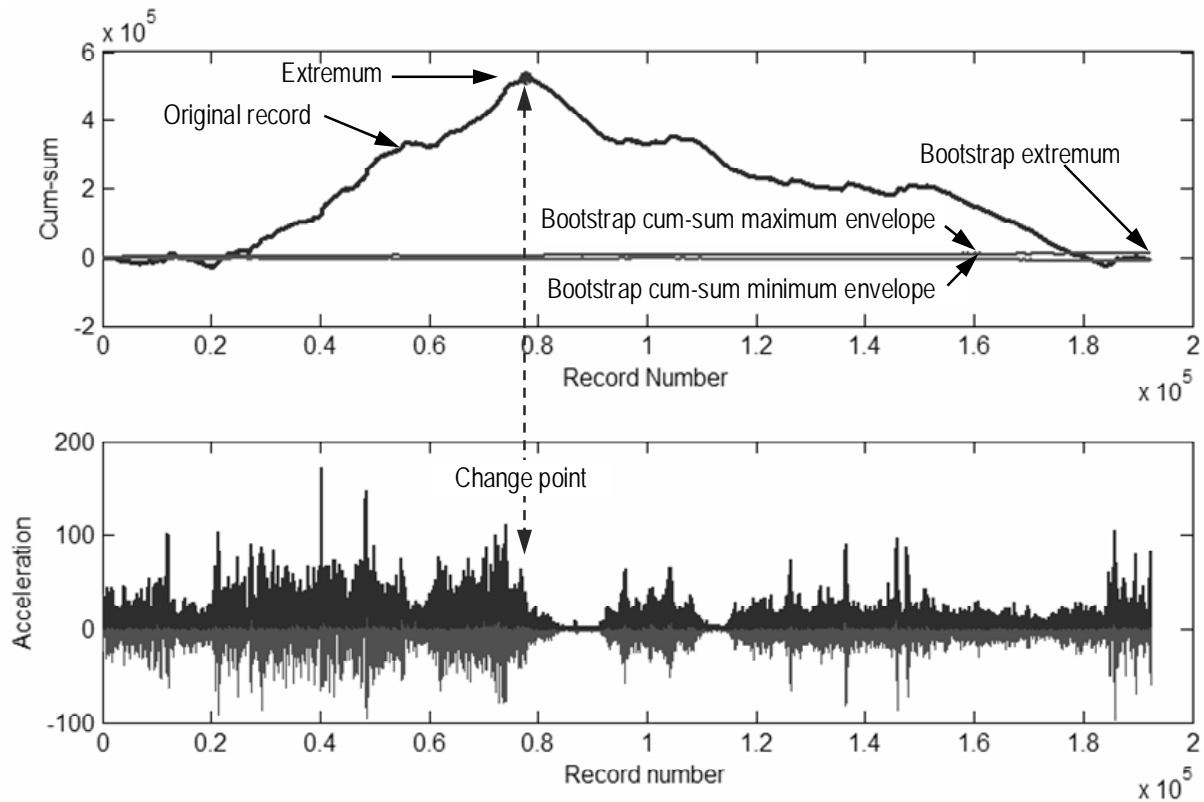


Figure 2. Determination of a change-point using the cum-sum / bootstrap algorithms.

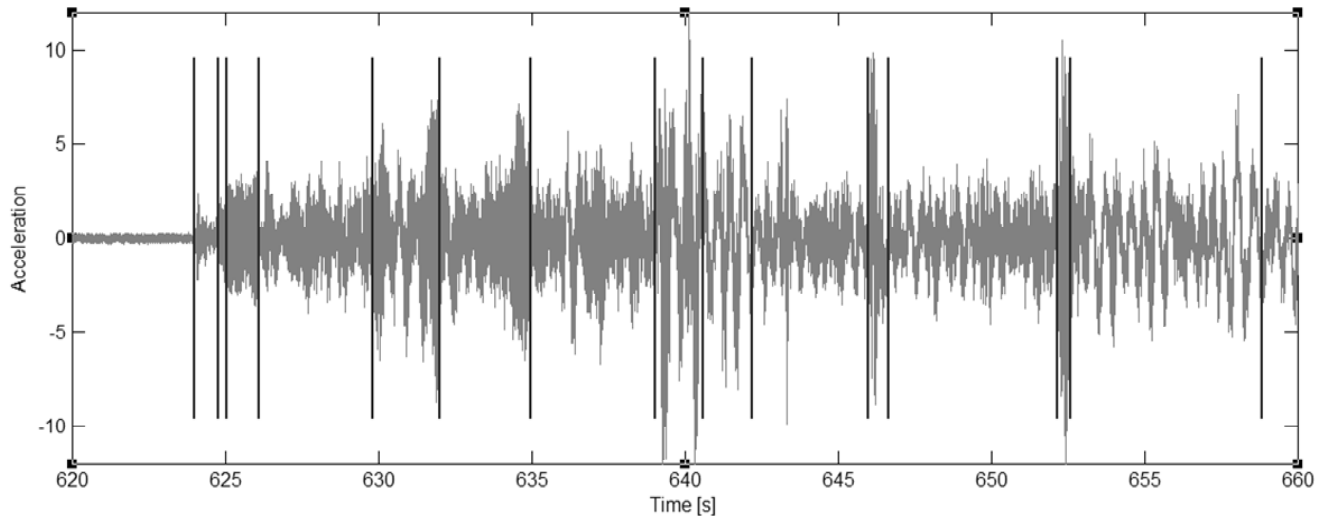


Figure 3. Identification of stationary segments (change-points shown in dark) using the cum-sum / bootstrap algorithm for a section from a typical vibration record.

The change-point detection algorithm, written in Matlab® (available from the author on request), was used to determine the length of stationary segments within a large number of typical non-stationary random vibration records. These include measured vibration records as well as numerically-generated records based on quarter-car models

and measured pavement profiles. The statistical distribution of segment lengths for each vibration record was computed with the aim at identifying similarities and trends for the development of an overall statistical model for segment lengths to be use for synthesis purposes. Examples of the segment length distribution for one typical measured

vibration record and one typical numerically-generated record are shown in Fig. 4 and 5 respectively.

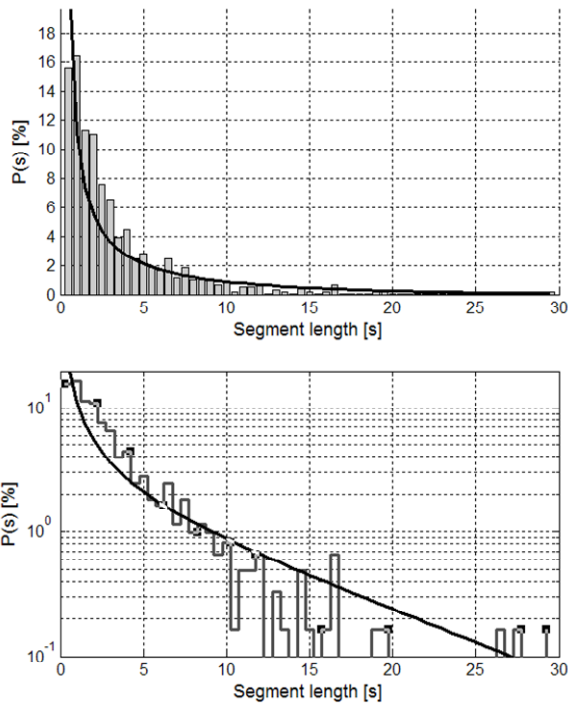


Figure 4. Statistical distribution of stationary segment lengths for a typical measured vibration record along with the hyperbolic line (dark) of best fit.

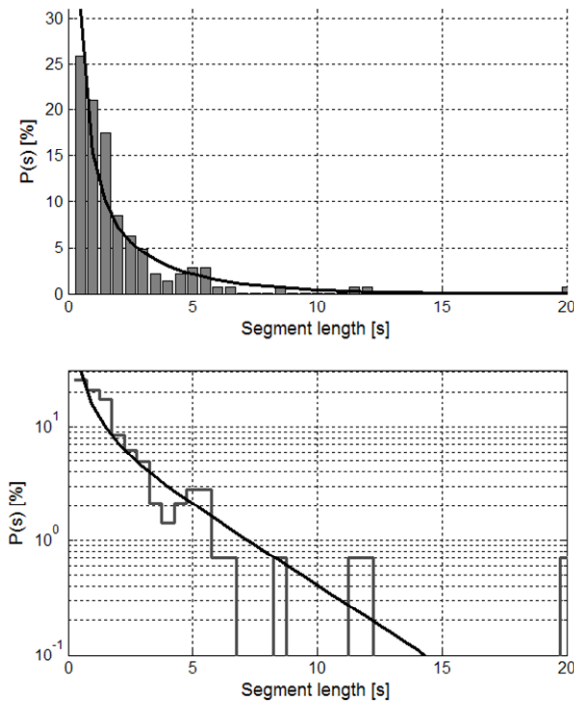


Figure 5. Statistical distribution of stationary segment lengths for a typical numerically-generated vibration record along with the hyperbolic line (dark) of best fit.

The statistical distributions of the segment lengths for all nine measured vibrations records and all thirteen numerically-generated records are shown separately in Fig.

6. It is interesting to note that the shape of the segment length distributions are generally comparable and exhibit an asymptotic – like decrease in probability of occurrence as the segment length increases. This behaviour was found to be competently modelled with a hyperbolic function in the form

$$p(s) = C / \sinh(ks) \quad (5)$$

where  $C$  and  $k$  are empirical constants obtained by a non-linear least squares regression method. This model was applied to the measured and numerically-generated vibration data sets separately yielding lines of best fit shown in Fig. 7.

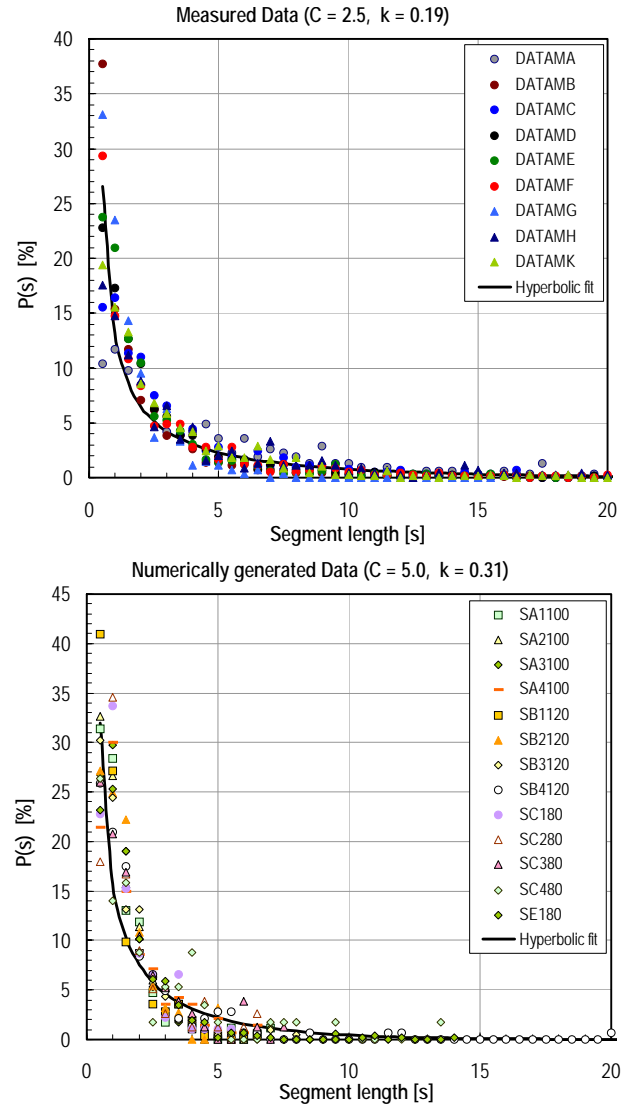


Figure 6. Statistical distributions of stationary segment lengths for all 9 typical measured vibration records (top) and 13 typical numerically-generated vibration records (bottom) along with the hyperbolic curve of best fit.

Given that the primary purpose of this analysis is to determine an overall rule by which the variations in segment lengths are distributed, there is no requirement to distinguish between measured and numerically-generated vibration data. In fact, when both sets are combined, there is little to

distinguish one from the other as shown in Fig. 7 which shows that a single probability density function in the form of Eqn. (4) with  $C \approx 4$  and  $k \approx 1/4$  is sufficient for describing the collective expected occurrence of stationary segment lengths for road vehicle vibrations.

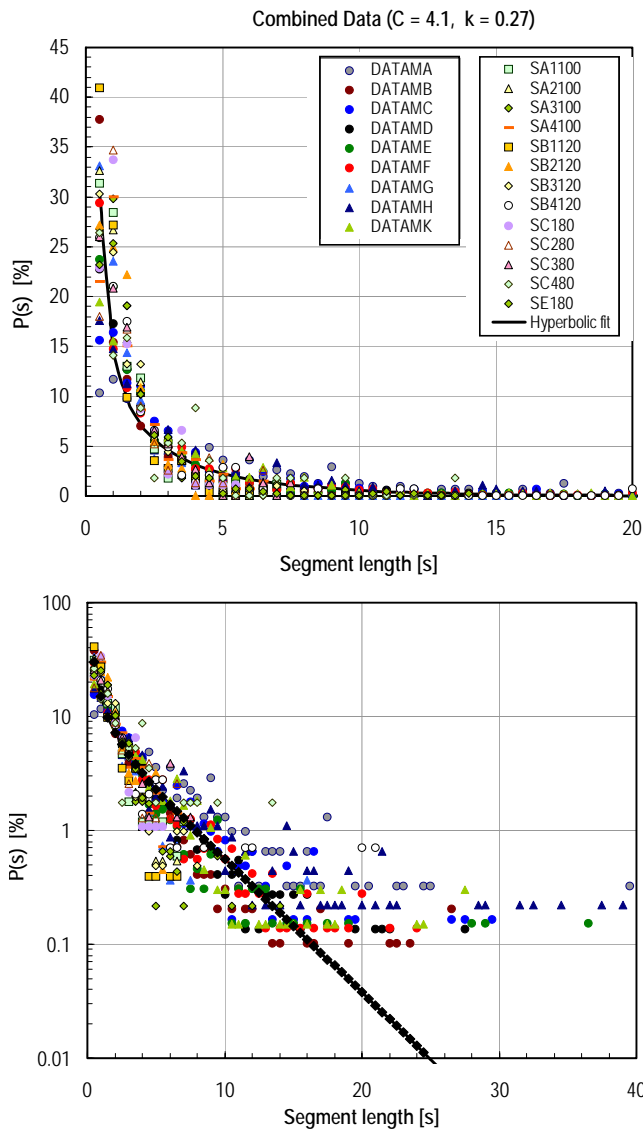


Figure 7. Statistical distributions of stationary segment lengths for all sample vibration records long with the hyperbolic curve of best fit. (Left: Linear scales, Right: semi-logarithmic scales for clarity).

### III. CONCLUSIONS

The effectiveness of a novel approach to detect significant change-points within records of non-stationary random vibrations produced by wheeled vehicles has been demonstrated. The paper has shown that the instantaneous magnitude, easily computed by means of the Hilbert

transform, can be used to apply the cumulative cum (cum sum) /bootstrapping algorithm to detect statistically significant change points within the process. The main relevance of the results is the ability to determine the length of stationary segments within the process segment as well as their statistical distribution. The similarity of form between the segment length probability densities obtained from a wide variety of conditions (routes and vehicles as well as experimentally measured and numerically-generated data from measured pavement topographies) is unexpected. This demonstrates that, in terms of analysis methodology, there is no need to differentiate between measured and numerically generated vibration data. Finally, the paper proposed a simple hyperbolic distribution model that quite satisfactorily characterises the probability density of segment lengths. This outcome is relevant when attempting to synthesize realistic, non-stationary random vibrations in the laboratory. This technique, to be reported at a later date, makes use of the segment length distribution function to create a modulation function which is then used to alter the overall root-mean-square of a synthesized Gaussian signal to reproduce the amplitude fluctuations that are manifest in the process.

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