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Distributed Self-Localisation in Sensor Networks using RIPS Measurements

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Abstract—**This paper develops an efficient distributed algorithm for localising motes in a large scale sensor network using radio interferometric positioning. The focus here is on finding exact solutions while using a relatively small number of measurements, where the effects of noise are largely ignored.**

Keywords: wireless sensor networks, localisation, Radio Interferometric positioning

1 Introduction

For many applications of wireless sensor networks the determination of the precise locations of the sensors or *motes* is a key to achieving the aim of the sensing application [3]. Knowledge of the locations of the sensors is usually crucial but often difficult to determine as the initial deployment may be random. Generally, cost and power limitations on the motes mean that they are not GPS-enabled; thus other techniques for localisation must be sought.

The localisation problem addressed in this paper is one where randomly deployed motes must determine their locations using only their abilities to communicate with other motes in the network. The networks we consider are generally multi-hop networks; in a typical low-powered sensor network individual motes will lack the energy necessary for long range communication and their communication ranges will be less than the area covered by the entire network. It follows that localisation algorithms based on classic multidimensional scaling (MDS) which depend on the availability of all pairwise distances between nodes in the network cannot be used for large scale low-powered networks. In addition, the algorithms we develop rely on far fewer measurements than are required for MDS methods, and hence have much better scaling and resource conservation properties.

There are two main types of multi-hop localisation [3]. In *centralised* localisation algorithms, global information can be used to improve the quality of position estimates. This has the disadvantage of requiring a powerful central node dealing with the data structures associated with a large network, and it exposes a single point of failure in the network. A *distributed* algorithm makes use of computing and communication capabilities across the motes with the advantages of natural load balancing and a lack of reliance on a single point of possible failure. In this paper we focus on distributed algorithms.

The sorts of distributed algorithms we develop in this paper are *collaborative (or cooperative) self-localisation* algorithms, which make use of local data, having communication limited to comparatively small neighbourhoods. Here we allow unknown-location devices to make measurements with known-location references, called *anchors* or beacons, and we additionally allow unknown-location devices to make measurements with other unknown-location devices. Such distributed algorithms are attractive because they are robust to network changes and node failures. The communication cost also scales well with increasing network size [4].

Localisation measurements are typically made using acoustic signals. These frequently have ultrasonic frequencies due to stealthy operations. Here a transmitter and receiver pair on each mote enables the sensor nodes to measure inter-node ranges using the time difference of arrival (TDoA) between the ultrasonic and RF signals. Such measurements utilising radio usually rely on received signal strength that is relatively accurate in short ranges with extensive calibration, but imprecise beyond a few metres [6]. Existing wireless sensor network localisation methods have either adequate accuracy or acceptable range, but not both at the same time. In this paper we will consider an alternative method, called the Radio Interferometric Positioning system (RIPS) method described in [2], using radio interferometry, which attains high accuracy and long range simultaneously. In addition it does not require any sensor on a mote other than the radio used for wireless communications. Instead of providing measurements of the distances between pairs of nodes, it provides measurements which are a function of two sets of pairwise distances. The technique relies on a pair of nodes emitting radio waves simultaneously

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at almost the same frequency. The resultant interference composite signal will have a low frequency envelope that can be measured by cheap and simple hardware. In [2] a centralised localisation algorithm is described where all measurements are sent to a central processor which them uses a genetic algorithm to simultaneously locate all the nodes. In addition, the network is deployed over a sufficiently small area that all nodes can communicate with one another. In the implementations described, all possible measurements are taken, thus the number of transmissions is $O(N^2)$ where N is the number of nodes. The algorithm is thus computationally demanding both in the number of measurements required and the solution method.

In this paper we present a distributed algorithm for accurate unambiguous localisation using RIPS measurements, in the absence of measurement noise. This approach is suitable for localisation of sensors in a surveillance region that is larger than the communication range. The algorithm requires that a small number of anchor nodes, in known locations, are available. The number of RIPS measurements is at most linear in the number of nodes, and usually much smaller, depending on the distribution and density of motes with respect to their maximum communication range. This results in a critical cost saving, as the power requirements required for transmissions are typically significantly more than that of local processing.

Note that although we ignore measurement noise in this paper, we nevertheless design our algorithms so as to attempt to minimise error propagation, where this can be done without compromising computational efficiency. A study of the error propagation inherent in this approach, and efficient methods of minimising such errors will appear in a future paper.

2 Preliminaries: A graph model

Let $M = \{m_i\}$ be the set of motes in a given sensor network. Each mote is assumed to be located at a point on the xy-plane; in other words, we restrict our attention to localisation in 2-dimensional space. We also assume that M contains at least three *anchors*, ie, motes whose positions are known. All other motes initially have unknown positions. Our aim is to determine the positions of all motes using measurements obtained from the Radio Interferometric Positioning System (RIPS) [2], where the number transmissions required for such RIPS measurements should be a small as possible. It is assumed in this paper that the measurement noise associated with RIPS trilateration is negligible in order to obtain lower bounds on the effectiveness of this approach. An explanation of how we compute distances and positions from RIPS measurements is given in Section 3.

For a given wireless sensor network (WSN), we can define the associated *transmission graph* $G_T = G_T(M)$, where $V(G_T) = M$ and $(m_i, m_j) \in E(G)$ if and only if the motes m_i and m_j are within transmission range of each other in the WSN.

Recall, from Graph Theory, that $S \subset M$ is said to be a *clique* if the subgraph of G_T induced by S is complete (ie, fully-connected). Furthermore, S is a *maximal clique* if, for every $m_i \in M \setminus S$, $S \cup \{m_i\}$ is not a clique.

We will see in the next section that three RIPS transmissions are sufficient to allow us to determine the positions of all motes within a clique of G_T relative to a small set of motes in the cliques known as *pseudo-anchors*. The algorithm we propose in this paper, first covers G_T by a set of overlapping cliques, then propagates the location information obtained by RIPS measurements in each clique through the network (via multiple hops) in a rapid and distributed manner in order to determine the locations of all motes. In the absence of noise a necessary condition for this to be possible is that M contains at least 3 anchors and G_T is connected.

The remainder of the paper is structured as follows. Section 3 describes the localisation of motes in a clique with respect to the pseudo-anchors in that clique, using RIPS measurements. Section 4 describes a distributed method for dividing the set of nodes into a collection of overlapping *c*liques in order to propagate this location information through the sensor network. Section 5 describes criteria and a distributed algorithm for choosing the pseudo-anchors in each clique. Finally, some computational results are given in Section 6.

3 Trilateration within a clique

Assume for the moment that we have a clique of G_T containing three anchor motes. We wish to locate the remaining motes in the clique using RIPS measurements, as discussed in [2]. The idea is to utilise two transmitters (two of the anchors) to create an interference signal and compare the phase offset at two receivers. Given a suitable choice of carrier frequencies, so that there is no ambiguity of phase over the range of transmission, this allows one to determine a sum of distance differences. In particular, if motes A and B are transmitters and C and D are receivers then the method effectively allows one to determine

$$d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC},$$

where d_{PQ} represents the distance between motes P and Q.

If we ignore errors due to noise, then the locations of the remaining motes can be *exactly* computed, in terms of the locations of the anchors. Suppose that A, B and C are three anchors, and that we wish to determine the location of D.An application of interferometric positioning with A

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and B as transmitters and C and D as receivers yields

$$d_{AD} - d_{BD} = d_{BC} - d_{AC} + d_{ABCD} := k_1 \qquad (1)$$

where k_1 is a known constant. Similarly, if we set A and C as transmitters and B and D as receivers we obtain

$$d_{AD} - d_{CD} = d_{CB} - d_{AB} + d_{ACBD} := k_2. \qquad (2)$$

As observed in [1], these are the only two independent measurements of this form that can be obtained from this configuration. For example, by setting B and C as transmitters and A and D as receivers we obtain Equation (2) minus Equation (1).

This means that we cannot directly compute d_{AD}, d_{BD} and d_{CD} by solving a system of linear equations. For the planar problem, however, the location of D only has two coordinates, so in theory the above equations should suffice for determining the location of D. Equations (1) and (2) each define a branch of a hyperbola in the plane, so the solution involves finding the intersection of two hyperbola.

It is shown in [1] that this problem is equivalent to solving an associated quadratic equation; hence the problem is exactly solvable, but may have up to two solutions. Geometrically, these solutions correspond to the intersections of the the two hyperbola branches defined by equations (1) and (2).

For motes in general position, this potential ambiguity in the solution can be resolved by performing one extra RIPS transmission. Suppose the clique contains a fifth mote E. Then the two transmissions corresponding to equations (1) and (2) allow us to narrow down the possible locations of E to at most two points. If we then take a third RIPS measurement transmitting from, say, A and D, then the associated hyperbola for receivers B and E corresponding to each of the two possible positions of D are independent of the previous hyperbola for E and consequently allow us to determine the locations of both D and E.

It follows that (with probability 1 — because the motes are assumed in general position) localisation of all motes in a clique S can be performed using 3 RIPS transmissions, providing $|S| \geq 5$ and S contains 3 anchors. Furthermore, if we replace some or all of these anchors by *pseudo-anchors* (ie, motes whose locations are not necessarily known but that are treated like anchors) then the 3 transmissions allow us to determine the locations of all elements of S in terms of the positions of the pseudo-anchors, ie, in terms of those variable parameters that determine the positions of the pseudo-anchors.

It now remains to propagate this location information through the network (via multiple hops) in a rapid and distributed manner in order to determine the location of all motes.

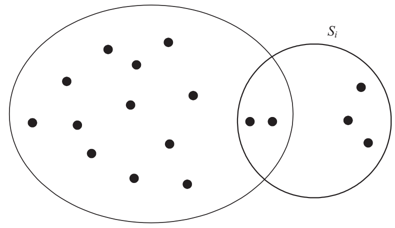

Figure 1: Illustration of a situation where localising motes in all cliques except S_i does not allow the remaining motes in S_i to be uniquely localised.

4 Division of M into cliques

The method we propose for propagating location information through the sensor network is to cover M by a collection of overlapping cliques. We first define some useful concepts.

Definitions: Given a collection of cliques of M: $\mathcal{C} = \{S_1, \ldots, S_k\}$, we define the propagation graph of $\mathcal{C}$ to be $G_P = G_P(\mathcal{C})$ such that $V(G_P) = \mathcal{C}$ and $(S_i, S_j) \in E(G_P)$ if and only if $|S_i \cap S_j| \geq 3$.

The set of cliques $\mathcal{C} = \{S_1, \ldots, S_k\}$ is said to be *admissible* if it satisfies the following conditions:

1. for each $S_i \in \mathcal{C}$, $|S_i| \geq 5$;
2. $\cup S_i = M$; and
3. $G_P(\mathcal{C})$ is connected.

Note that the conditions in the above definition for $\mathcal{C}$ being admissible are necessary conditions for being able to localise all motes by performing RIPS measurements in each clique and then propagating the resulting location information through G_P. In particular, Condition 1 is required to guarantee that we can localise all motes in each clique (as described in Section 3); Condition 2 is required in order to reach all motes in M; and Condition 3 is needed in order to ensure that motes in a clique containing no anchors can eventually be uniquely localised. This last claim follows from the fact that G_P is connected only if each clique intersects some other clique in at least 3 motes. Suppose on the contrary, that clique S_i containing no anchors only intersects one other clique, and that that intersection only contains two motes. Then, as illustrated in Figure 1, even if we can localise motes in all other cliques, knowing the relative positions of the motes in S_i may not allow us to localise the remaining motes in S_i as we can reflect the positions of the motes in S_i through the line containing the two motes in the intersection without changing the relative positions.

Hence, for efficient and effective propagation we aim to solve the following problem:

Clique Problem: Find a collection of cliques $\mathcal{C} = \{S_1, \ldots, S_k\}$ for M such that:

(i) $\mathcal{C}$ is admissible, and

(ii) each clique is as large as possible (ideally, a maximal clique), and

(iii) the maximum distance (in $G_P(\mathcal{C})$) of any clique in $\mathcal{C}$ from a clique in $\mathcal{C}$ containing an anchor is as small as possible.

Note that (i) guarantees that we can propagate over all motes, (ii) will help minimise the number of transmissions required, and (iii) will help minimise cumulative measurement errors.

The problem of constructing maximal cliques is known to be NP-hard, even in non-distributed settings. It follows that we require a heuristic approach to the optimisations in (ii) and (iii) that can be implemented in a distributed way. For a given mote m_i denote by $N(m_i)$ the set of all motes (including m_i) within transmission range of m_i, and call it the *set of neighbours* of m_i. We assume that each mote m_i is able to determine $N(m_i)$ and compare its cardinality $|N(m_i)|$ with $|N(m_j)|$ for any neighbouring mote m_j.

Near-maximal cliques can be constructed by the following simple greedy procedure ConstructClique which takes as input a set $S \subseteq M$ and two subsets $A \subseteq S$ and $B \subseteq S$ such that $|A| \geq 1$ and $|B| \geq 1$. It returns a clique in S that contains at least one element of A and at least $\min\{3, |B|\}$ elements of B (if the resulting clique has at least 5 elements).

ConstructClique(S, A, B)
$X \leftarrow \emptyset$
While S is not a clique or a subset of B
  If $|S \cap A| = 1$ and $|S| > 5$
    Then $X \leftarrow S \cap A$
  If $|S \cap B| > 3$
    Then choose $x \in S \setminus X$ such that $|N(x) \cap S|$ is minimum
    Else choose $x \in S \setminus (B \cup X)$ such that $|N(x) \cap S|$ is minimum
  $S \leftarrow S \setminus \{x\}$
Return S

The main algorithm for constructing a set of cliques $\mathcal{C}$ is as follows. It takes as input M, the set of motes, including knowledge of which motes are anchors. For any clique S, $d(S)$ will denote its distance in G_P from the closest clique containing an anchor mote.

SetOfCliques(M)

1. Set $\mathcal{C} \leftarrow \emptyset$, $d_a \leftarrow 0$. Label all motes as available.
2. Let M_{av} be the set of available motes.
   Choose an available anchor mote $m_a \in M_{\text{av}}$.
   $M_a \leftarrow N(m_a) \cap M_{\text{av}}$
   $S \leftarrow$ ConstructClique$(N(m_a), M_a, \{m_a\})$.
   Set $d(S) \leftarrow d_a$ and $\mathcal{C} \leftarrow \mathcal{C} \cup \{S\}$.
   Label all elements of S as not available.
3. While there is an available anchor, repeat Step 2.
4. Let $d_a \leftarrow d_a + 1$.
5. Let M_{av} be the set of available motes.
   Choose $T \in \mathcal{C}$ with $d(T) = d_a - 1$ and $x \in T$ such that $|N(x) \cap T| \geq 3$ and $|N(x) \cap M_{\text{av}}|$ is as large as possible.
   $M_x \leftarrow N(x) \cap M_{\text{av}}$
   $S \leftarrow$ ConstructClique$(N(x), M_x, N(x) \cap T)$.
   If S contains available motes and $|S| \geq 5$, set $d(S) \leftarrow d_a$ and $\mathcal{C} \leftarrow \mathcal{C} \cup \{S\}$.
   Label all elements of S as not available.
6. Repeat Step 5 until it no longer reduces $|M_{\text{av}}|$.
7. Repeat Steps 4 - 6 until $M_{\text{av}} = \emptyset$. Return $\mathcal{C}$ and d_a.

Note that the strategy of constructing as many cliques as possible before increasing d_a in the algorithm acts to minimise the maximum distance of any clique from an anchor clique. It is possible, however, that the algorithm may produce a set of cliques $\mathcal{C}$ such that $G_P(\mathcal{C})$ is not connected. The number of connected components $G_P(\mathcal{C})$ will be at most k, the number of anchors in M. In general, we would require only a small number of extra cliques to interconnect these components (usually, no more than $k-1$ cliques). In order to connect all cliques we now run the following algorithm, which takes as input $\mathcal{C}$ (the set of cliques from the previous algorithm).

ConnectCliques$(\mathcal{C})$

1. Choose a connected component of $G_P(\mathcal{C})$. Let $\mathcal{K} \subseteq \mathcal{C}$ be the set of cliques in that connected component.
2. Label all motes not in a clique of $\mathcal{K}$ as available
3. Let M_{av} be the set of available motes.
   Choose $T \in \mathcal{K}$ with $d(T) = d_a - 1$ and $x \in T$ such that $|N(x) \cap T| \geq 3$ and $|N(x) \cap M_{\text{av}}|$ is as large as possible.
   $M_x \leftarrow N(x) \cap M_{\text{av}}$
   $S \leftarrow$ ConstructClique$(N(x), M_x, N(x) \cap T)$.
   If S contains available motes and $|S| \geq 5$, set $\mathcal{C} \leftarrow \mathcal{C} \cup \{S\}$ and set $\mathcal{K} \leftarrow$ (all cliques in the same connected component of $G_P(\mathcal{C})$ as S).
4. Repeat Steps 2 - 3 until $M_{\text{av}} = \emptyset$. Return $\mathcal{C}$.

This procedure computes a good heuristic solution $\mathcal{C}$ to the Clique Problem. It is straightforward to see that these algorithms can be implemented in a distributed setting.

5 Choosing Pseudo-anchors

Having constructed a set of cliques $\mathcal{C}$ solving the Clique Problem we now have an efficient way to propagate location information throughout the network, via the propagation graph $G_P(\mathcal{C})$. However, within each clique we have the question of which motes to choose for the transmissions.

Pseudo-Anchor Problem: Given a clique in M, which 3 motes do we choose as pseudo-anchors to localise the rest?

Note that we may not have a completely free choice. If the clique contains an anchor it obviously should be chosen as one of the three pseudo-anchors. Also there may be an advantage in choosing a mote in the intersection of two cliques to be a pseudo-anchor for both cliques, as it will reduce the number of parameters in the whole system.

Beyond these more obvious criteria, it is clear that to reduce errors we should aim to choose motes that are as widely spread within the clique as possible. We can attempt to do this in a distributed way by using each mote's knowledge of its set of neighbours. Let "$\triangle$" denote *symmetric difference*. Then two motes m_i and m_j in a clique will tend to be far apart if $|N(m_i) \triangle N(m_j)|$ is large. In particular, for a uniform random distribution of motes the probability of the two most distant motes m_i and m_j in a given clique having the maximum value of $|N(m_i) \triangle N(m_j)|$ in the clique converges to 1 as the number of motes and their density increases.

Given a clique $S \in \mathcal{C}$ we can try to find $m_i, m_j, m_k \in S$ such that

$$\begin{aligned} F(i,j,k) \quad &:= \quad |N(m_i) \triangle N(m_j)| + |N(m_i) \triangle N(m_k)| \\ &\quad + |N(m_j) \triangle N(m_k)| \end{aligned}$$

is as large as possible. This will tend to ensure that m_i, m_j, m_k are nicely distributed close to the boundary of S.

We now describe an efficient heuristic algorithm for finding such a triple m_i, m_j, m_k. First observe that

$$\begin{aligned} F(i,j,k) \quad &= \quad 2(|N(m_i) \cup N(m_j) \cup N(m_k)| \\ &\quad - |N(m_i) \cap N(m_j) \cap N(m_k)|). \end{aligned}$$

For the given clique S, the following algorithm provides an efficient greedy approach to maximising $F(i,j,k)$.

MaximumSpread(S)

1. For each $m_i \in S$ the mote m_i computes $|N(m_i)| := K_i^1$.
   Choose m_i such that K_i^1 is maximum.
   Set $a(1) \leftarrow i$ (where i is the index such that K_i^1 is maximum.)
2. For each $m_i \in S \setminus \{m_{a(1)}\}$ the mote m_i computes $|N(m_i) \setminus N(m_{a(1)})| := K_i^2$.
   Choose $m_i \in S \setminus \{m_{a(1)}\}$ such that K_i^2 is maximum.
   Set $a(2) = i$.
3. For each $m_i \in S \setminus \{m_{a(1)}, m_{a(2)}\}$ the mote m_i computes $|N(m_i) \setminus (N(m_{a(1)}) \cup N(m_{a(2)}))| - |N(m_{a(1)}) \cap N(m_{a(2)}) \cap N(m_i)| := K_i^3$.
   Choose $m_i \in S \setminus \{m_{a(1)}, m_{a(2)}\}$ such that K_i^3 is maximum.
   Set $a(3) = i$.
4. Return $m_{a(1)}, m_{a(2)}, m_{a(3)}$.

The reason that $m_{a(1)}$, $m_{a(2)}$, $m_{a(3)}$ make a good choice of pseudo-anchors of S is as follows:

From the algorithm it is easy to see that

$$K_{a(1)}^1 + K_{a(2)}^2 + K_{a(3)}^3 = \frac{1}{2} F(a(1), a(2), a(3)).$$

Now note that in Step 1 we have chosen $K_{a(1)}^1$ so that it is maximum among all K_i^1's. Similarly, Step 2 ensures that $K_{a(1)}^1 + K_{a(2)}^2$ is maximum for the given choice of $a(1)$ in Step 1. Finally, Step 2 ensures that $K_{a(1)}^1 + K_{a(2)}^2 + K_{a(3)}^3$ is maximum for the given choice of $a(1)$ and $a(2)$ in the previous steps.

It follows that the above algorithm is a greedy (thought not, generally, optimal) algorithm for finding a set of pseudo-anchors $m_i, m_{j)}, m_k$ so that $F(i,j,k)$ is large. It is clear that it can be easily implemented in a distributed setting.

The above algorithm is linear and can be shown to have an approximation bound of 2/3. That is, if $F_{\text{MAX}} := \max(F(i,j,k))$, where the maximum is taken over all $m_i, m_j, m_k \in S$ then for motes $m_{a(1)}, m_{a(2)}, m_{a(3)}$ found via the above algorithm, we have

$$F(a(1), a(2), a(3)) \geq \frac{2}{3} F_{\text{MAX}}.$$

We can improve the bound to 7/9 by modifying the algorithm to a quadratic algorithm where $m_{a(1)}, m_{a(2)}$ are first chosen so that $K_{a(1)}^1 + K_{a(2)}^2$ is maximum (over any choice of $a(1), a(2)$) which can be done in quadratic time, and then finding $a(3)$ via Step 3 above. Proofs of these approximation bounds are not given here.

6 Computational Results

The behaviour of the distributed self-localisation scheme has been examined for the scenario shown in Figure 2, via a series of simulations. Anchor nodes were placed at the vertices of a square with side length 250m. The unknown nodes were randomly distributed in the region $[10, 240]^2$. Of particular interest was the number of cliques formed in the proposed approximate solution to the clique problem. The number of radio transmissions required to acquire RIPS measurements, given by three times the number of cliques, was compared to the number of radio transmissions required to gather all possible RIPS measurements (as used in [2]). Results were obtained for various numbers of unknown nodes and various communication ranges. Note that we have not included radio transmissions required to construct the cliques or determine the pseudo-anchors.

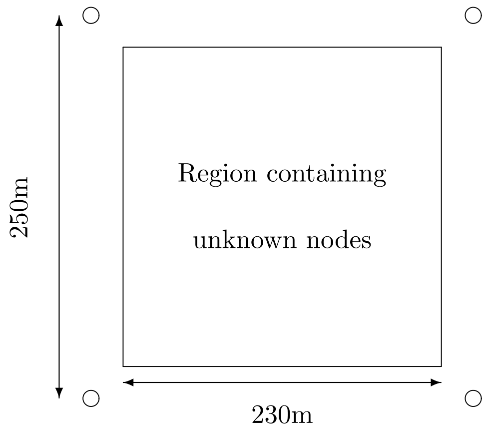

Figure 2: Localisation scenario. Circles indicate the anchor node locations. The unknown nodes are randomly distributed within the inner 230m × 230m square.

Tables 1 and 2 respectively show the number of radio transmissions for the proposed scheme and for the case where all possible RIPS measurements are collected. Results are shown for various numbers of unknown and various communication ranges. It can be seen that the proposed scheme requires an extremely small number of radio transmissions which remains essentially constant as the number of nodes increases for a fixed observation region and communication range. For a given surveillance region, decreasing the communication range will tend to decrease the size of the cliques therefore increasing the required number of cliques. For a random deployment of q motes, each with communication range R, the expected number of transmissions required to find all measurements is proportional to q^2R. The results shown in Table 2 indicate that collecting all measurements will be very difficult for any reasonably sized network.

Table 1: Number of radio transmissions required for measurement collection for the proposed scheme.

R (m)	Number of nodes		
	50	75	100
100	48	39	33
125	27	33	30
150	15	15	15

An important issue which is only partially addressed by the proposed localisation scheme is identifiability, that

Table 2: Number of radio transmissions required for collection of all possible measurements.

R (m)	Number of nodes		
	50	75	100
100	573	1203	2060
125	789	1672	2888
150	980	2120	3638

is, the capacity of the measurements to provide the information needed to unambiguously localise the motes. The proposed scheme guarantees local identifiability of the node locations; in other words if we know that the motes lie in a small region we can localise them. It does not guarantee global identifiability. In some cases, where the motes are sparsely distributed, it may be necessary to take additional measurements to ensure global identifiability. This raises two interesting questions for future research. First how do we ascertain global identifiability and second, for a scenario without global identifiability, which measurements should be taken to give global identifiability. We will return to these topics in future publications.

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