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Four Rotors Helicopter Yaw and Altitude Stabilization

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Abstract— **In this paper an appropriate four rotors helicopter nonlinear dynamic model for identification and control law synthesis is obtained via modelization procedure where several phenomena are included like gyroscopic effects and aerodynamic friction. The numerical simulations of the model obtained show that the control law stabilizes the four rotors helicopter with good tracking.**

I. Introduction

Autonomous Unmanned Air vehicles (UAV) are increasingly popular platforms, due to their use in military applications, traffic surveillance, environment exploration, structure inspection, mapping and aerial cinematography [1]. For these applications, the ability of helicopters to take off and land vertically, to perform hover flight, as well as their agility, make them ideal vehicles.

Four rotors helicopter Fig.1 have several basic advantages over manned systems including increased manoeuvrability[2], low cost, reduced radar signatures. Vertical take off and landing type UAVs exhibit further advantages in the manoeuvrability features. Such vehicles are to require little human intervention from take-off to landing. This helicopter is one of the most complex flying systems that exist. This is due partly to the number of physical effects (Aerodynamic effects, gravity, gyroscopic, friction and inertial counter torques) acting on the system [4].The idea of using four rotors is not new. A full-scale four rotors helicopter was built by De Bothezat in 1921[3].

Helicopters are dynamically unstable and therefore suitable control methods are needed to stabilise them. In order to be able to optimize the operation of the control loop in terms of stability, precision and reaction time, it is essential to know the dynamic behavior of the process wich can be established by a representative mathematical model.

II. Dynamic modeling of quadrotor

The mini quadrotor is a four rotors helicopter. Each rotor consists of propeller driven by a geared electrical DC motor. The two pairs of propellers, (1,3) and (2,4) Fig. 1, turn

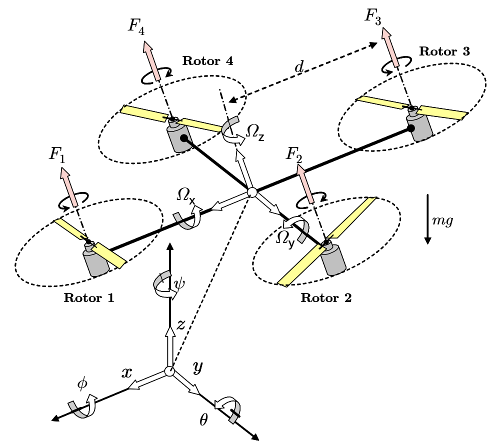

Fig. 1. Quadrotor helicopter

in opposite directions. Forward motion is accomplished by increasing the speed of the rear rotor while simultaneously reducing the forward rotor by the same amount. Left and right motions work in same way. Yaw command is accomplished by accelerating the two clockwise turning rotors while decelerating the counter-clockwise turning rotors.

The system is restricted with six degrees of freedom according to the earth fixed frame given respectively by position and the attitude [5]. The absolute position of center of mass of quadrotor is described by $\xi = [x, y, z]^T$ and its attitude by the three Euler's angles $\alpha = [\phi, \theta, \psi]^T$, these three angles are respectively pitch angle ($-\frac{\pi}{2} \leq \phi < \frac{\pi}{2}$), roll angle ($-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$) and yaw angle ($-\pi \leq \psi < \pi$).

The rotational matrix between the earth-fixed frame and the body-fixed frame can be obtained based on Euler angles is given as follows:

$$R = R_{z,\psi}.R_{y,\theta}.R_{x,\phi} \tag{1}$$

where

$$Rot_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix}$$

$$Rot_{y,\theta} = \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix}$$

$$Rot_{z,\psi} = \begin{bmatrix} C_\psi & -S_\psi & 0 \\ S_\psi & C_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with $S_{(.)}$ and $C_{(.)}$ represent $\sin(.)$ and $\cos(.)$ respectively.

$$R = \begin{bmatrix} C_\theta C_\psi & C_\psi S_\theta S_\phi - C_\phi S_\psi & C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\theta S_\psi & S_\theta S_\phi S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \tag{2}$$

The dynamic equations of quadrotor are described in the following sub-sections:

A. Translation dynamic model

The control of the qudrotor can be thought of as achieving force and torque balance. It will hover in the air when there is not net force in any degree of freedom. The smallest force will result in linear acceleration. Force balance for a stable hover is achieved when the sum of the thrust from the four rotors equals the weight of the quadrotor [6].

Motion is opposed by forces from three sources: gravity, inertia and air drag. Gravity opposes vertical motion, air drag provides damping to linear and rotory motion. As the drag force is proportional to velocity, drag forces are small except for those in oppostion to the rotation of the rotor.

Using the Newton's law [7] about translation motion, we obtain:

$$F_f + F_{dt} + F_G = m\ddot{\xi} \tag{3}$$

where m is the mass of quadrotor and F_f, F_{dt} and F_G are respectively the the forces generated by the propeller system, the drag force and the gravity force, such as:

$$F_{dt} = K_{dt}\dot{\xi} \tag{4}$$

where $K_{dt} = diag(K_{dtx}, K_{dty}, K_{dtz})$ is the translation drag coefficients.

$$F_G = mG \tag{5}$$

with $G = [0, 0, g]^T$ is gravity vector.

The forces generated by the propeller system of the quadrotor described in the earth-fixed frame are given by the following equations [2]:

$$F_f = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = R \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix} = \begin{bmatrix} C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\phi S_\theta S_\psi - S_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix} \sum_{i=1}^{4} F_i \tag{6}$$

In equation (6), F_i is the lift force generated by the rotor i and it's proportional to the square of the angular speed rotation ω_i [10], such as:

$$F_i = K_l \omega_i^2 \tag{7}$$

where K_l is the lift constant containing the air density ρ and the lift coefficient C_z and the blade rotor characteristics (diameter, step, profile, ...) and it's given as follows:

$$K_l = \frac{1}{2}\rho S C_z$$

Using the equation (3) the dynamic equation of translation becames:

$$m\ddot{\xi} = F_f - K_{dt}\dot{\xi} - mG \tag{8}$$

B. Rotation dynamic model

The main physical effects acting on a quadrotor are: aerodynamic effects, Inertial counter torques, aerodynamic friction and gyroscopic effects. Using the Newton's law about the rotation motion, the sum of moments is given as follow:

$$\tau_f - \tau_a - \tau_g = J\dot{\Omega} + \Omega \wedge J\Omega \tag{9}$$

where $J = diag(I_x, I_y, I_z)$ is the inertia matrix, this last is diagonal matrix due to the symetry of the quadrotor, the coupling inertia is assumed to be zero. And $\wedge$ denotes the product vector.

Ω is the angular speed expressed in body fixed frame [8],

$$\Omega = M\dot{\alpha} \tag{10}$$

with

$$M = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\phi C_\theta \end{bmatrix}$$

τ_f the moment developed by the quadrotor according to the body fixed frame is given by:

$$\tau_f = \begin{bmatrix} d(F_3 - F_1) \\ d(F_4 - F_2) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} \tag{11}$$

with d the distance between the quadrotor center of mass and the rotation axe of propeller and M_i the quadrotor moment developped about z axis.

$$M_i = K_d \omega_i^2$$

where:K_d is the drag coefficient of rotation

τ_a is the aerodynamic friction torque

$$\tau_a = K_{af}\Omega^2 \tag{12}$$

with $K_{af} = diag(K_{afx}, K_{afy}, K_{afz})$ is the aerodynamic friction coefficients

τ_g is the gyroscopic torque, the rotors turn at speed up to 2,500 rpm [6]. The axes of these motors (spin axes) are parallel to z axis of the plateform. When the quadrotor rolls or pitches it changes the direction of the angular momentum vectors of the four motors. The result is a gyroscopic torque (13) that attempts to turn the spin axis so that it aligns with rotation around the z axis.

$$\tau_g = \sum_{i=1}^{4} \Omega \wedge J_r W_i \tag{13}$$

with $W_i = \left[0, 0, (-1)^{i+1}\omega_i\right]^T$ ω_i is angular speed of rotor i, and J_r is the rotor inertia.

C. Rotor dynamic model

The equations governing the operation of the motor are given by:

$$\begin{cases} U = RI + L\frac{dI}{dt} + E \\ \Gamma_m = J_r\frac{d\omega}{dt} + C_s + \Gamma_r \end{cases} \tag{14}$$

where:

$$\begin{cases} E = K_e\omega \\ \Gamma_m = K_m I \\ \Gamma_r = K_r\omega^2 \end{cases} \tag{15}$$

The motor have a very small inductance, then the DC- motor model becomes:

$$(a_0 + a_1\omega + a_2\omega^2 + \dot{\omega})/b = U \tag{16}$$

with: $a_0 = \frac{C_s}{J_r}$, $a_1 = \frac{K_e K_m}{J_r R}$, $a_2 = \frac{K_r}{J_r}$ and $b = \frac{K_m}{J_r R}$.

The parameters are defined in Table I.

Symbol	definition
U	motor input
ω	angular speed
Ke, Km	electrical and mechanical torque constant
K_r	load torque constant
J_r	rotor inertia
Cs	solid friction

TABLE I
ROTOR PARAMETERS

III. LAW CONTROL DESIGN

Our main objective is to design a classic law control in order to stabilize the yaw angle and the altitude of the platform.. The flying machine we have used is a mini rotorcraft having a four rotors (*Draganflyer IV* not including electronics control) manufactured by *Draganfly* Innovations, Inc.(http://www.rctoys.com). The physical characteristics [9] are given in table II.

Weight (including the support)	$400g$
Blade diameter	$29cm$
Blade step	$11cm$
Distance between the motor and teh C.G	$20.5cm$
Motor reduction rate	$1:6$

TABLE II
ROTOR CHARACTERISTICS

A. Space state representation

The main objective of this subsection is to write the dynamic model in a form appropriate to control including the rotors dynamics , The system is given as follow:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)\Phi(x_2) - w = f_1(x_1) + g_1(x_1)v - w \\ \dot{x}_2 = f_2(x_2) + g_2 u \end{cases}$$

where:

$$f_1(x_1) = \left[\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}, \frac{K_{dtx}}{m}\dot{x}, \frac{K_{dty}}{m}\dot{y}, \frac{K_{dtz}}{m}\dot{z} - g, f_\phi, f_\theta, f_\psi\right]^T$$

$$x_1 = \left[x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}\right]^T$$

with:

$$\begin{aligned} f_\phi = {} & \frac{1}{I_x}\left[(I_{zy})C_\phi S_\phi\dot{\theta}^2 + (I_x + C_{2\phi}(I_{yz}))C_\theta\dot{\theta}\dot{\psi} + (I_{yz})C_\theta^2 C_\phi S_\phi\dot{\psi}^2\right] \\ & + \frac{1}{I_y}\Big[(I_x + I_{yz})(S_\phi^2 T_\theta\dot{\theta}\dot{\phi} - C_\phi S_\phi S_\theta\dot{\phi}\dot{\psi}) + I_{xz}C_\phi S_\phi S_\theta^2\dot{\psi}^2 \\ & -(I_{xy} - I_z)S_\phi^2 S_\theta T_\theta\dot{\theta}\dot{\psi}\Big] \\ & + \frac{1}{I_z}\Big[(I_{xy} + I_z)(C_\phi^2 T_\theta\dot{\theta}\dot{\phi} + C_\phi S_\phi S_\theta\dot{\phi}\dot{\psi}) - I_{xy}C_\phi S_\theta^2 S_\phi\dot{\psi}^2 \\ & -(I_{xy} - I_z)C_\phi^2 S_\theta T_\theta\dot{\theta}\dot{\psi}\Big] \end{aligned}$$

$$\begin{aligned} f_\theta = {} & \frac{1}{I_z}\Big[-(I_{xy} + I_z)(S_\phi C_\phi\dot{\theta}\dot{\phi} + C_\theta S_\phi^2\dot{\phi}\dot{\psi}) + I_{xy}C_\theta S_\theta S_\phi^2\dot{\psi}^2 \\ & +(I_{xy} - I_z)C_\phi S_\phi S_\theta\dot{\theta}\dot{\psi}\Big] \\ & + \frac{1}{I_y}\Big[(I_x + I_{yz})(S_\phi C_\phi\dot{\theta}\dot{\phi} + C_\theta C_\phi^2\dot{\phi}\dot{\psi}) + I_{xz}C_\theta C_\phi^2 S_\theta\dot{\psi}^2 \\ & -(I_{xy} - I_z)S_\theta S_\phi C_\phi\dot{\theta}\dot{\psi}\Big] \end{aligned}$$

$$\begin{aligned} f_\psi = {} & \frac{1}{I_y}\Big[(I_x + I_{yz})(S_\phi^2 S_{e\theta}\dot{\theta}\dot{\phi} - C_\theta S_\phi\dot{\phi}\dot{\psi}) + I_{xz}C_\theta S_\theta S_\phi\dot{\psi}^2 \\ & -(I_{xy} - I_z)S_\phi^2 T_\theta\dot{\theta}\dot{\psi}\Big] \\ & + \frac{1}{I_z}\Big[(I_{xy} + I_z)(C_\phi^2 S_{e\theta}\dot{\theta}\dot{\phi} + C_\phi S_\phi\dot{\phi}\dot{\psi}) - I_{xy}C_\phi S_\theta S_\phi\dot{\psi}^2 \\ & -(I_{xy} - I_z)C_\phi^2 T_\theta\dot{\theta}\dot{\psi}\Big] \end{aligned}$$

with:$S_{e\theta}$ is the abreviation of $\frac{1}{\cos\theta}$ and I_{xy}, I_{yz} ...are the abreviation of $I_x - I_y$,$I_y - I_z$...

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$$g_1(x_1) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{f_x}{m} & 0 & 0 & 0 \\ \frac{f_y}{m} & 0 & 0 & 0 \\ \frac{f_z}{m} & 0 & 0 & 0 \\ 0 & \frac{d.K_l}{I_x} & \frac{d.K_l}{I_y} T_\theta S_\phi & \frac{d.K_l}{I_z} T_\theta C_\phi \\ 0 & 0 & \frac{d.K_l}{I_y} C_\phi & -\frac{d.K_l}{I_z} S_\phi \\ 0 & 0 & \frac{d.K_l}{I_y} S_\phi S_{e\theta} & \frac{d.K_l}{I_z} C_\phi S_{e\theta} \end{bmatrix}$$

where: $\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\phi S_\theta S_\psi - S_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix}$

and

$$v = \begin{bmatrix} \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 \\ \omega_3^2 - \omega_1^2 \\ \omega_4^2 - \omega_2^2 \\ \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 \end{bmatrix}$$

$$w = \tau_a + \tau_g$$

The rotors dynamics is given by:

$$\dot{x}_2 = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \\ \dot{\omega}_4 \end{bmatrix} = \begin{bmatrix} f(\omega_1) \\ f(\omega_2) \\ f(\omega_3) \\ f(\omega_4) \end{bmatrix} + \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

The control of the yaw angle and the vertical position can be obtained by using the following control inputs:

$$U_z = K_{iz}\left(\int (Z_d - Z)\right) - K_z\left(Z + K_{dz}\dot{Z}\right) + offset$$
$$U_\psi = K_{i\psi}\left(\int (\psi_d - \psi)\right) - K_\psi\left(\psi + K_{d\psi}\dot{\psi}\right)$$

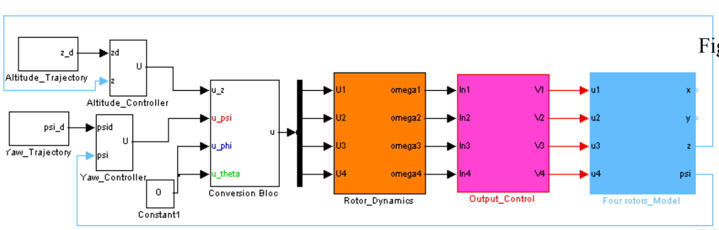

Fig. 2. Synoptic_shema

The simulate parameters are given as follow[11]

B. Dynamic parameters

$$J = diag(\ 3.8278\ ,\ 3.8278\ ,\ 7.1345\ ) \times 10^{-3} N.m/rad/s^2$$
$$K_{af} = diag(\ 5.567\ ,\ 5.567\ ,\ 6.354\ ) \times 10^{-4} N/rad/s$$
$$K_{dt} = diag(0.032\ ,\ 0.032\ ,\ 0.048) N/m/s$$

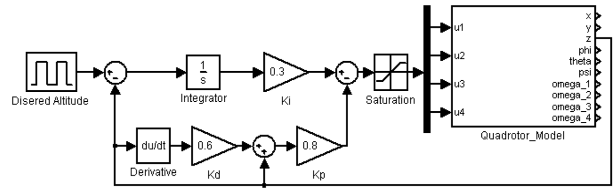

Fig. 3. z position controller

C. Static parameters

$K_l = 2.9842 \times 10^{-5} N/rad/s$. $K_d = 3.232 \times 10^{-7} N.m/rad/s$

D. Rotor parameters

$a_0 = 189.63$, $a_1 = 6.0612$, $a_2 = 0.0122$, $b = 280.19$ and $J_r = 2.8385 \times 10^{-5}.N.m/rad/s^2$.

E. Controler parameters

The desired trajectory is a square signal between $0.2\ m$ and $0.45\ m$.

The controller gain values are respectively ($K_z = 0.8$ $K_{dz} = 0.6$, $K_{iz} = 0.3$, $K_\psi = 0.1$, $K_{d\psi} = 0.1$ and $K_{i\psi} = 0$).

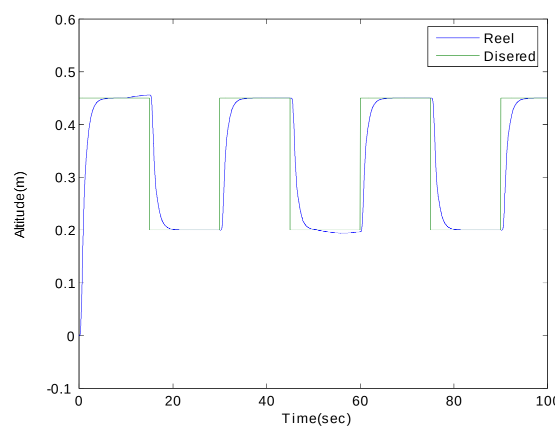

Fig. 4. Altitude Response

IV. Conlusion

In this paper we gave the complete model of a quadrotor UAVobtained via the Newton formalism, written in a form suited for identification of the pertinent dynamic parameters. In order to validate the obtained model via parameter estimation twe applied a law control to the estimated system. The obtained results of the regulation test show that the estimated model is very acceptable.

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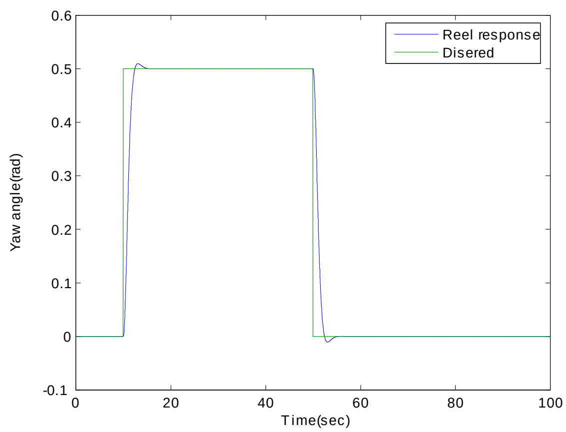

Fig. 5. Yaw angle response

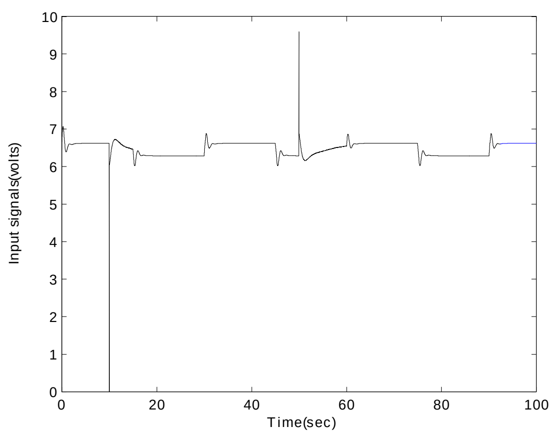

Fig. 6. Input signals

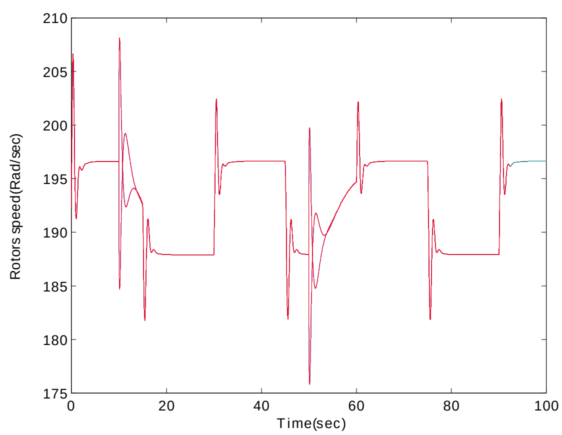

Fig. 7. Rotrs speed

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