A Note on Tornado Diagrams in Interval Decision Analysis

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Abstract—**The research efforts of the DECIDE Research Group have resulted in a decision tool capable of handling imprecise information in complex decision situations. Some of the research has been directed towards developing decision analytical algorithms and applying these algorithms in a graphical user interface. The decision tool takes intervals as well as comparative relations as input constraint, and is incorporating sensitivity analyses into the representation, instead of applying separate sensitivity analyses on top of the evaluation procedure. However, besides the built-in sensitivity analysis, a second form of sensitivity analysis could be useful in order to point out the most critical probabilities, values, or weights to the decision at hand. This paper deals with the problems and the implementation of interval tornado diagrams in a decision tool supporting interval probabilities, values, criteria weights, as well as comparative relations.**

Index Terms— **decision analysis, decision tree, imprecise information, tornado diagram.**

I. Introduction

In the market of decision analytic tools today, see e.g. [1], most of the software are incapable of handling imprecise information. Instead, the input needs to be specified in precise numbers, which often is considered unrealistic for decision-makers. The decision analytic tool described in this paper, *Decide*IT, coincides functionally with regular decision tree software, such as [2], [3], but also with multi-criteria software such as Expert Choice [4], since it handles probabilities, values, and weights in the same framework. However, *Decide*IT also handles imprecise information, not requiring a pointwise precision in the input parameters, but instead being capable of handling intervals as well as comparative statements between parameters, such as "greater than", "more important than", "equal to" or "between 20 and 40 more valuable than".

There have been quite extensive research efforts in the area of imprecise probabilities, see, e.g., [5], but few of the theories have been converted into practical applications. *Decide*IT has its roots in research on imprecise probabilities and has extended the ideas into interval values and interval criteria weights. The tool also contains an approach incorporating the basic sensitivity analysis into the representation [6]. However, beside the built-in sensitivity analysis, a second form of sensitivity analysis, a tornado diagram, could be useful to point out the most critical probabilities, values, or weights. This is valuable in cases where we need to allocate further investigation resources in order to discriminate the alternatives, yielding insights in which of the parameters we should put more effort into investigating.

II. The DecideIT Software

In *Decide*IT, see Fig. 1, the decision-maker can take advantage of both decision trees and multi-criteria models allowing the use of imprecise input in the form of interval and comparative statements, see [7] for a thorough presentation of the decision tool. In a multi-criteria model, each end node represents one criterion, which can be modeled in two ways; either directly in the multi-criteria model for criteria not containing probabilistic uncertainty, or in its own decision tree, which then are joined together in a criteria hierarchy. The multi-criteria decision problem can then be analyzed from a one-criterion perspective, or from an all-criteria perspective.
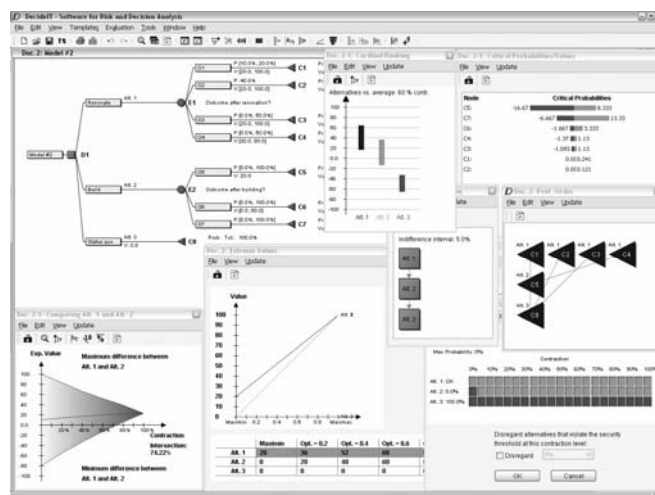

Fig. 1. DecideIT screenshot

The application consists of a graphical user interface, taking advantage of a set of algorithms capable of handling imprecise probabilities, values, and criteria weights, providing a good representation of the decision problem and the evaluation

results.

III. THE EMBEDDED SENSITIVITY ANALYSIS

Since the application takes imprecise input, the evaluations will also reflect that uncertainty, thus providing the decision-maker with expected value intervals that might overlap each other, i.e. one alternative might not dominate the others. The suggested solution is to examine in which parts of the intervals that we can obtain a dominating alternative, using an embedded sensitivity analysis. In order to explain this we must first introduce some concepts used in the tool.

A decision frame ($\mathbf{\Psi}$) is a model of the decision problem in the form of a decision tree, containing a set of alternatives ($\mathbf{A}$), a set of events ($\mathbf{E}$), a set of consequences ($\mathbf{C}$), and, in a multi-criteria problem, a set of criteria ($\mathbf{G}$). From $\mathbf{C}$, the value constraint set ($\mathbf{V}$) is created; a set of linear constraints containing the interval value statements of the leaf nodes together with the comparative value statements. From $\mathbf{E}$ the probability constraint set ($\mathbf{P}$) is created in a similar manner, but since the event nodes can be found on different depth in the decision tree $\mathbf{P}$ is a union of all local probability constraint sets, such that $\mathbf{P} = \cup \mathbf{P}_i$. Similarly, the same applies for the weight constraint set $\mathbf{W}$. Further, when dealing with probability and weight constraint sets an additional constraint must be taken into account for at each local constraint set; the normalisation constraint $\mathbf{P}_i = \Sigma\, p_i = 1$ and $\mathbf{W}_i = \Sigma\, w_j = 1$. In order for the decision-maker to be able to take advantage of this, the tool keeps track of all constraint sets, constantly performing consistency checks after a new statement has been added.

When performing a consistency check, the tool computes the orthogonal hull (Ξ) for each constraint set, yielding the consistent, i.e. the minimum and maximum, interval boundaries for each variable in $\mathbf{\Psi}$. The distribution over the orthogonal hull is, however, not uniform, but triangle shaped in its form. In addition to Ξ there is also a most likely point, a focal point (Φ), in the interval, see Fig. 2 for a visualisation.
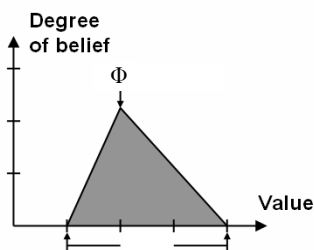

Fig. 2. The orthogonal hull (Ξ) and the focal point (Φ)

Φ is interpreted as the point where the belief mass are concentrated around, thus there are higher belief close to Φ than farther away. Φ can be retrieved in two ways, either by the decision-maker explicitly, or calculated by the tool as the centre of mass based on a uniform Ξ.

As mentioned earlier, since the application takes imprecise input, the evaluation result will also reflect that uncertainty, yielding overlapping expected value intervals, i.e. one alternative might not dominate the others. The probabilities (p),

values (v), and weights (w) are therefore subject to an embedded sensitivity analysis as well as a supplementary analysis, shown as a tornado diagram of the influences of the probabilities, values, and weights on the evaluation result. The embedded sensitivity analysis examines in which parts of the intervals that we can obtain a dominating alternative. This is made by introducing contraction levels, contracting the intervals, thus moving the interval boundaries closer towards Φ. The contraction level is $\tau \in [0, 1]$, where $\tau = 0$ preserves the boundaries, and where $\tau = 1$ transforms the constraint sets into single points, coinciding with Φ. The use of contraction levels can be seen as an embedded sensitivity analysis where the decision-maker gains a better understanding of the stability of the result and how important the interval boundary points are. For a more formal discussion on contraction levels, see [8].

IV. PERFORMING EXPLICIT SENSITIVITY ANALYSES

If the evaluation does not point out a single preferred alternative, the next step, to further discriminate the alternatives, could be to gather more information regarding the decision situation. In order to guide the allocation of analysis resources for further information gathering in an efficient way, the variables having the greatest impact on the expected value should be identified.

One established way of displaying a one-way sensitivity analysis of several variables in the same output window is by a *tornado diagram*. By showing the sensitivity ranges as bars and sorting the widest on top, the resulting picture resembles a tornado, see Fig. 3 for an example. The output can be interpreted such that value intervals having greater impact on the expected value are more critical, and information related to these consequences is important to investigate further.
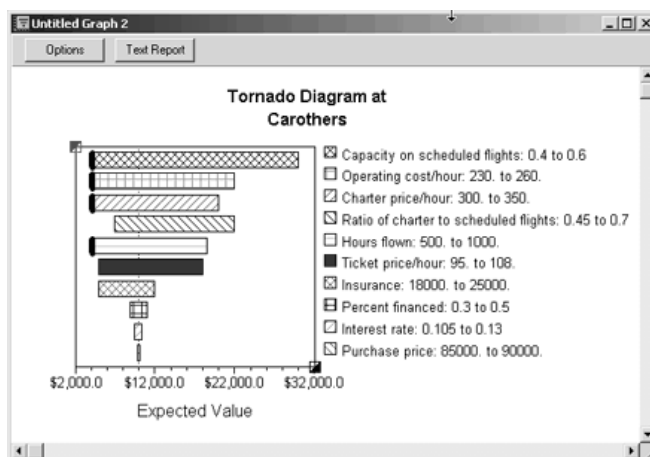

Fig. 3. A tornado diagram (TreeAge Pro Software)

The advantage of an interval approach is that the basis for calculating the critical values already is present, i.e. the interval width is given by the decision-maker already from the beginning. The problem, however, is that the parameters may include dependencies in the form of comparative value statements. Since we are aiming to provide the user with critical

probabilities and critical weights as well, the normalization constraint provides a further obstacle. Varying p_i, v_i, and w_i in isolation could yield incorrect results, due to possible comparative statements or normalization constraints.

Another problem is that a change in the probability of one node affects all the children of this node, which in turn affects the evaluation result. Similar problems are inherent when dealing with weights or comparative relations. Varying the value of one consequence might affect some other value due to value relations between them.

V. INTERVAL TORNADOS

In a pointwise specified decision problem (i.e. no intervals), a tornado consists simply of varying each of the variables, one at a time, either to their specified "best" and "worst" values or by a percentage, for example ±10%. In an interval decision tool, there is no obvious counterpart to the pointwise tornado diagram. In an interval evaluation, there is no single output expected value but rather an expected value interval within which the expected value falls when the input variables are kept within their interval input ranges and being consistent with the constraints. The meaning of an input interval constraint (or, to be more precise, the orthogonal hull) is the upper and lower bounds of the numbers that the variable could assume in the decision problem. But changing each variable within the interval, even letting it assume its lower and upper bounds, will not yield any new information since all possible consistent assignments of variables are already in the expected value range by definition.

To gain an overview of the sensitivity of the results of an interval decision evaluation, another approach is required in order to reflect the sensitivity of the output expected value to changes in the input variables. One approach is to study the sensitivity of the best representative of the expected value (the focal point).

The reference expected value (EU^Φ) is the expected value based on Φ. Using the focal point, EU^Φ is obtained for each alternative. EU^Φ is considered the best single representative for the expected value. Since it is calculated using the same expected value formula as all points in the expected value interval, it is a reasonable candidate for an anchor point in a sensitivity analysis. The analysis then studies the effects on the focal point of varying each variable, instantiating it with its lower and upper bounds, respectively. This way, the positive and negative impacts are obtained for each variable. The tornado diagram could also be based on the critical probabilities or the critical criteria weights, where the positive and negative impact, by varying p_i and w_i, is calculated in a similar manner. The positive impact on EU by varying v_i is defined as $\Omega_{v_i}^+ = EU_{v_i}^+ - EU^\Phi$, where $EU_{v_i}^+$ are calculated through varying v_i, retrieving new Φ, in order to yield the maximum EU. The negative impact on EU is similarly defined as $\Omega_{v_i}^- = EU_{v_i}^- - EU^\Phi$.

Each variable changes the focal point when assigned its lower and upper bound, not least because of dependencies between variables in an interval specified decision problem, such as comparisons ($v_{ij} > v_{kn}$) and normalisations ($\Sigma_j p_{ij} = 1$). The focal point is recomputed and the impact of the disturbance is then measured as the change in expected value for the recomputed focal point compared to the reference. Thus, the resulting impact range for each variable has the form $[\Omega_{x_i}^-, \Omega_{x_i}^+]$. Hence, collecting the intervals for each decision variable for each alternative is the operationalisation of degree of impact in an interval tornado analysis and the resulting impact intervals are displayed in a tornado diagram, sorted in decreasing interval width analogous to the pointwise decision problem. The tornado diagrams, yielding critical values and critical probabilities for decision trees, and critical weights and weighted critical values for multi-criteria models, have been implemented in the software *Decide*IT, see Fig. 4.
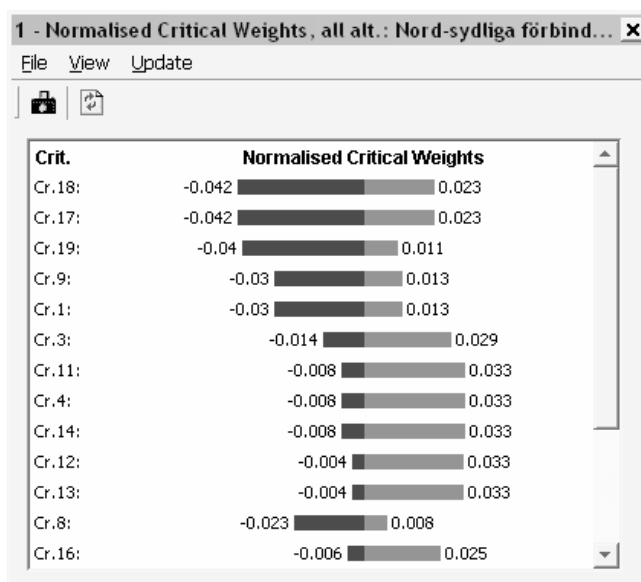

Fig. 4. Tornado diagrams showing the critical weights (DecideIT software)

A dark grey bar indicates that the expected value is influenced in a negative way, and a light grey bar indicates a positive influence on the expected value.

VI. CONCLUDING REMARKS

A tornado diagram can be used for displaying the sensitivity of probabilities and values in decision trees, and of weights and weighted values in multi-criteria models. The difference compared to traditional tornado diagrams is that interval ranges have already been specified by the decision-maker and taken account of in the resulting expected value interval. Furthermore, varying one variable at a time may not be possible due to dependencies between variables derived from, e.g., comparative relations and probability or weight normalisation constraints. We present a solution circumventing this problem, which has also been implemented in the software DecideIT.

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