Simplified Filtering Estimator for Spacecraft Attitude Determination from Phase Information of GPS Signals

S. Purivigraipong, Y. Hashida, and M. Unwin

***Abstract*—This paper presents an implementation of a simplified filtering estimator for satellite attitude determination using GPS (Global Positioning System) signals. The non-linear system of estimator was modelled using Euler angles parameterisation which applicable for real-time operation onboard-satellite. An approach was to keep a pitch state independent of roll and yaw. In order to cope with spacecraft dynamics, the moment of inertial tensor of spacecraft and torques caused by Earth's gravitation field were modelled to estimate the rate of change of angular velocity. Other advantage of the simplified filtering estimator was that the electrical path difference, line bias, between antenna chains caused an offset error in GPS measurements can be estimated. The developed filtering estimator was testing through simulated data, and flight data from real spacecraft.**

***Index Terms*—spacecraft attitude determination, GPS, line bias.**

I. Introduction

A deterministic algorithm such as QUEST [1] and TRIAD [1] can be used to determine attitude of spacecraft. However, since a new approach to linear filtering was proposed by Kalman [2], the Kalman filter has been used widely for space applications. By contrast to the deterministic algorithms, more accurate results could potentially be achieved. Nevertheless, the filtering requires the knowledge of the spacecraft dynamics.

In the traditional approach, spacecraft attitude determination depends on attitude sensors, such as magnetometers, Sun sensors, Earth sensors, inertial measurement units (IMU), and star sensors. The use of Kalman filtering based on quaternion parameterisation for attitude determination from traditional attitude sensor was presented in [3].

However, since the use of GPS has been successfully demonstrated in space navigation [4], a new approach is now available using GPS for attitude determination. The quaternion-based filtering estimator for spacecraft attitude determination from phase information of GPS signals presented in [5].

The use of quaternion parameters was suitable for manoeuvre operation and singularities avoidance. However, the way to model the estimator requires an extensive work on mathematics and modeling. Furthermore, for real-flight operation, the complexity could be arisen in the procedure of software implementation.

This paper presents an approach to model a simplified filtering estimator for flight operation. Using an Euler angles parameterisation, the pitch state was independent of roll and yaw. The advantage of this approach was that the system model was simplified and suitable for flight operation under small rotation angle.

II. Background

A. GPS Attitude Observable

The observable in GPS attitude determination is the carrier path difference, r (in length unit), between two antennas separated by the baseline length, l.

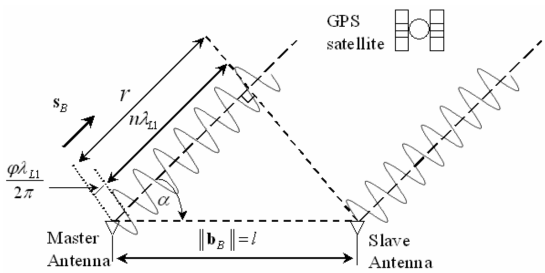

Figure 1. Carrier path difference for GPS attitude

A basic equation showing the relation between path difference and attitude matrix, $\mathbf{A}$, is expressed as

$$r = \bar{r} + n\lambda_{L1} = (\mathbf{s}_B \cdot \mathbf{b}_B) = \mathbf{b}_B^T \mathbf{A} \mathbf{s}_O \qquad (1)$$

where $\bar{r}$ is a *true modulo* path difference in length units, n is the *unknown* integer cycle, λ_{L1} is the *known* wavelength of GPS carrier frequency, $\mathbf{s}_B$ is the *unknown* unit vector directed to GPS satellite in body-fixed coordinates, and $\mathbf{b}_B$ is the

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known baseline vector in body-fixed coordinates, and $\mathbf{s}_O$ is the *known* line of sight unit vector to the GPS satellite in orbit-define coordinates.

In reality, the received phase measurements are perturbed by measurement noise (w), multipath and bias. The multipath signals are reflective signals from nearby objects surrounding the antenna. A bias error (called *line bias*), β, is a relative phase offset or phase delay between two antenna chains. Line bias is common to all measurements taken from a common pair of GPS antennas.

The full expression for path difference including integer cycles and measurement errors can be written

$$r_{RX} = (\overline{r} + w + \beta) + n\lambda_{L1} \tag{2}$$

where r_{RX} is the recovered path difference including measurement error.

B. Attitude Dynamics

It is supposed that a rigid body is moving in inertial coordinates. The motion can be described by the translation motion of its centre of mass, together with a rotation motion of the body about some axis through its centre of mass. The rotation motion is caused by the applied moment. The basic equation of attitude dynamics relates the time derivative of the angular momentum vector [6].

$$\mathbf{I}_{MOI}\dot{\boldsymbol{\omega}}_B^I = \left(\mathbf{N}_G + \mathbf{N}_M\right) - \boldsymbol{\omega}_B^I \times \left(\mathbf{I}_{MOI}\boldsymbol{\omega}_B^I\right) \tag{3}$$

where $\mathbf{I}_{MOI}$ is a moment of inertia tensor of spacecraft,
$\boldsymbol{\omega}_B^I$ is an angular rate vector referenced to the inertial frame, expressed in body-fixed coordinates,
$\mathbf{N}_G$ is a gravity-gradient torque vector,
$\mathbf{N}_M$ is a magnetorquers vector.

III. Simplified Model of Earth-Pointing Spacecraft Dynamics under Small Rotation Angles

This section describes spacecraft dynamics under small rotation angles using Euler angle representation. The Earth-pointing spacecraft is widely used for communication satellites and Earth observation satellites. The spacecraft rotates at one revolution per orbit in a near circular orbit with orbital angular rate, ω_o. The orbital rate vector can be written as

$$\boldsymbol{\omega}_o = \begin{bmatrix} 0 & -\omega_o & 0 \end{bmatrix}^T \tag{4}$$

The attitude angles are defined as roll, pitch and yaw which are treated as small errors about the velocity vector. The transformation matrix from orbit-defined coordinates to the body-fixed coordinates can be expressed as

$$\mathbf{A} \cong \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \tag{5}$$

where (ϕ, θ, ψ) denote roll, pitch and yaw angles, respectively.

The body angular rate vector referenced to orbit-defined coordinates can be derived from the rate of change of Euler angles (2-1-3 type)

$$\boldsymbol{\omega}_B^O = \begin{bmatrix} \omega_{Ox} & \omega_{Oy} & \omega_{Oz} \end{bmatrix}^T \cong \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \tag{6}$$

Where $\boldsymbol{\omega}_B^O$ is an angular rate vector referenced to the orbit-defined frame, expressed in body-fixed coordinates.

The body angular velocity vector referenced to inertial coordinate system can be derived

$$\boldsymbol{\omega}_B^I = \boldsymbol{\omega}_B^O + \mathbf{A}\boldsymbol{\omega}_o \equiv \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \mathbf{A}\begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\phi} - \omega_o\psi \\ \dot{\theta} - \omega_o \\ \dot{\psi} + \omega_o\phi \end{bmatrix} \tag{7}$$

where $\omega_o^2 = \mu_\oplus / \mathbf{R}_S^3$ is the orbital angular velocity of the spacecraft in the circular orbit of radius $\mathbf{R}_S$, and $\mu_\oplus$ is the Earth's gravitational constant. For example a typical orbit of a microsatellite at 800 km altitude is $\omega_o =$ 0.059 degree/second.

The zenith vector along the yaw axis in the orbit-defined coordinates is $(0,0,-1)$. Thus, the zenith vector in the body-fixed coordinates system, $\bar{\mathbf{z}}_B$, is

$$\bar{\mathbf{z}}_B = \mathbf{A}\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T = \begin{bmatrix} \theta & -\phi & -1 \end{bmatrix}^T \tag{8}$$

The simplified formulation of gravity-gradient torque, N_G, on the entire spacecraft can be expressed [6]

$$\mathbf{N}_G = 3\omega_o^2 \bar{\mathbf{z}}_B \times \left(\mathbf{I}_{MOI}\bar{\mathbf{z}}_B\right) \equiv 3\omega_o^2 \begin{bmatrix} \left(I_{zz} - I_{yy}\right)\phi \\ \left(I_{zz} - I_{xx}\right)\theta \\ 0 \end{bmatrix} \tag{9}$$

The above equation is simplified by linearisation for a spacecraft in a near circular orbit using small-angle approximation for ϕ and θ.

If we consider only the torque from Earth's gravitational field, the dynamic equation in the body-fixed coordinates system can be expressed as

$$\mathbf{I}_{MOI}\dot{\omega}_B^I = \begin{bmatrix} 3\omega_o^2\left(I_{zz}-I_{yy}\right)\phi - \left(I_{zz}-I_{yy}\right)\omega_y\omega_z \\ 3\omega_o^2\left(I_{zz}-I_{xx}\right)\theta + \left(I_{zz}-I_{xx}\right)\omega_x\omega_z \\ \left(I_{xx}-I_{yy}\right)\omega_x\omega_y \end{bmatrix} \tag{10}$$

where $\omega_B^I = \begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T$.

If the satellite has a symmetric structure in the x and y axes ($I_{xx} = I_{yy} = I_t$ = transverse inertia momentum), the dynamics equations are then rewritten as

$$\mathbf{I}_{MOI}\dot{\omega}_B^I \approx \begin{bmatrix} 3\omega_o^2\left(I_{zz}-I_t\right)\phi - \left(I_{zz}-I_t\right)\omega_z\omega_y \\ 3\omega_o^2\left(I_{zz}-I_t\right)\theta + \left(I_{zz}-I_t\right)\omega_z\omega_x \\ 0 \end{bmatrix} \tag{11}$$

From Equation (7), the first-order derivative of the body-fixed angular rate vector is

$$\dot{\omega}_B^I = \begin{bmatrix}\ddot{\phi} - \omega_o\dot{\psi} & \ddot{\theta} & \ddot{\psi} + \omega_o\dot{\phi}\end{bmatrix}^T \tag{12}$$

Substituting the component of angular velocity vector ω_B^I and its derivative $\dot{\omega}_B^I$ into Equation (11) yields

$$\begin{bmatrix}\ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi}\end{bmatrix} \approx \begin{bmatrix} 4(\kappa-1)\omega_o^2\phi + \kappa\omega_o\dot{\psi} \\ 3(\kappa-1)\omega_o^2\theta \\ -\omega_o\dot{\phi} \end{bmatrix} \tag{13}$$

where $\kappa = I_{zz}/I_t$

It can be seen that the pitch is separated from roll and yaw under small rotation angles. There is only a coupling term between roll and yaw.

IV. Simplified EKF (sEKF) Estimator

The assumptions of the sEKF estimator are listed as follows:

1) The spacecraft is nominally Earth pointing with either a certain spin rate in **Z** axis or 3-axis stabilised.
2) The spacecraft has a symmetric structure on **X** and **Y** axes ($I_{xx} = I_{yy} = I_t$ = transverse inertia momentum), and without any cross terms.
3) The orbit of the spacecraft is near circular with an almost constant angular rate.
4) The system noise model has zero mean.

A. State Vector

From Equation (13) in previous section, it is explicitly shown that pitch is independently separated from roll and yaw. The novel formulation identifies two state vectors that keep the pitch state independent of roll and yaw, and simplifies the general calculation.

The state vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, are defined as

$$\mathbf{x}_1 = \begin{bmatrix}\phi & \psi & \dot{\phi} & \dot{\psi}\end{bmatrix}^T \tag{14}$$

$$\mathbf{x}_2 = \begin{bmatrix}\theta & \dot{\theta} & \beta_1 & \beta_2\end{bmatrix}^T \tag{15}$$

B. System Model

The non-linear model is defined as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{w}(t) \tag{16}$$

where $\mathbf{f}(\mathbf{x}, t)$ is a non-linear system model,
$w(t)$ is a zero mean white system noise with covariance matrix **Q**

The difference between the actual state vector, $\mathbf{x}$, and estimated state vector, $\hat{\mathbf{x}}$, is defined as the state perturbation, $\Delta\mathbf{x}$

$$\Delta\mathbf{x}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \tag{17}$$

As it is assumed that $\Delta\mathbf{x}$ is small, the system model can be approximately derived from

$$\mathbf{f}(\mathbf{x}, t) \approx \mathbf{f}(\hat{\mathbf{x}}, t) + \mathbf{F}\cdot\Delta\mathbf{x} \tag{18}$$

where **F** is a linearised system model defined as

$$\mathbf{F} = \left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]_{\mathbf{x}=\hat{\mathbf{x}}} \tag{19}$$

As we consider only the torque from the Earth's gravitation field and from magnetorquers, then the dynamics equation of the system model is analytically simplified as

$$\dot{\mathbf{x}}_1 = \begin{bmatrix}\dot{\phi} \\ \dot{\psi} \\ \ddot{\phi} \\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix} \dot{\phi} & & & & \\ \dot{\psi} & & & & \\ 4(\kappa-1)\omega_o^2\phi + \kappa\omega_o\dot{\psi} & + & N_{Mx}/I_t & + & w_x \\ -\omega_o\dot{\phi} & + & N_{Mz}/I_z & + & w_z \end{bmatrix} \tag{20}$$

$$\dot{\mathbf{x}}_2 = \begin{bmatrix}\dot{\theta} \\ \ddot{\theta} \\ \dot{\beta}_1 \\ \dot{\beta}_2\end{bmatrix} = \begin{bmatrix} \partial\theta & & & & \\ 3(\kappa-1)\omega_o^2\theta & + & N_{My}/I_t & + & w_y \\ \dot{\beta}_1 & & & & \\ \dot{\beta}_2 & & & & \end{bmatrix} \tag{21}$$

Two discrete state transition matrices, Φ_1 and Φ_2, can be approximated for a short sampling period Δt

$$\Phi_1 \approx \mathbf{I}_{4\times4} + \mathbf{F}_1\Delta t \tag{22}$$

$$\Phi_2 \approx \mathbf{I}_{4\times4} + \mathbf{F}_2\Delta t \tag{23}$$

where $\Delta t = t_{(p+1)} - t_{(p)}$.

Therefore, the discrete state perturbation model is then given by

$$\Delta \mathbf{x}_{1(p+1)} = \Phi_{1(p)} \Delta \mathbf{x}_{1(p)} \quad (24)$$

$$\Delta \mathbf{x}_{2(p+1)} = \Phi_{2(p)} \Delta \mathbf{x}_{2(p)} \quad (25)$$

C. Measurement Model

A discrete non-linear measurement model is expressed as

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, t) + \mathbf{m}(t) \quad (26)$$

where $\mathbf{h}(\mathbf{x}, t)$ is a non-linear output model,
$\mathbf{m}(t)$ is a zero mean white measurement noise with scalar covariance R.

The linearised innovation error model is given by

$$\Delta \mathbf{r}_{(p)} = \mathbf{z}_{(p)} - \mathbf{h}_{(p)}(\hat{\mathbf{x}}, t) = \mathbf{H}_{(p)} \cdot \Delta \mathbf{x}_{(p)} + \mathbf{m}_{(p)}(t) \quad (27)$$

where $\Delta \mathbf{r}_{(p)}$ is an innovation vector at epoch p, an observation matrix $\mathbf{H}_{(p)}$ is defined as

$$\mathbf{H}_{(p)} = \left[\frac{\partial \mathbf{h}_{(p)}}{\partial \mathbf{x}} \right]_{\mathbf{x}=\hat{\mathbf{x}}} \quad (28)$$

The attitude matrix (2-1-3 type) for small angles in roll and pitch, but unlimited yaw rotation is used to calculate the predicted path difference.

$$\hat{\mathbf{A}}_{\Theta} = \begin{bmatrix} \cos\hat{\psi} & \sin\hat{\psi} & -\hat{\theta}\cos\hat{\psi} + \hat{\phi}\sin\hat{\psi} \\ -\sin\hat{\psi} & \cos\hat{\psi} & \hat{\theta}\sin\hat{\psi} + \hat{\phi}\cos\hat{\psi} \\ \hat{\theta} & -\hat{\phi} & 1 \end{bmatrix} \quad (29)$$

At epoch p, an observation matrix for each estimator is then obtained from

$$\mathbf{H}_{1(p)} = \left[\mathbf{b}_B^T \left(\frac{\partial \hat{\mathbf{A}}_{\Theta(p-1)}}{\partial \phi} \right) \mathbf{s}_{O(p)} \quad \mathbf{b}_B^T \left(\frac{\partial \hat{\mathbf{A}}_{\Theta(p-1)}}{\partial \psi} \right) \mathbf{s}_{O(p)} \quad 0 \quad 0 \right] \quad (30)$$

$$\mathbf{H}_{2(p)} = \left[\mathbf{b}_B^T \left(\frac{\partial \hat{\mathbf{A}}_{\Theta(p-1)}}{\partial \theta} \right) \mathbf{s}_{O(p)} \quad 0 \quad -1 \quad -1 \right] \quad (31)$$

D. Innovation Computation

The innovation is computed as the scalar difference between recovered path difference $\breve{r}$ and predicted path difference $\hat{r}^-$.

$$\delta r = \breve{r} - \hat{r}^- \quad (32)$$

Where δr is an innovation for one measurement.

At epoch p, knowledge of the quaternion-attitude, $\hat{\mathbf{A}}_{\Theta}$, from the previous epoch (p-1) is required to estimate the predicted path difference, $\hat{r}^-_{(p)}$.

$$\hat{r}^-_{(p)} = \mathbf{b}_B^T \hat{\mathbf{A}}_{\mathbf{q}(p-1)} \mathbf{s}_{O(p)} \quad (33)$$

For all GPS data at epoch p, the innovation is stacked into a vector $\Delta \mathbf{r}_{(p)}$.

E. Covariance Matrices

The error covariance matrices $\mathbf{P}_1$ and $\mathbf{P}_2$, are computed by

$$\mathbf{P}_{1(t)} = \left\langle \Delta \mathbf{x}_1 \cdot \Delta \mathbf{x}_1^T \right\rangle \quad (34)$$

$$\mathbf{P}_{2(t)} = \left\langle \Delta \mathbf{x}_2 \cdot \Delta \mathbf{x}_2^T \right\rangle \quad (35)$$

Note that, two estimators operate simultaneously. The original (8×8) covariance matrix is replaced with the (4×4) $\mathbf{P}_1$ and (4×4) $\mathbf{P}_2$ matrices.

V. Test Results

A. Simulated Results

Simulation results presented in this paper are based on a three-axis stabilised satellite in a circular orbit, 64.5 degrees inclination, and altitude 650 km. The nominal simulation parameters are given in Table 1.

Table 1: Nominal Simulation Parameters

parameter	x axis	y axis	z axis
moment of inertia (kg m^2)	40.0	40.0	40.0
initial attitude (degrees)	0.0	0.0	0.0
initial angular velocity (deg/s)	0.0	-0.06	0.0
baseline coordinates $\mathbf{b}_1$ (mm)	167.7	625.7	0.0
baseline coordinates $\mathbf{b}_2$ (mm)	-625.7	-167.7	0.0
line bias of baseline $\mathbf{b}_1$ (β_1) (mm)	90		
line bias of baseline $\mathbf{b}_2$, (β_2) (mm)	50		

The attitude dynamics of three-axis stabilised satellite was simulated. The six hours of simulated GPS measurements were used as the input file. It was important to note that in this paper, the measurement error is assumed as white Gaussian with 5 mm rms [7].

The setup parameters for the filtering estimator were shown in Table 2.

Table 2: Setup parameters for qEKF estimator

parameter	value	dimension
system noise variance, **Q**	1.0e-6	mixed dimension (rad^2 and rad^2/sec^2)
measurement noise variance, R	6.4e-5	metre^2
initial guess of attitude angles	0.0	degree
initial guess of angular velocities	0.0	degree/second

Using the filtering estimator, the estimated attitude disparity (estimated attitude from filtering compared to the reference attitude from simulation) in roll is plotted in Figure 2. The computed one-sigma rms of difference between reference attitude and estimated GPS attitude is shown in Table 3.

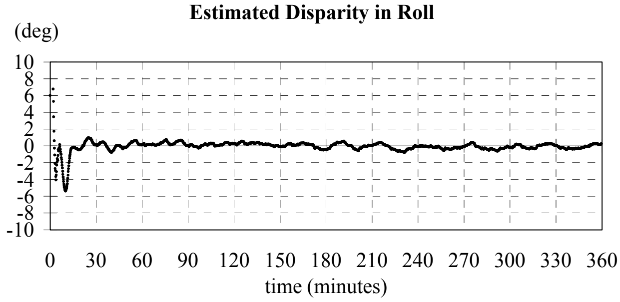

Figure 2. attitude disparity in roll

Table 3: One-sigma rms of difference between estimate and reference attitude (case: simulation)

disparity in roll	disparity in pitch	disparity in yaw
0.3°	0.2°	0.2°

As shown in the simulated results, the attitude error compared to reference solution is small than one degree. The estimated angular velocity in Y-axis also closes to the simulated velocity (-0.06 deg/sec) as shown in Figure 3. The estimated line bias is shown in Figure 4.

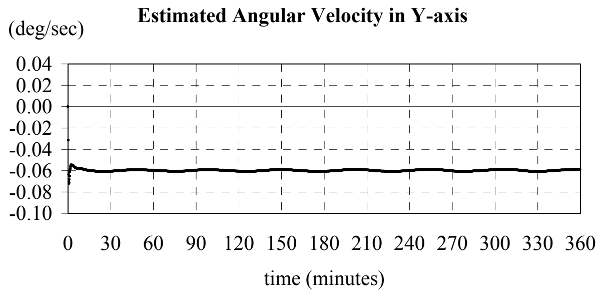

Figure 3. Estimated angular velocity in Y axis

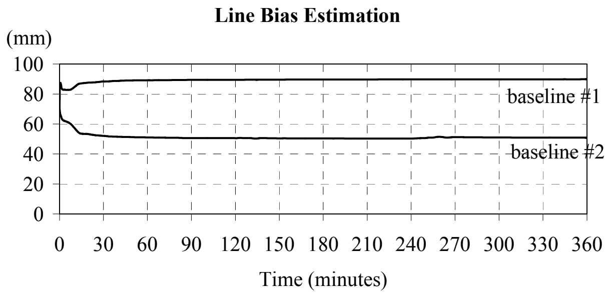

Figure 4. Estimated line bias for both antenna-baselines

As shown in Figure 4, the figure of estimated line bias for both baselines is close to the nominal in the Table 1.

B. Flight Results

This section shows an estimated attitude from real GPS data. A set of phase difference measurements was logged on UoSat-12 minisatellite [8], on 13th January 2000, for 200 minutes.

In January 2000, the UoSat-12 was operated in momentum bias mode. The spacecraft attitude was maintained by magnetic firing and torques generated by a reaction wheel in Y-axis. The logged data of wheel speed commanded by ADCS (Attitude Determination and Control System) is shown in Figure 5.

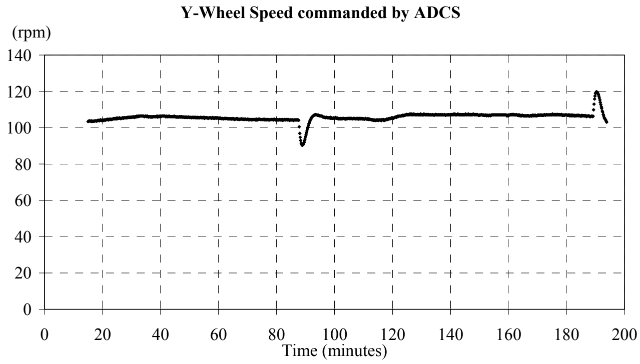

Figure 5. Logged data of Y-wheel speed

As can be seen that the nominal wheel speed was 100 rpm (revolution per minute) approximately. At the 90th minute and 190th minute, spacecraft was commanded to operate under manoeuvre in pitch.

The ADCS attitude on the UoSat-12 was derived from magnetometers and horizon-sensor measurements, and used as the reference attitude in evaluating the attitude derived from GPS sensing. The logged data of the computed ADCS attitude is shown in Figure 6.

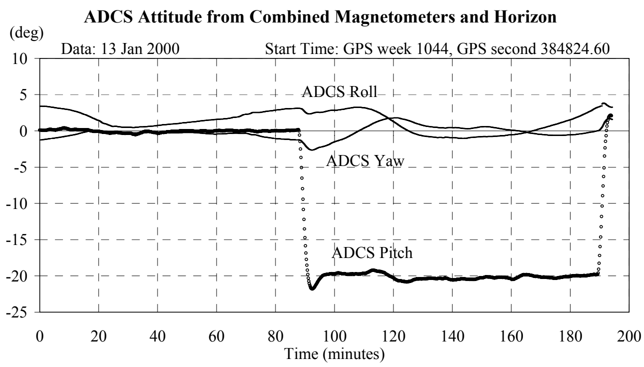

Figure 6. Logged data of UoSat-12 ADCS attitude

As shown in Figure 6, the rotation in roll and yaw of UoSat-12 was controlled to within 4 degrees, but the interesting thing is the change in pitch. UoSat-12 was manoeuvred to -20 degrees pitch, from the 90th minute to the 190th minute.

Using only GPS data, in acquisition process, a new ambiguity search is performed to estimate and verify initial attitude solution for 5 minutes. A detailed description and results is presented in [9].

In the following process, the filtering estimator was performed to estimate attitude from GPS data collected from two orthogonal antenna-baselines. The initialised elements of R and $\mathbf{Q}$ were given in Table 2.

Figure 7 shows a comparison between the ADCS pitch and GPS pitch estimated from sEKF estimator. As can be seen, the estimated attitude using GPS measurements is very close to ADCS attitude. The disparity is less than 1 degree rms. The computed one-sigma rms of difference between ADCS attitude and estimated GPS attitude is shown in Table 4.

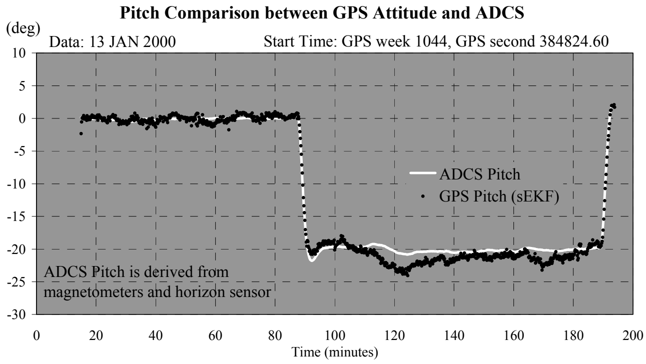

Figure 7. Pitch comparison between ADCS and GPS

Table 4: One-sigma rms of difference between GPS attitude and ADCS attitude (case: flight data)

disparity in roll	disparity in pitch	disparity in yaw
0.4°	0.9°	0.8°

Figure 8 shows an estimated angular velocity in Y-axis using GPS measurements through sEKF estimator is very close to angular velocity computed by ADCS. The rapidly changes in angular velocity at 90th minute and 190th minute are caused by the operation of Y-wheel as shown in Figure 5.

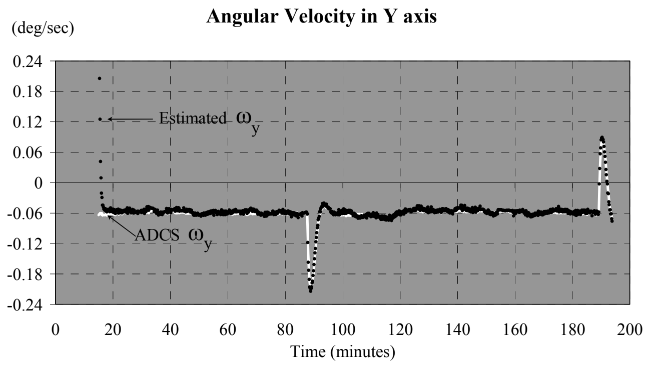

Figure 8. Estimated angular velocity in Y-axis

The estimated line bias of carrier phase difference for both baselines is shown in Figure 9. The line bias varied with time with a small drift rate. However, temperature may cause variation in line bias. If the temperature significantly changes, the variation of line bias may be significantly larger or smaller in a short period.

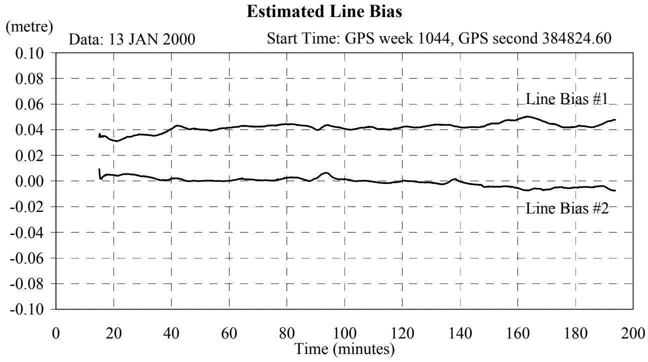

Figure 9. Estimated line bias of GPS attitude observable

VI. Conclusions

All results of attitude estimations from simulated data and flight data were proven that the implemented sEKF estimator provides remarkable results compared to reference solutions. Furthermore, these results also show that phase information of GPS signals potentially provided attitude information within 1 degree (one-sigma rms, compared to reference attitude).

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