

# Dynamic Behaviour and Significance of Neuron Networks with Varying Degrees of Random Coupling

Bijilash Babu and Marissa Condon *IAENG*

**Abstract**—Networks of coupled dynamical systems exhibit many interesting behaviours such as spatio-temporal chaos, pattern formation and synchronization. Such networks can be used to model a large variety of biological and physical systems. This contribution will focus on the 2-dimensional nonlinear map that describes the behaviour of individual neurons. The aim is to understand the coupling behaviour of this particular map and the dynamics of such coupled neuron systems. Various coupling schemes such as nearest neighbor coupling, random couplings and small-world networkings are numerically investigated. In particular, the paper will examine the transition of asynchronously firing neurons to a synchronous state. This is important in that it has similarities to the perceived behaviour of neurons in individuals during epileptic seizures.

**Index Terms**— neuron networks, coupled map, small-world network, spatio-temporal synchronization.

## I. INTRODUCTION

Coupled dynamical systems [1] exhibit many interesting features such as spatio-temporal chaos, pattern formation and synchronization. As such, these systems are suitable for modeling a large variety of biological systems such as the nervous system, cardiac cells and several chemical reactions [2-3]. While the dynamical properties of many of these systems have been explored and studied to a great extent, the dynamics of coupled neuron networks has not received as much focus[4]. Because of the large numbers of neurons in the brain, often the available computational infrastructure puts constraints on the investigative tasks that may be performed. In this contribution, the 2-dimensional neuron map in Chialvo and Apkarian [6] and as given in eqn. 1, is employed to study the collective dynamics of some “neuron networks”.

The principal aim is to study the transition of the dynamical state of the lattice when the network properties are changed. In particular, we wish to determine what kind of network properties give rise to synchronized firing and how changes in such properties affect the occurrence of synchronisation. Such an occurrence is believed to be involved when an individual has an epileptic seizure, - i.e.

the seizure is due to the transition of the neurons from a state of chaotic firing to a state of synchronized firing [7]. Normal brain activity is deemed to be a chaotic system with neurons

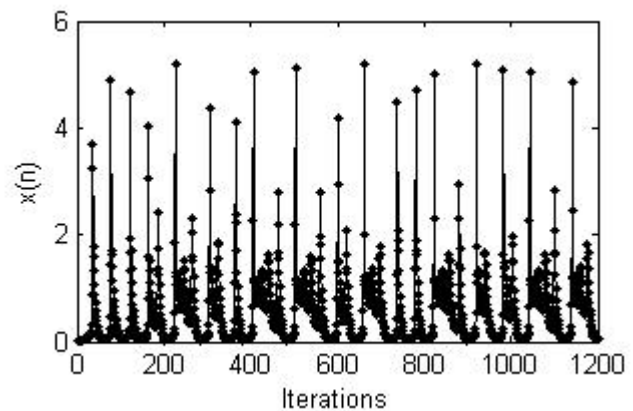
firing without any synchronization. In this work, it will be seen that a varying of the coupling arrangements of the network can affect the transition from an asynchronous state to a synchronous state in a neuron network although the parameters in each neuron model remain constant.

## II. NEURON DYNAMICS

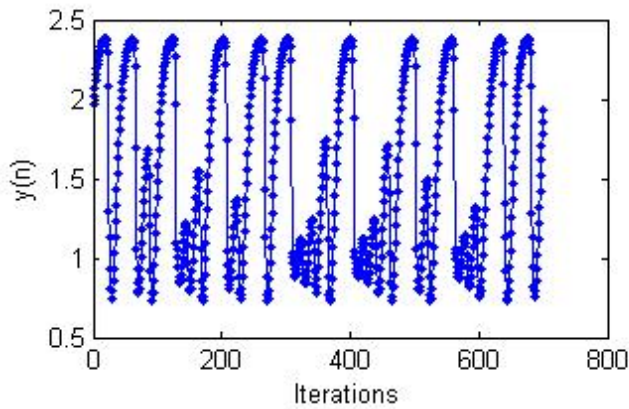
The neuron is an excitable system [5]. In general, the dynamics of an excitable system can be described by two state variables termed the “potential” and the “recovery” variables. In eqn. 1,  $x$  and  $y$  are the state variables related to neuron membrane potential and recovery current. The subscripts refer to the iteration step. The model has four parameters:  $a$  determines the time constant of reactivation,  $b$  determines the rate of inactivation and  $c$  determines the maximum amplitude of the recovery current. The parameter  $k$  may be a constant bias or a time-dependent external stimulation.

$$\begin{aligned}x_{n+1} &= x_n^2 \exp(y_n - x_n) + k \\y_{n+1} &= ay_n - bx_n + c\end{aligned}\quad (1)$$

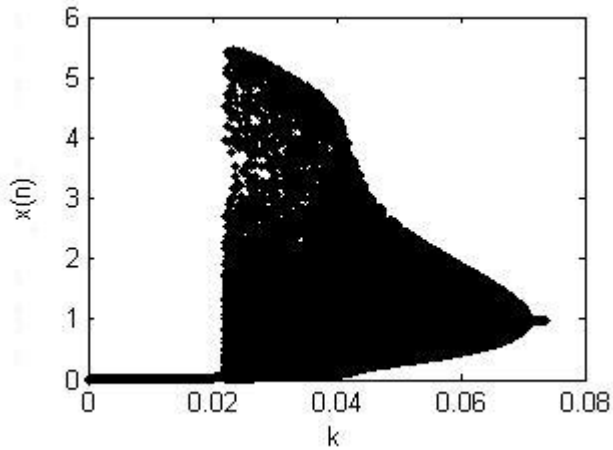
The dynamical behaviour of the system defined in eqn. 1 is shown in Figs. 1 and 2 which shows plots of  $x$  and  $y$ . The parameters in eqn. 1 are chosen so as to make the dynamics completely chaotic as evident from Figs. 1 and 2 ( $a = 0.89$ ,  $b = 0.18$ ,  $c = 0.28$ ,  $k = 0.03$ ). The bifurcation diagram of the  $x$  variable is shown in Fig. 3 in which  $k$  is varied from 0.02 to 0.08. Below  $k \sim 0.02$ , the system is stable. The region near  $k = 0.048$  is of interest. In this region which is shown enlarged in Fig. 4, the system behaviour changes from chaotic behaviour to quasi-periodic behaviour. Thus, the effects of parameter values on the neuron behaviour is evident.



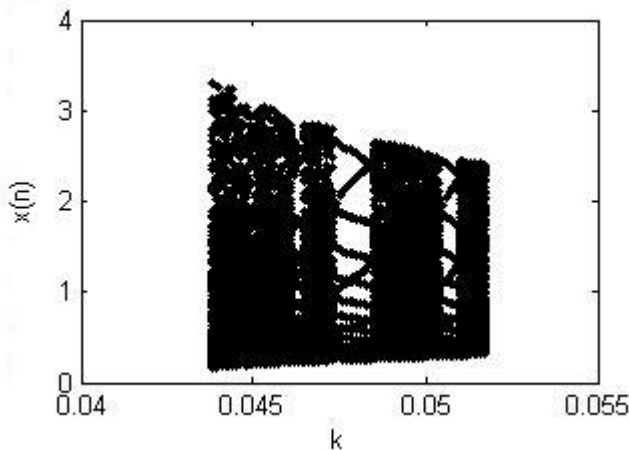
**Fig. 1.** The evolution of the  $x$  variable for a random initial condition



**Fig. 2** The evolution of the  $y$  variable for an initial condition of zero.



**Fig. 3** The bifurcation diagram of the  $x$  variable with  $k$  varying from 0.0 to 0.08.



**Fig. 4.** The bifurcation diagram enlarged between  $k = 0.045$  and  $0.051$ .

### III. COUPLED NEURON NETWORKS

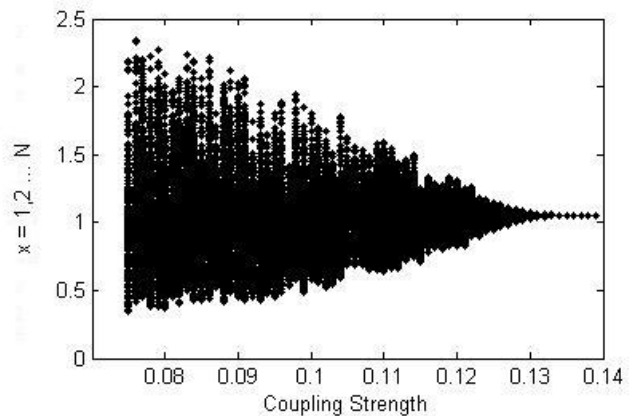
However, our main focus in this contribution is on *coupled* neuron networks. To study the collective dynamics of neuron

connections, the Chialvo map in eqn. 1 is coupled to form a Coupled Map Lattice (CML) [9] as given in eqn. 2. Several different coupling schemes will be considered and for each, the lattice is a ring of size  $N$  – (i.e.  $N$  neurons are considered).  $\varepsilon$  is the coupling strength.

$$x_{n+1} = f(x_n(i)) + \varepsilon/2 \{x_n(i+1) + x_n(i-1)\} \quad (2)$$

In particular, the focus will be on coupling schemes that closely resemble or match the actual neuronal connections in the brain. These particular systems exhibit spatio-temporal behavior when the *parameters* in eqn. 1 are varied [5]. However, in this contribution, the parameters in eqn. 1 are kept **constant**. What is varied is the coupling strength,  $\varepsilon$ , and the type of connections. It is the spatio-temporal pattern arising from the various coupling schemes that is our interest.

The first coupling scheme that we consider is the nearest neighbor coupling scheme [9]. In this coupling scheme, the  $i_{th}$  cell is coupled with its immediate neighbours as in eqn. 2. The value of the coupling strength  $\varepsilon$  is varied and the critical bifurcation points are observed. The bifurcation diagram is shown in Fig. 5. The general form of this diagram is not very different in nature from the bifurcation diagram of a single map – Fig. 3. Thus for strict neighbour coupling, the dynamics are very similar to those of the uncoupled lattice.

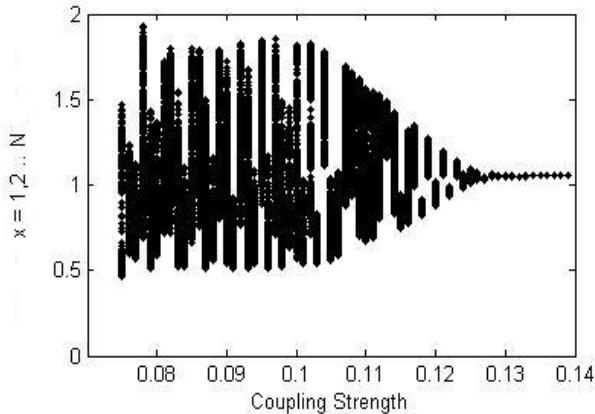


**Fig. 5** Bifurcation diagram of a lattice of size  $N$  with only nearest neighbor coupling

The second coupling scheme considered is where every cell is coupled to its nearest neighbors as before but in addition, it is also coupled to one random cell which can be anywhere in the lattice.

$$x_{n+1} = f(x_n(i)) + \varepsilon/3 \{x_n(i+1) + x_n(i-1) + x_n(random)\} \quad (3)$$

The bifurcation diagram for the system defined by eqn. 3 is shown in Fig. 6. It clearly shows a spatio-temporal pattern resulting from the random coupling. These spatio-temporal patterns indicate that there is a variation in the dynamics of the lattice when there is coupling to a random cell and a variation in the coupling strength.

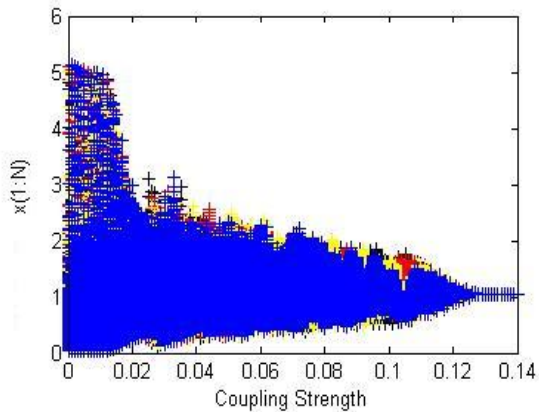


**Fig. 6** Bifurcation diagram of a lattice of size  $N$  with nearest neighbor coupling and one random coupling.

The scheme defined by eqn. 3 can be extended to include coupling from a few more neighbours with the inclusion of one random connection.

$$x_{n+1} = f(x_n(i)) + \frac{\epsilon}{(2n+1)} \left\{ \begin{array}{l} x_n(i-n) + x_n(i-n+1) + \\ \dots x_n(i+n-1) + x_n(i+n) + x_n(\text{random}) \end{array} \right\} \quad (4)$$

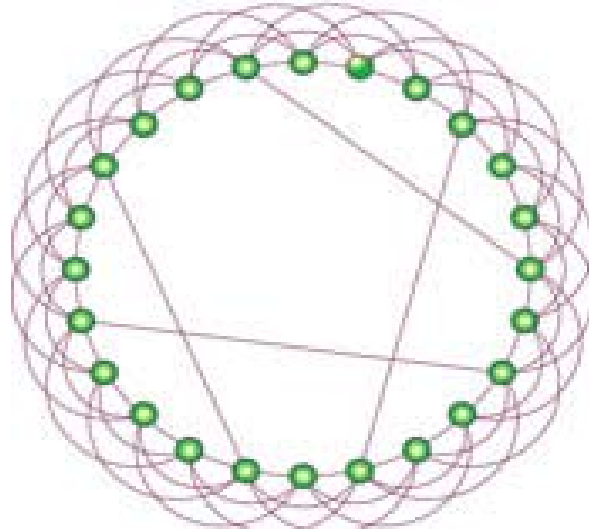
The scheme in eqn. 4 has  $n$  cells coupled on both sides plus one random coupling. Fig. 7 shows the bifurcation diagrams for  $n$  varying from 1 to 4 where the black colour shows  $n=1$  and the blue colour shows  $n=4$ . It is evident that adding more couplings of neighbours has no effect on the bifurcation point of the lattice. Thus, the number of neighbour couplings does not have a major impact on the dynamics of a neuron system.



**Fig. 7** Bifurcation diagram of a lattice with  $n$  couplings on both sides of the cell and one random coupling. Black color shows  $n=1$  and blue shows  $n=4$ .

This leads onto the most important coupling scheme - the small-world coupling scheme[4]. In this scheme as in the previous scheme, each cell is coupled with  $n$  cells on both sides. However, in addition, a few of these connections may be

rewired to random cells. This rewiring probability is denoted as  $p$ . This form of coupling is termed 'small-world' when  $p$  is small  $\sim 0.01$ . Fig. 8 shows a typical small-world network.



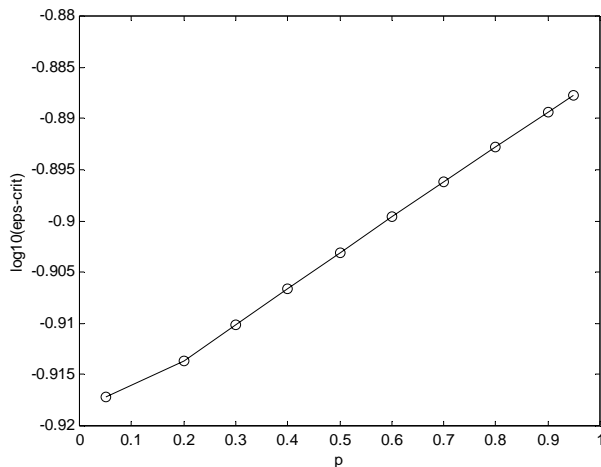
**Fig. 8** Small-world network, Picture Courtesy: Nature 410, 268-276 (8 March 2001)

Small-world networking is seen in many biological networks and also in the neural network of the nematode worm *Caenorhabditis elegans* (C-elegans)[4]. However, as in [9], we shall consider the complete range of  $p$  from 0 to 1 and not just  $p \sim 0.01$ . (We shall use the term 'small-world' to describe the network arrangement whereby a connection is rewired with a probability of  $p$  regardless of the actual value of  $p$ .)

Eqn. 5 represents such a small-world network with  $n$  nearby couplings and where one of these connections may be rewired to a random cell with a probability  $p$ .

$$x_{n+1} = f(x_n(i)) + \frac{\epsilon}{(2n)} \left\{ \begin{array}{l} x_n(i-n+j) + \dots + \\ \dots x_n(i+n-l) + \dots + \\ x_n(\text{random rewiring}) \end{array} \right\} \quad (5)$$

As an example,  $n$  is set equal to 5. The lattice is tested for  $p$  values varying from 0.0 to 1. For this system,  $k = 0.03$ .  $\epsilon_{cr}$  is the critical bifurcation point where the transition from the asynchronous to the synchronous state happens. In Fig. 9 the logarithm of the  $\epsilon_{cr}$  is plotted against  $p$ . The plot shows that there is a linear relationship between the logarithm of the critical bifurcation point and  $p$ , where  $p$  is the probability that one of the nearby couplings will be rewired to a random cell. Thus as  $p$  increases, then the higher the value of  $\epsilon_{cr}$  that the transition from asynchronous behaviour to synchronous behaviour occurs. Similar results were achieved with different lattice sizes (different values of  $N$ ).



**Fig. 9** A plot of the transition point from asynchronous behavior to synchronous behaviour in the coupled network with  $p$  ranging from 0-1.

#### IV. CONCLUSIONS AND COMMENTS

The brain is a very complex structure and mathematical modelling of it is in its infancy. A preliminary study has been carried out in this contribution to model the collective dynamics of neurons. Results show that straight-forward coupling of neighbouring neurons has little effect on the bifurcation diagram of the 'potential' variable  $x$  and from this we infer that straightforward coupling has little effect on the dynamics of the individual neurons. However, in contrast, with the **random rewiring** of neighbour couplings in the network, there is a pronounced linear relationship between the point at which the transition from asynchronous to synchronous behaviour occurs and the probability of a random rewiring of a connection. When a healthy brain moves to epileptic stage, it is thought that there is a change in the neuronal connections and that this gives rise to the transition from asynchronous firing to synchronous firing. The model described in this work describes this phenomenon. Further work is required to confirm the connections between the mathematical model results and biological results.

#### ACKNOWLEDGMENT

The authors would like to thank Prof. S. Sinha for her discussions on the topic.

#### REFERENCES

- [1] K. Aihara and G. Matsumoto: 'Chaos', Manchester University Press, Manchester.
- [2] S. H. Strogatz and I. Stewart: 'Coupled oscillators and biological synchronisation', *Scientific American*, **269** (6), pp. 102-109, 1993.
- [3] Y Kuramoto, 'Chemical Oscillations, Waves and Turbulence', Springer Publishers, Berlin, 1984.

- [4] D.J. Watts and S H Strogatz: 'Collective dynamics of 'small-world' networks', *Nature*, 1998, **Vol. 393**, pp. 440-442.
- [5] D. R. Chialvo, 'Generic Excitable Dynamics on a Two-Dimensional Map', *Chaos, Solitons and Fractals*, 1995, **5**, 461-479.
- [6] D.R. Chialvo and A.V. Apkarian: 'Modulated Noisy Biological Dynamics: Three Examples', *Journal of Statistical Physics*, **Vol. 70**, 1993.
- [7] H. Blumenfeld, 'Cellular and network mechanisms of spike-wave seizures', *Epilepsia*, 2005, **Vol. 46**.
- [8] Strogatz, S.H.: 'Exploring complex networks.' *Nature* **410**, pp. 268-276.
- [9] S Sinha, 'Random coupling of chaotic maps leads to spatiotemporal synchronization', *Phys review E* , **Vol. 66**, 2002.