

# Fuzzy Estimator Design for the Control Systems with Unknown Time-Delay

Y. J. Huang, *Member, IAENG*, T. C. Kuo, *Member, IAENG*, and H. Y. Tseng

**Abstract**—In this paper, a fuzzy estimator based on the Smith predictor for a time-delayed system is proposed. The purpose of this work is to predict the unknown plant time-delay by using a fuzzy logic controller. A classical controller is first designed for ideal non-time-delay systems. The developed fuzzy estimator measures the unknown plant time-delay. Once the estimation process completed, the control system performs an ideal non-time-delay system. The validity and effectiveness of the proposed method is shown with simulation results. Improved response can be obtained.

**Index Terms**—fuzzy logic controller, time-delayed system, Smith predictor.

## I. INTRODUCTION

Conventional controllers such as PI and PID are not effective controllers for time-delayed systems. For this reason, it has been a significant issue for designing suitable controllers for time-delayed systems.

Smith predictor is an effective compensator and used widely in large time-delayed systems. Some studies about applications of the Smith predictor were carried out [1, 2]. Based on the basic structure of the Smith predictor, a controller designed for a non-time-delayed system can be adopted for a time-delayed system if there is no difference between the model time-delay and the plant time-delay. Therefore, obtaining an accurate model time-delay is very important. If there is time-delay difference between the practical plant and the system model, unstable phenomenon may occur.

In [3, 4], the traditional control concepts to analyze the transfer function and to explain how to decide the parameters such as the control gain, the model time-delay and the time constant, were presented. Another literature [5] concerned with the incompatibility between a practical plant and its mathematical model. Furthermore, eliminating the complex disturbance externally is introduced in [6]. An asymmetrical

relay feedback test method was proposed by [7] to identify an actual plant. The equivalent model which approximates a second order transfer function was obtained. For this reason, the transfer function of the plant in [7] was different from its model, and it didn't need to assume the model time-delay equivalent to the plant time-delay. However, in general, the Smith predictor must be given equivalent time-delay between the plant and the model. In [8], an adaptive control loop which can automatically adjust the model parameters to match the time varying plant parameters was proposed. It used gradient type method to minimize the error square between the outputs of the model and the plant. Another different application with a Smith predictor was an adaptive vector forgetting factor algorithm as proposed in [9]. The actual plant is equivalent to its mathematical model through on-line identification for a time-delayed system.

It is very important that an actual plant time-delay is corresponding to its model time-delay in a Smith predictor. It may also go unstable with incorrect estimation of time-delay. In this paper, we propose a fuzzy logic controller (FLC) to predict the unknown plant time-delay and to on-line tune the model time-delay accordingly. The advantage is the capability of avoiding divergence which is resulted from incompatible time-delay by using a Smith predictor. The controller design for time-delayed systems becomes simple. In final section, the simulation results reveal the feasibility of the proposed method.

## II. CONTROL SYSTEM STRUCTURE

The whole control scheme is shown in Fig. 1, which includes the plant, the Smith predictor (within dash line frame), and the fuzzy logic controller (FLC) for adjusting the model time-delay. The Smith predictor loop within dashed line is designed to cancel the time-delay characteristic of the original feedback signal. The main advantage here is that only a classical controller design without time-delay is considered. In other words, a classical controller design with time-delay is taken as without time-delay once the model time-delay is precisely estimated by FLC. In Fig. 1,  $G_m(s)$  stands for the mathematical model of the plant  $G(s)$ . Assume that the unmodelled parameters are ignored, i.e.,  $G_m(s) \cong G(s)$ . From the system block diagram, we can obtain the following equations,

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$$y = G(s)e^{-T_d s} \cdot u(s), \quad (1)$$

$$y_m = G_m(s)e^{-Ts} \cdot u(s), \quad (2)$$

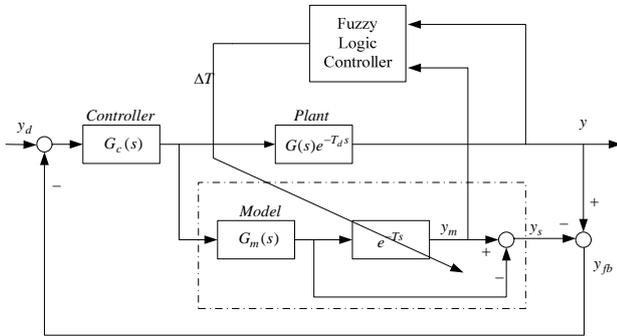


Fig. 1. Control system structure.

where  $T$  is the model time-delay,  $T_d$  is the plant time-delay,  $y$  is the output of the plant,  $y_m$  is the output of the model, and  $u$  is the control input. The responses of  $y$  and  $y_m$  are hopefully similar according to (1) and (2). But it is asynchronous, even to be divergent if  $T_d \neq T$ . In this case, the Smith predictor is invalid. Define an area as  $A(k) \equiv \int_0^k |y(k) - y_m(k)| dk$ . It is obvious that the area  $A(k)$  increases since the response curves of  $y$  and  $y_m$  do not overlap. As long as  $T = T_d$ , the response curves of  $y$  and  $y_m$  overlap each other, and  $A(k)$  will stay the same thereafter. Thus, the outputs  $y_s$  and  $y_{fb}$  can be described as

$$y_s = y_m - G_m(s) \cdot u(s), \quad (3)$$

$$y_{fb} = y - y_s,$$

$$\begin{aligned} &= G(s) \cdot e^{-T_d s} u(s) - [G_m(s) \cdot e^{-Ts} - G_m(s)] \cdot u(s), \\ &\cong G(s) \cdot u(s). \end{aligned} \quad (4)$$

From (4), the original feedback signal  $y$  is replaced by  $y_{fb} = G(s) \cdot u(s)$  when  $T \cong T_d$ . Thus, the controller design can be treated as well as the one for dealing with a non-time-delayed system. In addition, since the Smith predictor is a powerful compensator for time-delayed systems, it is obvious that the instability appearance can be eliminated. Hence, this paper proposes an investigation of updating the model time-delay  $T$  to approximate the plant time-delay  $T_d$  by using FLC. Through the merits of the Smith predictor, the controller design can be simplified to the case of

non-time-delayed system. The FLC design is presented in the next section.

### III. FUZZY LOGIC CONTROLLER DESIGN

First, let the initial model time-delay  $T$  be zero. Fig. 2 shows two response curves without overlapping. One is  $y$ , and the other is  $y_m$ . It is desired that the two responses overlap each other, i.e.,  $T$  approaches to  $T_d$ . Therefore, a tuning  $T$  is essential to follow  $T_d$ . In this work, the FLC is designed to adjust  $T$  according to the area  $A(k)$ . The area  $A(k)$  is increasing in the beginning. Once the responses of  $y$  and  $y_m$  overlap,  $A(k)$  is fixed. Fig. 2 shows the response of  $A(k)$  when the model time-delay is not tuned yet, i.e.,  $T \neq T_d$ . For this reason, the two separate responses have no overlap, then  $A(k)$  increases rapidly. Nevertheless, the above result is undesirable. Actually, the synchronous output responses are expected through adjusting  $T$  by using FLC. Once  $T \cong T_d$ ,  $A(k)$  does not increase but stop on a certain constant as shown in Fig. 2. At this point, it seems reasonable to recognize that  $A(k)$  increases more and more slowly if  $y_m$  approaches  $y$ . On the contrary,  $A(k)$  always increases if  $y_m$  and  $y$  are not the same. Furthermore, another feature is that the response of  $A(k)$  will tend to be horizontal when  $T \cong T_d$ . According to the above statements, the fuzzy rules can be established as follows.

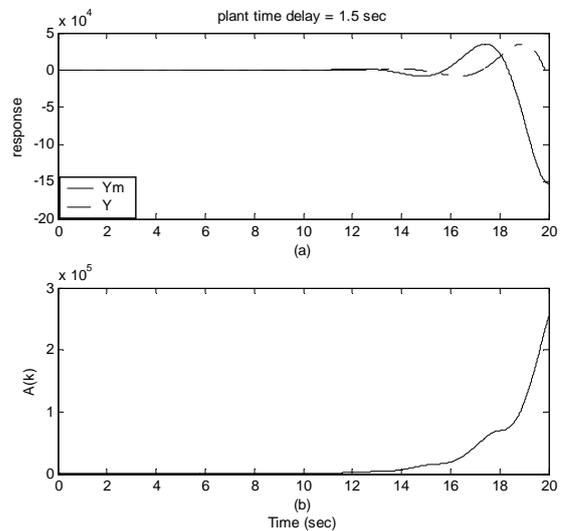


Fig. 2. Response of  $y_m$  and  $y$  and the sum of area before tuning the model time-delay.

The basic structure of fuzzy logic system is shown in Fig. 3. The four principal components of the fuzzy logic system are: a fuzzifier, a fuzzy rule base, an inference engine, and a defuzzifier. The fuzzifier performs the fuzzification module so that the measurement values are converted into fuzzy numbers

and degrees of membership functions. Hence, it can be defined as a mapping from a crisp input space to fuzzy set labels. All the fuzzy set membership functions adopted here are triangular-shaped functions, as shown in Fig. 4. The fuzzy rule base is constructed by using several IF-THEN statements and displays a simple mapping relation between the input and the output. The rule notation form is presented as below

$$\text{Rule } i: \text{ IF } e \text{ is } M_j \text{ and } \Delta e \text{ is } N_k \text{ THEN } \Delta T \text{ is } O_l, \quad (5)$$

where the fuzzy output is the variation of the model time-delay, noted as  $\Delta T$ . The fuzzy inputs  $e$  and  $\Delta e$  mean the error of area and the variation of the error  $e$  at every sampling time, respectively. The iterative calculation of  $e$  and  $\Delta e$  is as follows:

$$e(k+1) = A(k+1) - A(k), \quad (6)$$

$$\Delta e(k+1) = e(k+1) - e(k). \quad (7)$$

In (5),  $M_j, j=1,2,3$ ,  $N_k, k=1,2,3$ , and  $O_l, l=1,2,3$ , perform the fuzzy sets of  $e$ ,  $\Delta e$ , and  $\Delta T$  respectively. Besides, the features described previously are introduced to set up a series of fuzzy rules as shown in Table I, where each fuzzy set labelled NB, NM, N, Z, P, S, M, B, PM, and PB denotes negative big, negative medium, negative, zero, positive, small, medium, big, positive medium, and positive big, respectively.

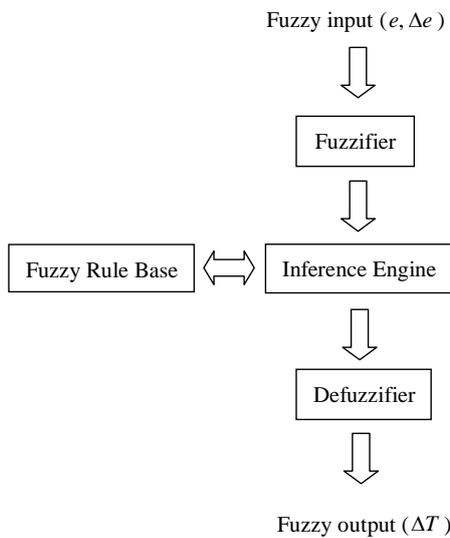


Fig. 3. Fuzzy logic system structure.

An inference engine plays a main role in a fuzzy logic system. Some of the fuzzy rules are fired to determine a weighted output through the module of the fuzzy inference. It is namely THEN-part. The Min-Min-Max inference method of Mamdani [10] is applied to generate the degree of consequence. Then, the output value remains a fuzzy number and needs to be transformed to the form of a crisp number by using the center of gravity method in the module of defuzzification, which can be defined as

$$\Delta T = \frac{\sum_{q=1}^r y_q \cdot \mu_q(y_q)}{\sum_{q=1}^r \mu_q(y_q)}, \quad (8)$$

where  $r$  is the quantitative number and  $Y = \{y_1, y_2, \dots, y_r\}$ , where  $y_q$  denotes  $q$ th value, and  $\mu_q(y_q)$  is the fired degree from  $y_q$ .

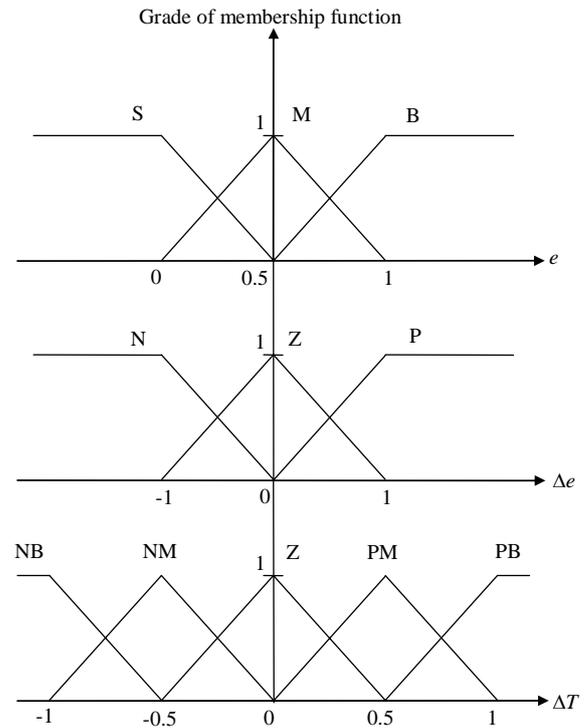


Fig. 4. Membership functions for the input and output.

Table I. Fuzzy rule base.

$e \backslash \Delta e$	S	M	B
N	Z	PM	PB
Z	Z	Z	Z
P	Z	NM	NB

#### IV. SIMULATION RESULTS

To illustrate the performance of the proposed method, the following example is shown. The sampling time is set to be 0.01 sec in the simulations.

Consider a transfer function [11] described by the following form of equation,

$$G(s) = \frac{21.2}{s^3 + 8s^2 + 15s} \quad (9)$$

Choose a suitable PD controller for the above plant,

$$G_c(s) = k_p + k_D s = 2.46 + s \quad (10)$$

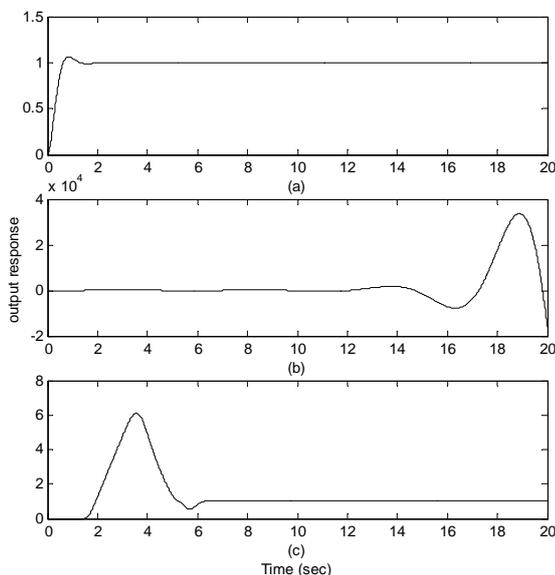


Fig. 5. Output responses for no time-delay, with time-delay, and with time-delay and FLC.

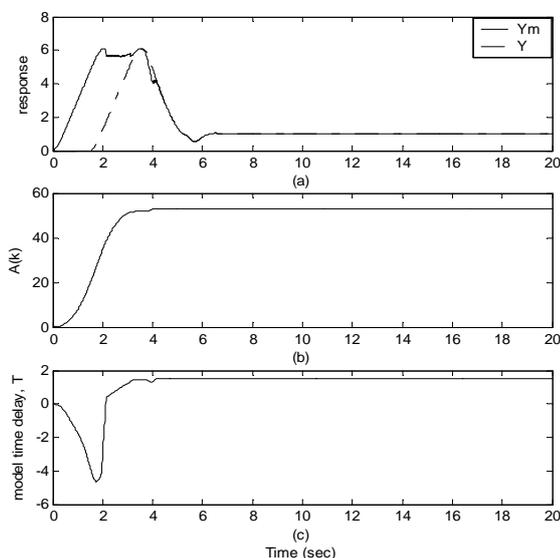


Fig. 6. Responses of  $y_m$  and  $y$ ,  $A(k)$ , and the model time-delay  $T$ .

In (10),  $G_c(s)$  is designed considering no time-delay. The stability of the system is guaranteed. Next, add the plant time-delay  $T_d=1.5$  sec into (9). Because  $G_c(s)$  is designed for non-time-delayed system and the model time-delay  $T$  has not been tuned, the system is divergent. Then, the proposed method is applied to improve the system performance. Fig. 5 illustrates

the output responses of above different cases. The control system becomes stable again and the error is zero. In Fig. 6, it is obvious that  $A(k)$  trends to be a horizontal line,  $T$  approaches  $T_d$ , and  $y_m$  approaches  $y$  in finite time with the application of the proposed method.

## V. CONCLUSIONS

A practical model time-delay can be determined successfully by using the proposed fuzzy logic controller. Either for a linear system or a nonlinear system, the proposed method is a powerful assistant compensator for time-delayed systems. Controller design may become simpler.

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