Suboptimal Filter for Multisensor Linear Discrete-Time Systems with Observation Uncertainties

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Abstract-**The focus of this paper is the problem of recursive estimation for uncertain multisensor linear discrete-time systems. We herein propose a new suboptimal filtering algorithm. The basis of the proposed algorithm is the fusion formula for an arbitrary number of local Kalman filters. The proposed suboptimal filter fuses each local Kalman filter by weighted sum with scalar weights. This filter can be implemented in real time because the scalar weights do not depend on current observations in distinction to the optimal adaptive filter. The examples given, demonstrate the effectiveness and high precision of proposed filter.**

IndexTerms-**Linear discrete-time system, multisensor, Kalman filter, fusion formula**

I. INTRODUCTION

The consideration focused herein is the estimation of the state of a linear system with multisensor environment with uncertainties. Though there are many methods available for such kind of systems in the structure adaptation [1]-[3], we chose, for this paper, the partition method and Lainiotis-Kalman filter (LKF). It is composed of segregation of the original nonlinear filter into a collection of much simpler local Kalman filters (KF's), where each local filter uses its own system model corresponding to each possible parameter value [1],[4]. The weighted sum of the local KF's provides the optimal fusion estimate of the state of LKF. The problem with the LKF is that the optimal scalar weights depend on sensor observations which complicates the implementation of the LKF in real-time, knowing that the dimension of state vector and the number of sensors are large.

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In [5], [6], we proposed to fuse the local KF's by a weighted sum with matrix weights, which are independent of sensor observations, and hence can be pre-computed. However, for high-order systems the matrix weights are also complicated to use.

This paper concentrates on uncertain multisensor systems and put forward the idea to fuse the local KF's using scalar weights independent from observations. Furthermore, the new filter helps in more than one way by assisting in minimizing the computation time and facilitating real-time state estimation, especially for large number of sensors.

The paper is structured as follows. In Section 2, we present the formulation of estimation problem for multisensor linear systems with observation uncertainties. Section 3 gives the optimal filter for the above system based on the LKF for all stacked sensors. In Section 4, we propose the suboptimal filter (SF), which represents a weighted sum of the local KF's with scalar weights depending only on time instance. Each local KF is fused by the minimum mean-square criterion. Section 5 tests the SF numerically. Conclusions are made in Section 6.

II. PROBLEM FORMULATION

Consider the following model of a multisensor system with observation uncertainties:

$$x_{k+1} = F_k x_k + G_k v_k, \quad k = 0,1,2,... \tag{1}$$

$$y_k^{(i)} = \theta^{(i)} H_k^{(i)} x_k + w_k^{(i)}, \quad i = 1,..., N, \tag{2}$$

where, as standard, $x_k \in \Re^n$ is the state, $v_k \in \Re^r \sim N(0, Q_k)$ is the normally distributed system noise. The system includes N sensors, $y_k^{(i)} \in \Re^{m_i}$ is the observation vector of i[th] sensor, and $w_k^{(i)} \in \Re^{m_i} \sim N(0, R_k^{(i)})$ is the normally distributed observation error. The system noise v_k and observation errors $w_k^{(1)},..., w_k^{(N)}$ are mutually uncorrelated. The initial state x_0 is normal, $x_0 \sim N(\bar{x}_0, P_0)$. All the local filters (sensors) are

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working on the same state vector, therefore the estimated state x_k has no superscript "i".

There are a number of applications, where the probability that the observations contain only noise is non zero. Therefore we assume, that the unknown parameters $\theta^{(i)}$, $i = 1,..., N$ come from the set $\{0,1\}$. The aim is to estimate x_k.

In order to estimate a state of such system optimally, we can use the LKF [1], [4].

III. THE OPTIMAL LAINIOTIS KALMAN FILTER

Bayesian approach forms the basis of the LKF in which the unknown parameter $\theta^{(i)}$ is assumed to be random with *prior* known probabilities

$$p_0^{(i)} = p\left(\theta^{(i)} = \theta_0^{(i)} = 0\right),\ \ p_1^{(i)} = p\left(\theta^{(i)} = \theta_1^{(i)} = 1\right), \\ p_0^{(i)} + p_1^{(i)} = 1,\ \ i = 1,..., N. \tag{3}$$

Let us collect all scalar parameters $\theta^{(1)},...,\theta^{(N)}$ into vector. Then we obtain the unknown parameter vector $\Theta \in \Re^N$, which takes $L = 2^N$ values, i.e.,

$$\Theta = \left[\theta^{(1)} \quad \dots \quad \theta^{(N)}\right]^T,\quad \theta^{(i)} = \begin{cases} \theta_0^{(i)} = 0, \\ \theta_1^{(i)} = 1. \end{cases} \tag{4}$$

With the above as prelude, we can rewrite the multisensor system model (1), (2) in the form,

$$x_{k+1} = F_k x_k + G_k v_k, \\ y_k = \widetilde{H}_k(\Theta) x_k + w_k, \tag{5}$$

where

$$y_k = \left[y_k^{(1)},\dots,y_k^{(N)}\right]^T,\ \ w_k = \left[w_k^{(1)},\dots,w_k^{(N)}\right]^T, \\ \widetilde{H}_k(\Theta) = \left[\theta^{(1)}H_k^{(1)},\dots,\theta^{(N)}H_k^{(N)}\right]^T, \\ y_k, w_k \in \Re^m,\ \ \widetilde{H}_k(\Theta) \in \Re^{m\times n},\ \ m = m_1 + \cdots + m_N. \tag{6}$$

Provided that the parameter vector Θ belongs to the discrete space (4), i.e., $\Theta = \Theta_i$, $i = 1,...,2^N$, the optimal LKF $\hat{x}_k^{opt}$ represents the weighted sum of the local KF's (estimates)

$$\hat{x}_k^{(i)} \equiv \hat{x}_k(\Theta_i),\quad i = 1,..., L = 2^N \tag{7}$$

matched to the linear system (5), (6) at fixed

$$\Theta = \Theta_i = \left[\theta_{i_1}^{(1)} \quad \dots \quad \theta_{i_N}^{(N)}\right]^T,\ \ i_1,...,i_N = 0,1. \tag{8}$$

We have

$$\hat{x}_k^{opt} = \sum_{i=1}^{L} \widetilde{c}_k^{(i)} \hat{x}_k^{(i)}, \tag{9}$$

where $\hat{x}_k^{(i)}$ represents the local Kalman estimate determined by the standard KF equations [1], [7]:

$$\hat{x}_k^{(i)} = F_k \hat{x}_{k-1}^{(i)} + K_k^{(i)}\left[y_k - \widetilde{H}_k^{(i)} F_k \hat{x}_{k-1}^{(i)}\right],\ \ \hat{x}_0^{(i)} = \overline{x}_0, \\ M_k^{(i)} = F_{k-1}P_{k-1}^{(ii)}F_{k-1}^T + G_{k-1}Q_{k-1}G_{k-1}^T,\ \ P_0^{(ii)} = P_0, \\ K_k^{(i)} = M_k^{(i)}\widetilde{H}_k^{(i)^T}\left[\widetilde{H}_k^{(i)}M_k^{(i)}\widetilde{H}_k^{(i)^T} + R_k\right]^{-1}, \\ P_k^{(ii)} = \left(I_n - K_k^{(i)}\widetilde{H}_k^{(i)}\right)M_k^{(i)},\ \ \widetilde{H}_k^{(i)} = \theta^{(i)}H_k^{(i)}, \\ R_k = \mathrm{diag}\left[R_k^{(1)} \quad \cdots \quad R_k^{(N)}\right],\ \ i = 1,..., L, \tag{10}$$

and the weights

$$\widetilde{c}_k^{(i)} = p(\Theta_i | Y_k),\ \ Y_k = \{y_0,..., y_k\},\ \ i = 1,...,2^N \tag{11}$$

correspond to *a posteriori* probabilities of Θ_i given Y_k. They are calculated through the recursive Bayesian formula [1], [4]. As discussed above, the efficiency of the LKF (7)-(11) depends on the dimension of the problem, since it requires calculations of a large number of *a posteriori* probabilities $p(\Theta_i|Y_k)$, $i = 1,...,2^N$ in real-time. In this paper we devise the alternative SF for the system (1), (2). This filter does not require calculations of a *posteriori* probabilities $p(\Theta_i|Y_k)$ at each time instance $k > 0$. The obtained suboptimal filtering algorithm reduces the computational burden and on-line computational requirements considerably.

IV. THE SUBOPTIMAL FILTER

The SF is similar to the optimal LKF, as likewise the LKF it also represents the state estimate as a weighted sum of the local KF's (7), however, unlike LKF the SF's weights does not depend on current observations y_k, but only on time instances $k > 0$. As a result, the weights can be pre-computed thus reducing computational complexity and provides an opportunity to design the SF, which can be easily applied in real-time, especially in high dimension problems. According of this proposal, we have

$$\hat{x}_k^{sub} = \sum_{i=1}^{L} c_k^{(i)} \hat{x}_k^{(i)}, \qquad \sum_{i=1}^{L} c_k^{(i)} = 1, \tag{12}$$

where, $c_k^{(1)},\dots,c_k^{(N)}$ are the scalar weights independent of current observations y_k. They depend only on time instance k and determined by the mean-square error (MSE) criterion:

$$J = E\left\| x_k - \hat{x}_k^{sub} \right\|^2 \to \min_{c_k^{(i)}}. \tag{13}$$

Theorem. (i) *The weights* $c_k^{(1)},\dots,c_k^{(L)}$ *satisfy the linear algebraic equations*

$$\sum_{i=1}^{L} c_k^{(i)} \mathrm{tr}\left(P_k^{(ij)} + P_k^{(ji)} - P_k^{(iL)} - P_k^{(Li)}\right) = 0, \tag{14}$$
$$j = 1,\dots,L-1, \quad c_k^{(1)} + \cdots + c_k^{(L)} = 1.$$

and they can be explicitly written out in the following form

$$c_k = \frac{A_k^{-1} e}{e^T A_k^{-1} e}, \quad c_k = \left[c_k^{(1)} \quad c_k^{(2)} \dots, \quad c_k^{(L)}\right]^T, \tag{15}$$

where $A_k = \left[A_{ij,k}\right]$, $A_{ij,k} = \mathrm{tr}\left(P_k^{(ij)}\right)$ and $e = \left[1 \quad 1 \quad \dots 1^T\right]$

(ii) *The overall error covariance*

$$P_k^{sub} = E\left(\left(x_k - \hat{x}_k^{sub}\right)\left(x_k - \hat{x}_k^{sub}\right)^T\right) \tag{16}$$

is given by

$$P_k^{sub} = \sum_{i,j=1}^{L} c_k^{(i)} c_k^{(j)} P_k^{(ij)}, \quad P_k^{(ij)} = E\left(\tilde{x}_k^{(i)} \tilde{x}_k^{(j)^T}\right), \tag{17}$$
$$\tilde{x}_k^{(i)} = x_k - \hat{x}_k^{(i)}, \quad i = 1,\dots,L.$$

In (13), $\mathrm{tr}(A)$ is the trace of a matrix A.

The proof of Theorem is given in Appendix A.

Note that formulas (14)-(17) depend on the local error covariances $P_k^{(ii)}$ determined by the Riccati equations (10), and the local cross-covariances $P_k^{(ij)}$, $i \neq j$, which satisfy the following recursive equation:

$$P_k^{(ij)} = (I_n - K_k^{(i)} \tilde{H}_k^{(i)})$$
$$\times (F_{k-1} P_{k-1}^{(ij)} F_{k-1}^T + G_{k-1} Q_{k-1} G_{k-1}^T)(I_n - K_k^{(j)} \tilde{H}_k^{(j)})^T, \tag{18}$$
$$P_0^{(ij)} = P_0, \quad i,j = 1,\dots,L; \quad i \neq j,$$

where $K_k^{(i)}$ *stands for the local Kalman gains* (10).

The derivation of (18) is given in Appendix B.

Thus, the suboptimal filter can be completely formed by the local Kalman estimates and covariances $\hat{x}_k^{(i)}, P_k^{(ii)}$ (see (10)), the local cross-covariances $P_k^{(ij)}$, $i \neq j$ (see (18)), and the fusion equations (14), (15).

Remark 1 *(Computational complexity*). In general, the both results, namely, linear equations (14) and expression (15) are equivalent, being the implicit and explicit forms of the solution, respectively. However, from the computational point of view, when the number of sensors N is large or the local cross-covariance matrices $P_k^{(ij)}$ are ill-conditioned, the linear equations (14) may be more preferable than the explicit expression (15).

Remark 2 *(Real-time implementation)*. It is very interesting to note that once the observation schedule is settled, the real-time implementation of the SF requires only the computation of the local Kalman estimates $\hat{x}_k^{(1)},\dots,\hat{x}_k^{(L)}$ and the final fusion suboptimal estimate $\hat{x}_k^{sub}$, since the local Kalman gains $K_k^{(i)}$, the local error cross-covariances $P_k^{(ij)}$, and the weights $c_k^{(i)}$ can be pre-computed, as they are dependent only on the noise statistics and system matrices, and on the values Θ_i of the parameter Θ, which are the part of system model (1),(2), (4), and not on the present observations Y_k.

Remark 3 *(Parallel implementation)*. Another important advantage of the SF is that each local estimate $\hat{x}_k^{(i)}$ is found independently of other estimates $\hat{x}_k^{(j)}$, $j = 1,\dots,L;\ j \neq i$, and thus; can be evaluated in parallel, because of the fact that Θ takes a finite number of values (8), the local Kalman estimates (10) are separated for each value of $i = 1,\dots,L$.

V. EXAMPLES

Example 1. Consider a scalar linear system

$$x_{k+1} = a x_k + v_k, \quad k = 0,1,2,\dots, \tag{19}$$

where $a = \mathrm{const}$, $v_k \sim N(0,q)$, $x_0 \sim N\left(\bar{x}_0, \sigma^2\right)$.

The observation model contains three sensors:

$$y_k^{(i)} = \theta^{(i)} x_k + w_k^{(i)}, \quad i = 1,2,3, \tag{20}$$

where $w_k^{(i)} \sim N(0, r_i)$, $i = 1,2,3$, and the unknown parameters $\theta^{(i)}$, $i = 1,2,3$ take only two values with equal prior probabilities, i.e.,

$$\theta^{(i)} = \begin{cases} \theta_0^{(i)} = 0, & p\left(\theta_0^{(i)}\right) = 0.5, \\ \theta_1^{(i)} = 1, & p\left(\theta_1^{(i)}\right) = 0.5. \end{cases} \tag{21}$$

Here each sensor takes two modes, which are, $\theta^{(i)} = 1$ (signal-present) and $\theta^{(i)} = 0$ (signal-absent). Then the vector parameter $\Theta = \left[\theta^{(1)} \quad \theta^{(2)} \quad \theta^{(3)}\right]^T$ takes $L = 8$ values, $\Theta = \Theta_i$ as given below:

$$\begin{aligned} &\Theta_1 = [1 \quad 1 \quad 1]^T,\ \Theta_2 = [1 \quad 1 \quad 0]^T,\ \Theta_3 = [1 \quad 0 \quad 1]^T, \\ &\Theta_4 = [1 \quad 0 \quad 0]^T,\ \Theta_5 = [0 \quad 1 \quad 0]^T,\ \Theta_6 = [0 \quad 1 \quad 1]^T, \\ &\Theta_7 = [0 \quad 0 \quad 1]^T,\ \Theta_8 = [0 \quad 0 \quad 0]^T. \end{aligned} \tag{22}$$

The model parameters are set to $a = 0.9$, $q = 0.05$, $\bar{x}_0 = 5$, $\sigma^2 = 3$, $r_1 = .5$, $r_2 = 1$, $r_3 = 2$.

We compare the optimal LKF and SF:

$$\hat{x}_k^{opt} = \sum_{i=1}^{8} \tilde{c}_k^{(i)} \hat{x}_k^{(i)}, \quad \hat{x}_k^{sub} = \sum_{i=1}^{8} c_k^{(i)} \hat{x}_k^{(i)}, \quad \hat{x}_k^{(i)} = \hat{x}_k(\Theta_i). \tag{23}$$

The system (19)-(21) is simulated for all values of the parameter (22). The Figs. 1-3 present the time histories of the LKF and SF characteristics for the first case, $\Theta = \Theta_1$. Such time histories are perfect analogy for the other cases.

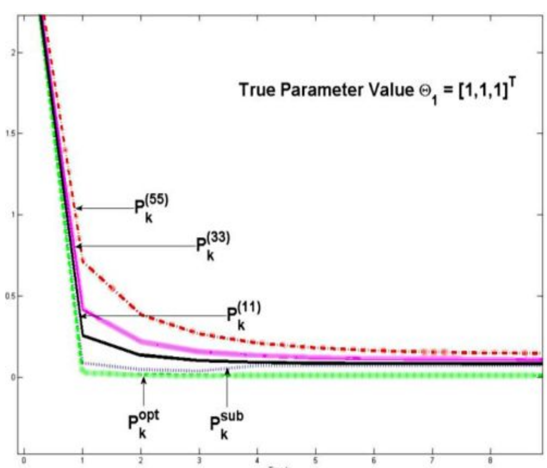

Fig.1 Comparison of MSEs for LKF and SF

Fig.1 shows the overall MSE's $P_k^{opt} = E\left(x_k - \hat{x}_k^{opt}\right)^2$ and $P_k^{sub} = E\left(x_k - \hat{x}_k^{sub}\right)^2$, and three local MSE's $P_k^{(ii)} = E\left(x_k - \hat{x}_k(\Theta_i)\right)^2$ corresponding to the different values of the parameter $\Theta = \Theta_1, \Theta_3, \Theta_5$, but among them only the value $\Theta = \Theta_1$ is the true value in (20).

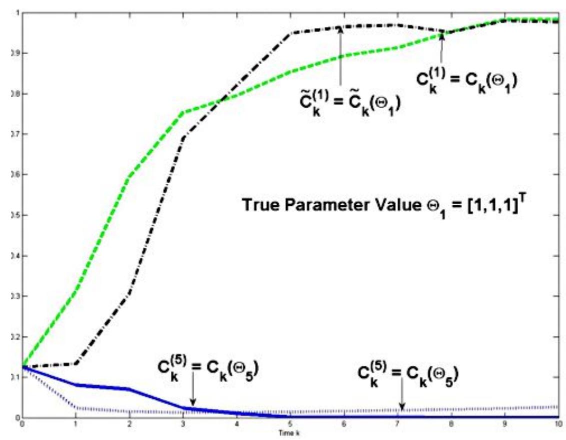

Fig. 2 Comparison of the weights $\tilde{c}_k^{(i)}$ and $c_k^{(i)}$

From Fig.1 shows us that P_k^{opt} and P_k^{sub} are very close, due to the verity that optimal and suboptimal weights corresponding to the true value $\Theta = \Theta_1$ coincides very closely to each other, i.e., $c_k^{(1)} \approx \tilde{c}_k^{(1)}$ (see Fig. 2). It is very apparent from Fig.3, showing the comparison of the optimal and suboptimal estimates that performance of the SF is matching the optimal one which further proves that SF is a good alternative for LKF.

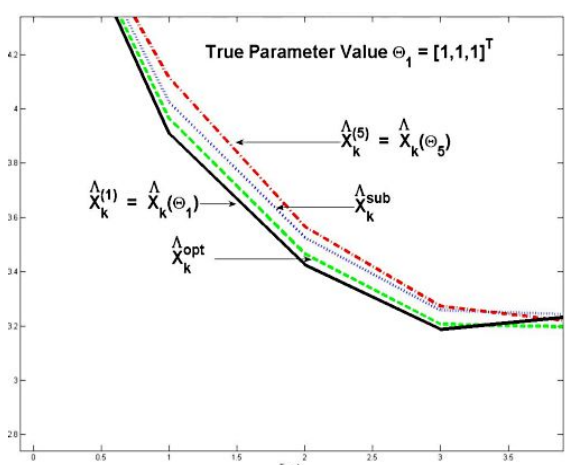

Fig.3 Optimal and suboptimal estimates

Example 2. Consider the 2-dimensional system

$$x_{k+1} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.07 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_k, \quad k = 0,1,2,... \quad, \tag{24}$$

where $x_k = [x_{1,k} \quad x_{2,k}]^T$, $v_k \sim N(0,q)$, $x_0 \sim N(\bar{x}_0, P_0)$.

We are measuring the position $x_{1,k}$ and velocity $x_{2,k}$ using two sensors,

$$y_k^{(1)} = \theta^{(1)} x_{1,k} + w_k^{(1)}, \quad y_k^{(2)} = \theta^{(2)} x_{2,k} + w_k^{(2)}, \tag{25}$$

where $w_k^{(i)} \sim N(0,r), \quad \theta^{(i)} \in \{0,1\}, \quad i = 1,2.$

In this case the vector parameter $\Theta = [\theta^{(1)} \quad \theta^{(2)}]^T$ takes $L = 4$ values with equal prior probabilities, $\Theta = \Theta_i$, as given below:

$$\Theta_1 = [1 \quad 1]^T, \ \Theta_2 = [1 \quad 0]^T, \ \Theta_3 = [0 \quad 1]^T, \ \Theta_4 = [0 \quad 0]^T. \tag{26}$$

We compare the optimal LKF and SF. The model parameters are set to

$$\begin{aligned} &q = 0.2, \quad r = 1, \\ &\bar{x}_0 = [5.0 \quad 0.0]^T, \quad P_0 = \mathrm{diag}\,[1.0 \quad 2.0]. \end{aligned} \tag{27}$$

Figs 4-6 present the time histories of the filter characteristics for the true value of $\Theta = \Theta_1$. This time history is similar for the other values of Θ.

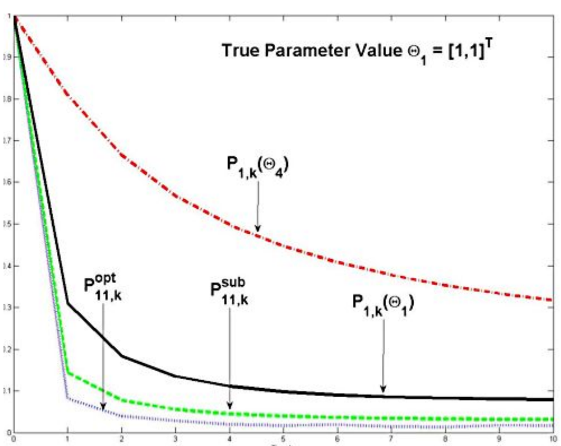

Fig.4 Comparison of MSEs for position

In Figs. 4 and 5 we show the overall optimal $P^{opt}_{11,k}$ and suboptimal $P^{sub}_{11,k}$ MSE's, and also two local MSE's for the position and velocity, respectively. The local MSE's $P_{i,k}(\Theta_1)$ and $P_{i,k}(\Theta_4)$ for position $(i = 1)$ and velocity $(i = 2)$ correspond to the values $\Theta = \Theta_1$ and $\Theta = \Theta_4$, respectively.

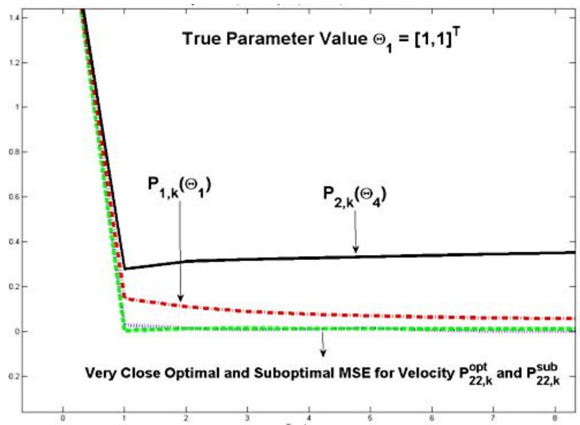

Fig.5 Comparison of MSEs for velocity

The observation of Figs. 4 and 5 reveals very clearly that the differences between the optimal MSE ($P^{opt}_{ii,k}$, $i = 1,2$) and suboptimal MSE ($P^{sub}_{ii,k}$, $i = 1,2$) are negligible. It is worth mentioning that in Fig. 4 the local MSE $P_{1,k}(\Theta_4)$ corresponding to only noise observations is very poor since LKF's and SF's weights $\tilde{c}_k^{(4)}$ and $c_k^{(4)}$, corresponding to Θ_4 are very small (see Fig. 6). Fig. 4 also shows that in the steady state regime the local MSE $P_{1,k}(\Theta_1)$ is also close to the optimal one $P^{opt}_{11,k}$.

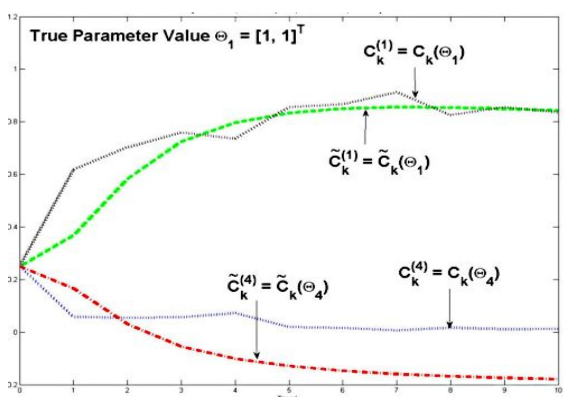

Fig.6 Comparison of the weights $\tilde{c}_k^{(i)}$ and $c_k^{(i)}$

VI. CONCLUSION

In this paper, we have designed a new SF for uncertain multisensor linear discrete-time systems. This filter represents a linear combination of the local KF's with weights depending only on time instance. Each local KF is fused by the minimum mean-square error criterion. The proposed filter has a parallel structure and as a result of that is suitable for parallel processing. Simulation results demonstrate the relative loss of accuracy of the SF as compared to the optimal LKF.

APPENDIX A: PROOF OF THE THEOREM

Using (12) the criterion (13) can be rewritten as

$$\begin{aligned} J &= \mathrm{tr}E\left(\left(x_k - \hat{x}_k^{\mathrm{sub}}\right)\left(x_k - \hat{x}_k^{\mathrm{sub}}\right)^T\right) \\ &= \mathrm{tr}E\left(\left(\sum_{i=1}^{L} c_k^{(i)}\left(x_k - \hat{x}_k^{(i)}\right)\right)\left(\sum_{j=1}^{L} c_k^{(j)}\left(x_k - \hat{x}_k^{(j)}\right)^T\right)\right) \\ &= \sum_{i,j=1}^{L} \mathrm{tr}\left(c_k^{(i)}c_k^{(j)}E\left(x_k - \hat{x}_k^{(i)}\right)\left(x_k - \hat{x}_k^{(j)}\right)^T\right) = \mathrm{tr}\left\{\sum_{i,j=1}^{L} c_k^{(i)}c_k^{(j)}P_k^{(ij)}\right\}. \end{aligned} \tag{A.1}$$

The formula (A.1) gives the overall covariance (17). Next substituting $c_k^{(L)} = 1 - \left(c_k^{(1)} + \cdots + c_k^{(L-1)}\right)$ into (A.1) we obtain

$$\begin{aligned} J &= \sum_{i,j=1}^{L-1} c_k^{(i)}c_k^{(j)}\mathrm{tr}\left(P_k^{(ij)}\right) + \left(1 - \sum_{h=1}^{L-1} c_k^{(h)}\right)\mathrm{tr}\left(P_k^{(LL)}\right) \\ &+ \left(1 - \sum_{h=1}^{L-1} c_k^{(h)}\right)\sum_{i=1}^{L-1} c_k^{(i)}\mathrm{tr}\left(P_k^{(hL)} + P_k^{(Lh)}\right). \end{aligned} \tag{A.2}$$

Differentiating each summand of the criterion J in (A.2) with respect to $c_k^{(1)},\dots,c_k^{(L-1)}$ and then setting the result to zero, we obtain the linear algebraic equations (14) for the unknown weights $c_k^{(1)},\dots,c_k^{(L)}$.

Applying the Lagrange multiplier method for minimization the criterion J under the restriction $c_k^{(1)} + \cdots + c_k^{(L)} = 1$, we obtain

$$\Phi = J + \lambda\left(\sum_{i=1}^{L} c_k^{(i)} - 1\right). \tag{A.3}$$

Setting $\partial\Phi/\partial c_k^{(i)} = 0$, $i = 1,\dots,N$ and $\partial\Phi/\partial\lambda = 0$, and after simple manipulations we obtain (15).

This completes the proof of the Theorem.

APPENDIX B: DERIVATION OF EQUATION (18)

The derivation of (18) is based on the recursive equations for the state x_k and estimate $\hat{x}_k^{(i)}$. Using (1), (2), and (10) we obtain recursive equations for the local error $\tilde{x}_k^{(i)} = x_k - \hat{x}_k^{(i)}$, i.e,

$$\begin{aligned} \tilde{x}_k^{(i)} &= F_{k-1}x_{k-1} + G_{k-1}v_{k-1} - F_{k-1}\hat{x}_{k-1}^{(i)} \\ &- K_k^{(i)}\left[y_k - \tilde{H}_k^{(i)}F_{k-1}\hat{x}_{k-1}^{(i)}\right] = F_{k-1}\tilde{x}_{k-1}^{(i)} \\ &- K_k^{(i)}\left[\tilde{H}_k^{(i)}\left(F_{k-1}x_{k-1} + G_{k-1}v_{k-1}\right) + w_k^{(i)}\right. \\ &\left. - \tilde{H}_k^{(i)}F_{k-1}\hat{x}_{k-1}^{(i)}\right] + G_{k-1}v_{k-1} = \left(I_n - K_k^{(i)}\tilde{H}_k^{(i)}\right)F_{k-1}\tilde{x}_{k-1}^{(i)} \\ &+ \left(I_n - K_k^{(i)}\tilde{H}_k^{(i)}\right)G_{k-1}v_{k-1} - K_k^{(i)}w_k^{(i)}. \end{aligned} \tag{B.1}$$

According to the assumptions that the error $\tilde{x}_{k-1}^{(i)}$, and white noises v_{k-1} and $w_k^{(i)}$ are mutually uncorrelated the equation (B.1) yields the Lyapunov recursive equation (18) for the local cross-covariances $P_k^{(ij)} = E\left(\tilde{x}_k^{(i)}\tilde{x}_k^{(j)^T}\right)$, $i \neq j$.

This completes the derivation of (18).

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