Signature Analysis of Mechanical Watch by Reassigned Scalogram

S. Su, R. Du

Abstract— **This paper presents a new method for the signature analysis of mechanical watch movements. Contrary to the existing method, it analyzes the time-frequency features of mechanical watch movement through a combination of reassigned scalogram and Finite Element Analysis (FEA). By mapping the signal into a 2D domain of time and frequency, the reassigned scalogram reveals the frequency components of the shocks at different time. On the other hand, FEA gives the modal frequencies of the movements. By comparing the frequency components at different shocks to the modal frequencies of the movement, possible malfunctions of the movement can then be detected. A number of demonstration examples are included.**

***Index Terms*— Signature analysis, Mechanical watch, Reassigned scalogram, Finite element analysis, Fault diagnosis.**

I. INTRODUCTION

A mechanical watch is a mechanical device for timekeeping. Appeared in the middle of 16^{th} century, the mechanical watch is one of most complex mechanical mechanisms ever invented. Its operation still fascinates millions of people around the world today. As a matter of fact, it is still a billion dollars business nowadays. Different watches make different sounds. The basic "tick tick" sound comes from the impacts inside the escapement which is a feedback regulator that determines the timekeeping accuracy [1].

The escapement is composed of the balance wheel, the pallet fork and the escape wheel, which meshes with the fourth wheel through the escape pinion (see Fig. 1). One complete tick of the escapement can be divided into three phases: unlocking, impulse and drop [1]. Therefore, a good watch should have 3 main audible sounds, corresponding to the three phases (see Fig. 2). During the unlocking phase, the impulse pin on the balance wheel strikes against the pallet fork notch and at the same time, the escape wheel falls back a small angle along the entry pallet jewel. After unlocking, the escape wheel tooth gives an impulse to the entry pallet jewel and the other side of the pallet fork notch catches up with the impulse pin and pushes the balance wheel forward. During the drop phase, the escape wheel tooth drops onto the exit pallet jewel and the pallet fork drops clockwise until it hits the banking pin. As the intensity of the shocks in the drop phase is the largest, the vibration of the escape wheel may cause vibration on the fourth wheel. The instantaneous sound frequencies correspond to the modal frequencies of the vibrating parts, including the balance wheel, the pallet fork, the escape wheel and the fourth wheel.

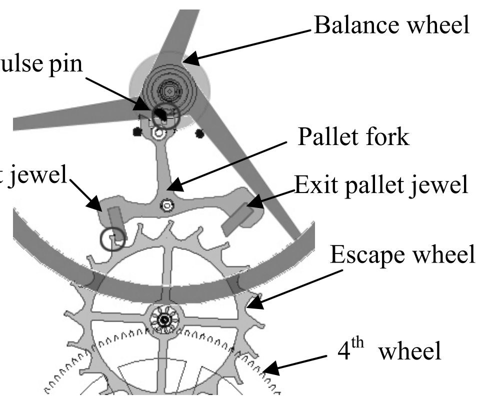

Fig. 1 The escapement

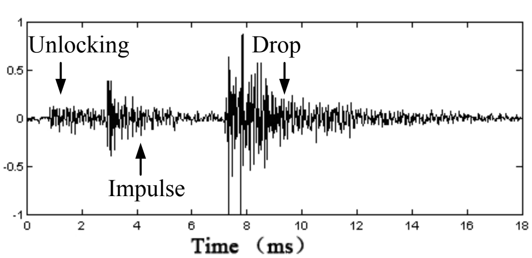

Fig. 2: A sound signal of a mechanical watch

Commercial systems make use the feature of watch sound and have developed some time-domain methods to estimate the accuracy of the watch and detect faults [2], [3]. This method disregards the fruitful information in the frequency domain. It works well on some faults but not robust for other faults, for example: (a) not enough clearance between the horns and the impulse pin; (b) too much endshake in the pivot of the balance wheel; (c) fork horn touching the impulse pin (knocking). The difference between these three faults in the diagnosis interpretation is only the magnitude of the additional waveform (the fourth peak). However, the intensity of shocks in different watch is different. Even for the same watch, shocks may not be uniform. Therefore, one fault can easily be mistaken as another by using this diagnosis method (see Fig. 3).

Since the sound is caused by the vibration of the escapement

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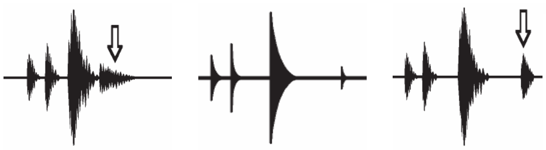
parts, we are motivated to analyze the sound from the aspect of vibration frequencies. Therefore, we propose reassigned wavelet scalogram (or reassigned scalogram), a kind of Time-Frequency Distribution (TFD) [4], [7]. By comparing the results to the modal frequencies of the escapement parts that are found by means of Finite Element Analysis (FEA) [5], we can then find much more information about the mechanical movement and diagnose possible malfunctions robustly.



(a) (b) (c)

Fig. 3 time-domain fault diagnosis

II. Reassigned Wavelet Scalogram

Signature analysis is a powerful tool for accessing the quality of mechanical watch movement. The key point is how to extract useful features from the sound signals for diagnosis. Time-frequency analysis is the most popular method for analyzing non-stationary signals. It transfers a one-dimensional time-domain signal, $x(t)$, into a two-dimensional function of time and frequency, $TFD(t, f)$. Hence, it can characterize signals over the time-frequency domain. There are a number of different TFDs, each one has its own advantages and drawbacks. Positive TFDs do not produce cross-terms, but the limitation of the Heisenberg inequality makes the tradeoff between the time resolution and frequency resolution unavoidable [4]. Bilinear TFDs, such as Wigner-Ville Distribution (WVD), provide good time and frequency resolution but their practical application is restricted by the presence of spurious cross-terms [8]. Reduction of interference terms by smoothing WVD in time or frequency will cause the loss of time-frequency concentration.

A. Wavelet Scalogram

Over the past few decades, wavelet theory has become one of the fast evolving signal processing tools for its desirable properties. Wavelet scalogram, the squared modulus of the continuous wavelet transform, is used to extract time-frequency features of signals for fault diagnosis [7], [11]. The scalogram is defined as:

$$SC_x(t,a;\psi) = \left|CW_x(t,a;\psi)\right|^2 \tag{1}$$

where, CW_x is the continuous wavelet transform of x [7], [13]. The idea of the continuous wavelet transform is to project a signal x on a family of wavelets deduced from the mother wavelet by translation and dilations (or compressions):

$$CW_x(t,a;\psi) = \int_{-\infty}^{+\infty} x(\tau)\frac{1}{\sqrt{a}}\psi^*\left(\frac{\tau-t}{a}\right)d\tau \tag{2}$$

where ψ^* is the conjugate of the complex of the wavelet ψ, a the scale factor. By definition, the wavelet transform is more a time-scale distribution than a time-frequency distribution. However, the scale, a, is related to frequency by the relationship $a = (\omega_c / \omega)$. ω_c is the center frequency of mother wavelet. The energy spread of ψ corresponds to a Heisenberg box centered at $(t, \omega_c / a)$ with the time spread proportional to a and the frequency spread proportional to $1/a$ [7]. The area of the box remains a constant. In other words, the wavelet transform uses short windows at high frequencies and long windows at low frequencies. Therefore, the frequency resolution becomes poorer whereas the time resolution becomes better as the analysis frequency grows.

Instead of using its usual definition, the scalogram can be equivalently expressed in the form of the affine class [9]:

$$SC_x(t,a;\psi) = \iint WV_x(s,\xi)WV_\psi\left(\frac{s-t}{a},a\xi\right)\frac{dsd\xi}{2\pi} \tag{3}$$

where WV is the WVD. From (3), we can see that the scalogram results from an affine smoothing of the WVD of x. The smoothing kernel is the WVD of wavelets.

B. Reassignment of Scalogram

Reassignment is a post-processing technique that can improve the readability of TFD with both good time and frequency concentration of the signal components and no misleading cross-terms [9], [10], [11]. As stated above, the value that the scalogram takes at a point (t, ω) cannot be seen as the energy of the point, but the weighted average of energy located in the domain within the essential support of the smoothing kernel. In other words, an entire distribution of values is summarized to a number which is assigned to the geometric center (t, ω) of this domain. However, the energy distribution is not always geometrically symmetric. Moving this value to the center of gravity $(\hat{t}, \hat{\omega})$ of the distribution is much more meaningful and this is the essence of reassignment. The reassigned scalogram is defined as:

$$RSP_x(t',\omega';h) = \iint SP_x(t,\omega;h)\delta(t'-\hat{t}_x(t,\omega))\delta(\omega'-\hat{\omega}_x(t,\omega))\frac{dtd\omega}{2\pi} \tag{4}$$

The coordinates of center of gravity are given by:

$$\hat{t}_x(t,\omega) = t - \Re e\left\{\frac{STFT_x(t,\omega;Th)\cdot STFT_x^*(t,\omega,h)}{\left|STFT_x(t,\omega,h)\right|^2}\right\} \tag{5}$$

$$\hat{\omega}_x(t,\omega) = \omega + \Im m\left\{\frac{STFT_x(t,\omega;Dh)\cdot STFT_x^*(t,\omega,h)}{\left|STFT_x(t,\omega,h)\right|^2}\right\} \tag{6}$$

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where, $Th = t{\cdot}h(t)$ and $Dh = dh(t) / dt$, $\Re$ and $\Im$ denote the real part and the imaginary part respectively.

III. Modal Frequencies the Escapement

As mentioned before, the frequencies of the watch sound are corresponding to the vibration frequencies. As each shock lasts very shortly, it can be approximated as impulse force. The damping ratio is very small and negligible, so the vibration of escapement parts can be given by the impulse response of undamped system [5]:

$$x(t) = A\cos(\omega_n t - \phi) \tag{7}$$

where, A is the magnitude of the vibration determined by the initial conditions, ω_n the modal frequency of vibration. It is seen that the frequencies of vibration correspond to the modal frequencies of the system.

In order to analyze the sound by its frequencies, we need to find the modal frequencies of mechanical watch parts first. In general, the motion of a mechanical system can be expressed as a 2nd order differential equation [5]:

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{R(t)\} \tag{8}$$

where, M is the equivalent mass matrix, K is the stiffness matrix, C is the damping matrix and $R(t)$ is the external load matrix. When evaluating the modal frequency of the system, $\{R(t)\}$ is set to zero. Also, because the damping is usually rather small, equation (1) can be simplified as:

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = 0 \tag{9}$$

Let $\omega_n^2 = \lambda$, the eigen-function of equation (9) is found:

$$\det([K] + \lambda[M]) = 0 \tag{10}$$

Solving equation (3) will then give the modal frequencies.

Practically, the FEA is used for finding approximate solution of differential equations [6]. There are numerous commercial software for FEA such as ANSYS®, Algor® and etc. In our research, we use SolidWorks® to evaluate the modal frequencies of the escapement and the fourth wheel. The simulation results include the mode shapes and modal frequencies. Fig. 4 shows mode shape of the 5th modal frequencies of the balance wheel, at which the amplitudes are the largest. The modal frequencies (0~24kHz) of the balance wheel, pallet fork, escape wheel and the fourth wheel are summarized in Table 1.

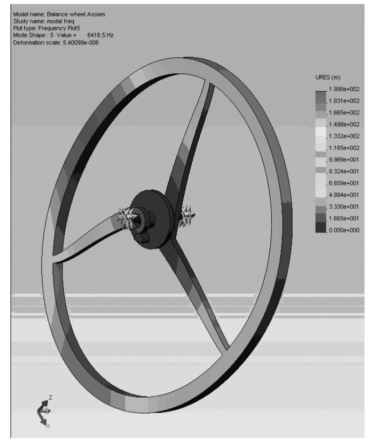


Fig. 4

Table. 1 Modal Frequencies of Mechanical Parts

Mechanical Parts	Modal Frequencies (Hz)
Balance wheel	481, 1752, 1753, 2488, 6417, 6419, 6766, 6768, 18331, 18892, 20223, 20228, 21537
Pallet fork	16779, 21150, 22212
Escape wheel	9117, 11262, 11271, 12667, 18728, 19425
Fourth wheel	3752, 3755, 4361, 5646, 6645, 6658, 14177, 14178

IV. Performance Evaluation of Scalogram and Reassigned Scalogram

A. Performance on Synthetic Watch Signal

Based on the modal frequencies of the watch parts (see Table 1), a signal (sampling rate at 48kHz) is generated as shown in Fig. 5. This signal is very similar to a real watch signal and is used to evaluate the performance of scalogram and reassigned scalogram on analyzing watch signals.

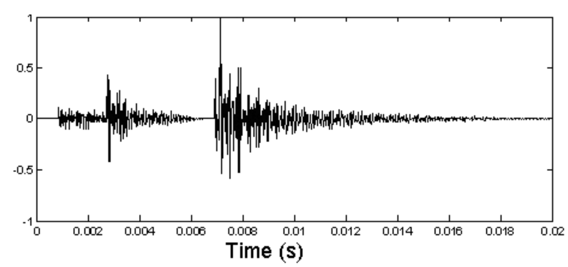
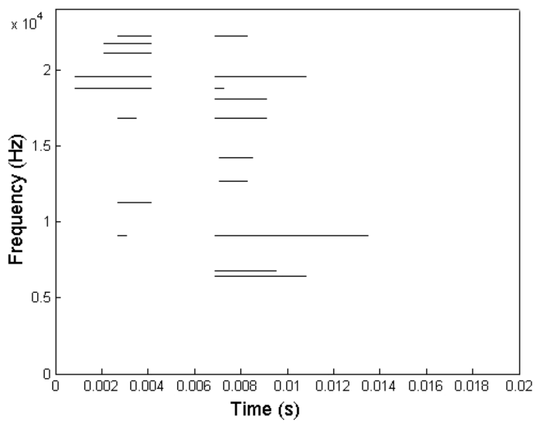
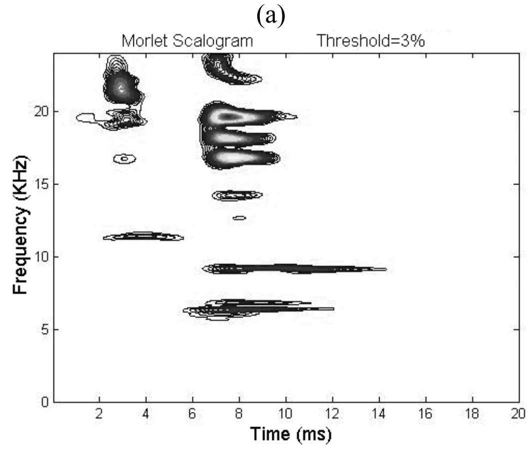


Fig. 5 Synthetic Watch Signal

Fig. 6 shows the idealized TFD, scalogram and reassigned scalogram using Morlet wavelet. From the idealized modal, we can see that there are different frequency components at different time instants; some frequency components are very close to each other; and there are more high frequency components than low frequency components. Therefore, readability of a TFD is very important in analyzing the watch signal. It is seen that that the resolution especially the high frequency resolution is not good enough in the Morlet scalogram. For example, the frequencies 21150Hz, 21537Hz and 22212Hz from 2ms to 4ms are very close and mixed together. The time and frequency resolution of reassigned Morlet scalogram is much better than that of the Morlet scalogram. The difference of frequency resolution between high and low frequency is not so obvious after reassignment. The reassignment provides a TF plot which resembles the idealized model to a greater extent. The reassigned Morlet scalogram provides excellent time and frequency resolution although few interference fringes still exist (pointed with arrows in Fig. 6).



(a)



(b)

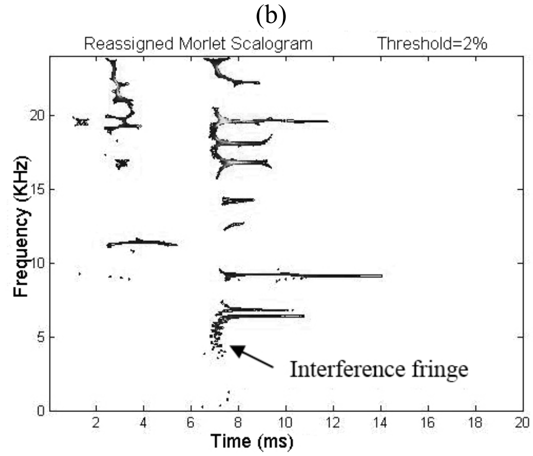


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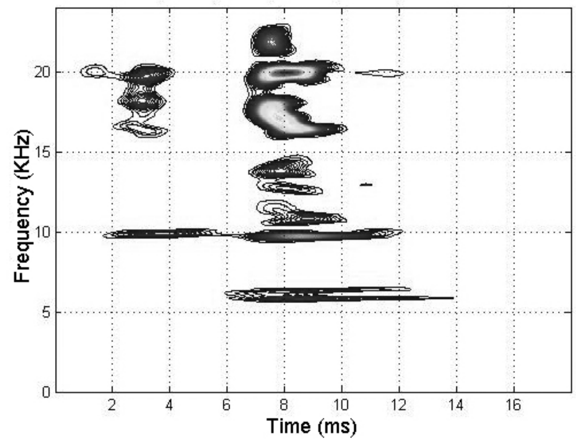
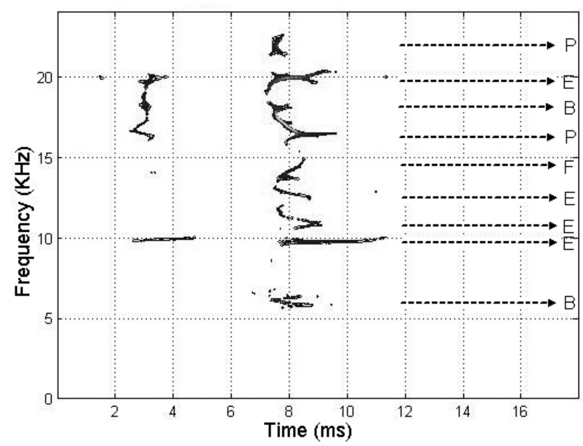
Fig. 6 (a) Idealized TFD; (b) Morlet scalogram; (c) Reassigned Morlet scalogram

B. *Performance on Real Watch Signal*

The performances of the scalogram and reassigned scalogram are also evaluated using a real watch signal shown in Fig. 2. The watch is a good watch whose parts have the same dimension as those used in FEA. The results are shown in Fig. 7.



(a)



(b)

Fig. 7 (a) Morlet scalogram; (b) Reassigned Morlet scalogram

Because of the poor resolution, many frequency components especially high frequency components can hardly be read for the Morlet scalogram (see Fig. 7(a)). On contrary, the interdependency between resolution and frequency is dramatically reduced after reassignment. Nearly all of the frequency components in the reassigned Morlet scalogram can match their counterparts among the modal frequencies given in Table 1 (B represents balance wheel, P represents pallet fork, E represents escape wheel and F represents fourth wheel). It is seen that in the shocks in the third phase, the main modes of the balance wheel are the 5th, 6th, 9th and 10th; the main vibration modes of the escape wheel are the 2nd, 3rd, 4th and 6th; the main vibration modes of the pallet fork are the first three modes; the 7th mode of the fourth wheel vibration also contributes to

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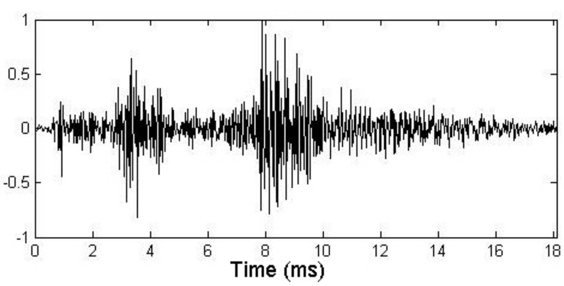
the sound in this phase. It can be seen that time-frequency analysis reveals the insight of the escapement motion [12].

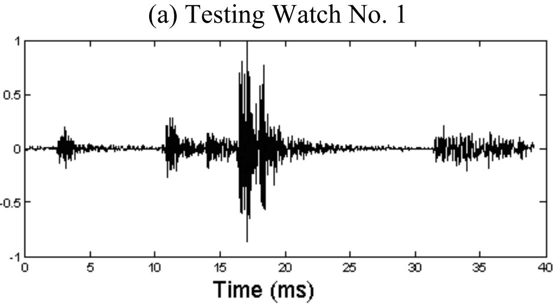
V. Fault Diagnosis

As mentioned before, time-domain analysis does not work robustly in diagnosing certain faults of the mechanical watch. Since reassigned Morlet scalogram reveals the shocks at different time of the movement, we therefore propose time-frequency analysis. We use the examples in Fig. 3 to show the advantages of time-frequency analysis.

Fig. 8 shows the signals of two testing watches and both of them have some faults. The clearance between the horns and the impulse pin is too small in testing watch No. 1. For testing watch No. 2, there are too much endshake in the pivot of the balance wheel. However, their time-domain signals are not like the diagnosis interpretations shown in Fig. 3(a) and (b). The signal of the first testing watch resembles the signal of a good watch very much. After comparing the reassigned Morlet scalogram of the testing watch (Fig. 9(a)) and that of the good watch (Fig. 7(b)), we can tell their differences. It is seen from Fig. 9(a), during the third phase, the frequency of the balance wheel (5th modal frequency) appears twice, the first one at about 8.2ms and the other is at about 12ms. This implies that there is an additional shock at about 12ms and in reality. This is a shock between the impulse pin and the pallet fork notch. Subsequently, the shock causes vibration of the escape wheel as a result of the destitution of clearance between the horns and the impulse pin.

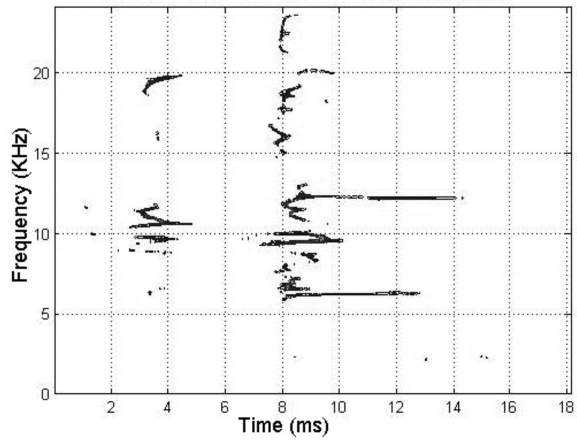


(a) Testing Watch No. 1

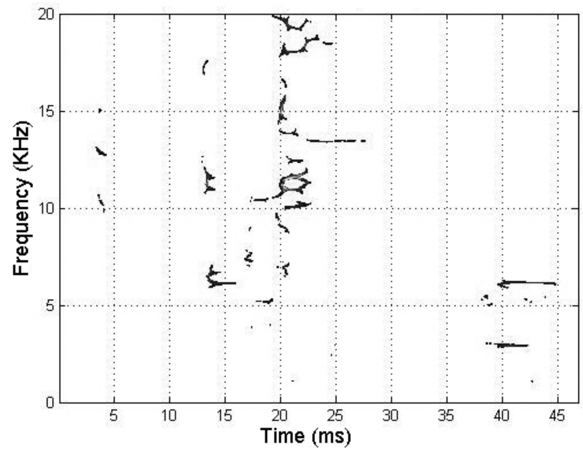


(b) Testing Watch No. 2

Fig. 8 Signals of Two Testing Watches



(a) Testing Watch No. 1



(b) Testing Watch No. 2

Fig. 9 Reassigned Morlet Scalogram of Two Testing Watches

From Fig 9(b), we can see that there is one additional peak appearing in the time-domain signal. With the help of Fig. 9(b), one can easily tell that this watch does not suffer from faults (a) and (c) in Fig. 3, because there are only modal frequencies of the balance wheel (4th and 5th modal freuquencies). This additional peak is caused by excessive axial end-shaking in the pivot of the balance wheel [3].

Other types of faults can also be detected by analyzing reassigned Morlet scalogram in a similar manner.

VI. Conclusion

This paper presents the signature analysis of mechanical watch using sound signals. The following conclusions can be drawn:

(a) The sound frequencies of a mechanical watch movement correspond to modal frequencies of the balance wheel, the pallet fork, escape wheel and the fourth wheel.

(b) Time-frequency analysis can be done by a combination of FEA which gives the modal frequencies and reassigned scalogram which gives the frequencies of the mechanical watch components.

(c) Reassigned spectrogram provides excellent time-frequency resolution with few misleading interference terms. Therefore, it is useful in detecting defects and errors of the mechanical watches.

(d) Time-frequency analysis reveals the insights of the escapement motion and has an obvious advantage over time-domain analysis in diagnosing possible faults of the escapement.

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