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X-Ray Image Restoration in the Wavelet Domain

Y. Laib dit leksir, H. Bendjama, A.Allag.

Abstract—**Wavelet expansions and wavelet transforms have proven to be very efficient and effective in analyzing a very wide class of signals and phenomena. Wavelet expansion allows a more accurate local description and separation of signal characteristics. While Fourier coefficient represents a component that lasts for all time, a wavelet expansion coefficient represents a component that is itself local and is easier to interpret. In this work, we have used images obtained by the microfocus radioscopy system for the quality control of the metallization step of silicon solar cells. In many systems, the observed image can result from the convolution of the true image and the point spread function (PSF) contaminated by noise from various sources. The goal of this paper is to investigate the discrete wavelet transform (DWT) and its application to X-Ray image denoising.**

Index Terms— **discrete wavelet transform (DWT), image denoising, Wavelet, X-ray.**

I. INTRODUCTION

In many applications, such as astronomy, remote sensing, medical imaging, military detection, public security, and video technology, images are the main sources of information. But, due to some reasons, observed images are degraded. The degradations are mainly caused by blur and noise. The aim of image restoration is to obtain restored image which should be as close as the original image. [1]

The use of wavelets for the task of image restoration and enhancement is a relatively new but rapidly emerging concept since their appearance in the image processing literature. Although there has long been the view that a non stationary approach may improve results substantially over those using stationary assumptions, the idea of multiresolution has not been a prevalent one. Instead, past adaptive restoration techniques, for example, have examined the problem in the spatial domain, using various local measures to describe the type of activity near a pixel. However, a number of researchers are now beginning to analyze enhancement problems, and restoration and recovery problems from the multiresolution/subband perspective [2], where a new matrix formulation of a wavelet-based subband decomposition was presented. This formulation allows for the computation of the decomposition of both the signal and the convolution operator in the wavelet domain. This permits the conversion of any linear single channel space-invariant filtering problem into a multichannel one. In particular, this approach can be used to restore a single channel image with any multichannel image restoration routine, like the approach to color image restoration followed in.

Laboratoire de Traitement du Signal et de l'Image (LTSI), Centre de Recherche Scientifique et technique en Soudage et Contrôle, Bp 64 Route de dely Ibrahim Chéraga Alger-Algérie*Tél/Fax : +213.21.36.18.50 E-mail :* yaziddl@yahoo.fr .

Also utilizing the wavelet concept, in [3], a new spatially adaptive restoration approach which uses a multiscale Kalman smoothing filter was discussed. The paper is basically split into three parts.

The first part of this paper deals with image restoration formulation. The second part extends to the concepts to two dimensional signal analysis with discrete wavelet transform (i.e. images). Finally, the applications of the DWT to image-restoration are investigated and compared with Fourier-based techniques.

II. BACKGROUND

A. *Image restoration formulation*

The process of image degradation and restoration can be described by the model which is shown in Figure 1. Where f(x, y) represents an original image and g(x, y) is the degraded image. In the model, n(x, y) represents an additive noise introduced by the system, and h(x, y) is the point spread function of the blur, while f'(x, y) and p(x, y) are restored image and restoration system function, respectively. The degraded image can be modeled by:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y)$$

Where * stands for convolution. Due to processing the image in digital form, the above equation can be written in the matrix-vector form. The expression of the equation (1) in frequency domain by the Fourier Transform is:

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Therefore, if we design a restoration filter:

$$P(u,v) = 1 / H(u,v)$$

Then:

$$F'(u,v) = \frac{F(u,v)H(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)}$$

Given the degraded image *g(x, y),* if we can estimate the *h(x, y),* namely PSF, the restored image will be obtained. Based on this theory, it is crucial to identify the true PSF, a very difficult task for the limitation of prior knowledge [1].

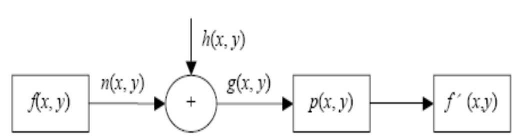

Fig.1. The model of degradation and restoration

III. WAVELET ANALYSIS

Wavelet-based analysis of signals is an interesting, and relatively recent, new tool. Similar to Fourier series analysis, where sinusoids are chosen as the basis function, wavelet analysis is also based on a decomposition of a signal using an orthonormal (typically, although not necessarily) family of basis functions. Unlike a sine wave, a wavelet has its energy concentrated in time. Sinusoids are useful in analyzing periodic and time-invariant phenomena, while wavelets are well suited for the analysis of transient, time-varying signals [4].

A wavelet expansion is similar in form to the well-known Fourier series expansion, but is defined by a two-parameter family of functions

$$f(t) = \sum_k \sum_j a_{i,j} \psi_{j,k}(t) \quad (5)$$

where *j* and *k* are integers and the functions $\psi_{j,k}(t)$ are the wavelet expansion functions. As indicated earlier, they usually form an orthogonal basis. The two-parameter expansion coefficients $a_{j,k}$ are called the discrete wavelet transform (DWT) coefficients of f(t) and Equation (5) is known as the synthesis formula (i.e., inverse transformation). The coefficients are given by:

$$a_{i,j} = \int f(t) \psi_{j,k}(t) dt \quad (6)$$

The wavelet basis functions are a two-parameter family of functions that are related to a function $\psi(t)$ called the generating or mother wavelet by:

$$\psi_{j,k}(t) = 2^{j/2} \psi\left(2^j t - k\right) \quad (7)$$

where *k* is the translation and *j* the dilation or compression parameter. Therefore, wavelet basis functions are obtained from a single wavelet by translation and scaling. There is, however, no single and universal mother wavelet function. The mother wavelet must simply satisfy a small set of conditions and is typically selected based on the signal processing problem domain. Almost all useful wavelet systems satisfy the multi-resolution condition. This means that given an approximation of a signal f(t) using translations of a mother wavelet up to some chosen scale, we can achieve a better approximation by using expansion signals with half the width and half as wide translation steps. This is conceptually similar to improving frequency resolution by doubling the number of harmonics (i.e., halving the fundamental harmonic) in a Fourier series expansion. It is important to note that the wavelet functions never actually enter into the calculation of the discrete wavelet transform. The computation of the transform may be formulated as a filtering operation with two related FIR filters [5].

IV. IMAGE DENOISING

In this part we compare the denoising capabilities of DWT and Fourier Transform. As discussed in the abstract, DWT enjoys an amazing energy compaction property and economical representation of piece-wise smooth signals and images. Based on this, the transform coefficients of a noisy image are thresholded by a factor of α times the noise variance i.e. all the transform coefficients (Fourier and Wavelet) which are less than $\alpha . \sigma^2$ are set to zero. Values of α for Wavelet Transform was varied from 3 to 500 and it gave very good results in terms of the subjective quality of the denoised image. For Fourier transform coefficients, values of α which were experimented with were $3N\sigma$, $3N^2\sigma$ and the subjective quality is not as good as it is for Wavelet Transform.

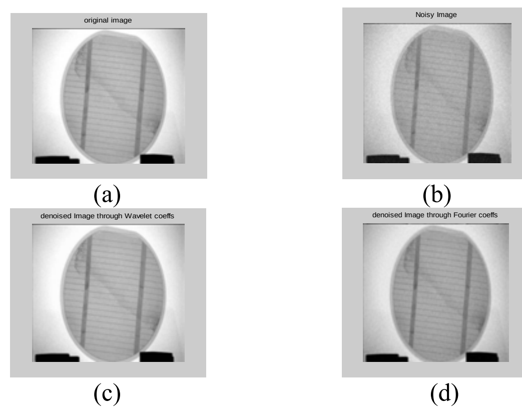

Fig.2. (a) Solar cell: original image; (b) noisy image; (c) denoised image with Wavelet coefficients; (d) denoised image with Fourier coefficients.

Also, we have taken a portion of the solar cell and we had tried to restore this image. The result is shown here:

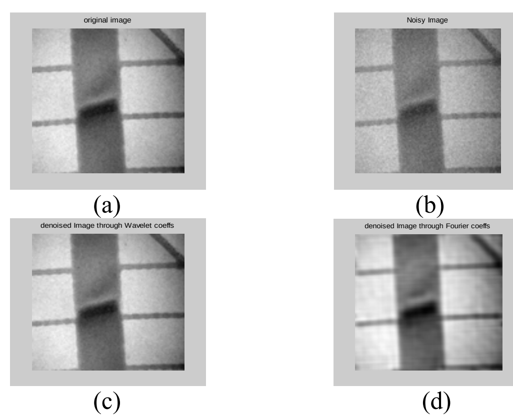

Fig.4. (a) The experiment image. (b) The noisy image. (c) The restored image by Fourier transform approach. (d) The restored image by discrete wavelet transform.

Roughly same number of coefficients were set to zero for both the transforms. For Fourier transform 56287 coefficients were set to zero while 59590 coefficients were set to zero for Wavelet transform. This again highlights the economical representation of signals by wavelet Transform.

This method is based on taking the discrete wavelet transform (DWT) of an image, passing this transform through a threshold, which removes the coefficients below a certain value, then taking the inverse DWT. They are able to remove noise and achieve high compression ratios because of the concentrating ability of the wavelet transform.

V. CONCLUSION

From the obtained results, we can know that the restoring images by the Fourier transform approaches are not satisfactory. Some details are missed and there are some blurs existing in the restored image, while the wavelet transform approach can obtain the better result. Also we deduce that wavelet domain provides economical representations for a wide variety of signals and among all orthogonal transforms, the wavelet transform can capture the maximum signal energy using any fixed number of coefficients.

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