New Index Base Iterative Controller

M. Roopaei, M. Zolghadri

***Abstract*—iterative control methodologies nowadays are represented as powerful tools to control of complex dynamics. Our new controller which based on particular case of iterative learning control attempts to find the best control value in each time step by trying virtually in index domain to reach the desired value of that time in regards to fixed final value. ILC's methods needed to have desired trajectory at any time to reach fixed final state value but this assumption always not reasonable, so in our algorithm this problem is solved.**

***Index Terms*— Iterative learning control, Differential Equations, Optimal Control.**

I. Introduction

In last ten years, many researchers have begun to focus their effort on learning control systems (LCS's) because this type of control technique is capable of progressively improving system performance.

Many researchers have purposed various learning control scheme. An interesting approach among these schemes for tracking control is the iterative learning control (ILC), which was originally, introduced n 1984 by "Arimito". The objective of "ILC" is to determine a control input iteratively, resulting in the plants' ability to track the given reference signal or output trajectory over a finite time interval. "ILC" uses the repetitive nature of the process to progressively improve the tracking performance. The control inputs are iteratively after each operation using the error measurements in the previous cycle. These controllers are able to deal with dynamic systems with imperfect knowledge of dynamic structures and/or parameter operating repetitively over a fixed time interval. ILC has been further explored and is now one of the appealing fields of research in control systems. Section II presents our new iterative control Scheme and conclusion is included in section III.

II. Iterative Controller Design

In this paper, a new type of learning method has been introduced as expressed below:

$$U^{i+1} = U^{i} + q\Delta e^{i} \qquad (1)$$

Where q is called learning factor and iterative index or i indicates a movement in a virtual axis called index axis while a system is considered to be fixed at a specific time t. this virtual movement continues until the appropriate controller would be achieved (the controller when implementing to the system at the fixed time t, causing the state variables to reach their desired values $x_d(t+1)$ at time t+1.

If there would be stability problem, the system has a tendency to lead the state variables to the origin. Of course this is possible (we show that our controller has ability to reach the desired values of states in the first time transient step with a large amount of iteration), but by applying a large amount of control and the controller shows behavior like as impulsive. It is obvious that this kind of controller is not feasible. To obtain a feasible controller for our new iterative learning method, we limit the movement in the index axis in each time transient step. For example consider stabilization problem in the linear following system:

$$\begin{aligned} &\dot{x} = ax + bu \\ &T = total\ time\ or\ final\ time = 0.5s \qquad (2) \\ &a = -2, b = 10, x(T) = 10, x(0) = 1 \end{aligned}$$

At first time we run our algorithm with no bounds on controller and as mentioned before this algorithm try to lead the dynamic with one step of time to the desired fixed final state which is with high magnitude (impulse shape).

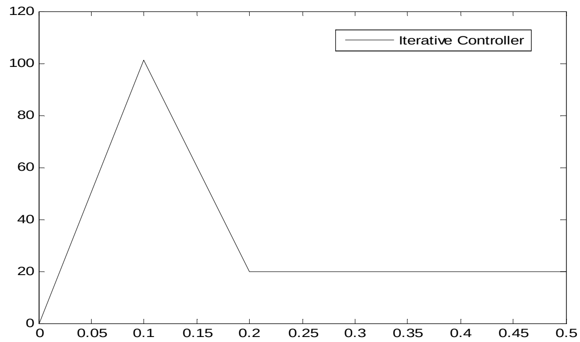

Fig.1- Iterative Controller with no Bounds

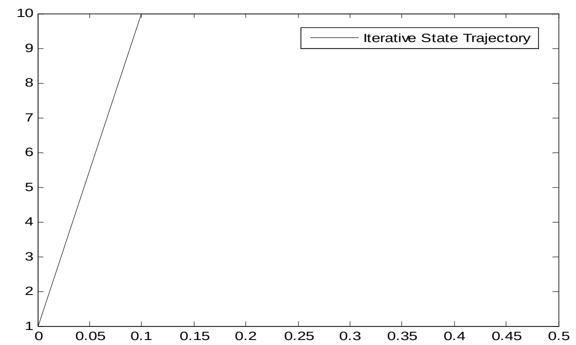

Fig. 2- State Trajectory

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To get ride of impulsive behavior of system, our proposed controller is bounded in iterations and its behavior is compared with optimal controller designed for fixed final state and minimum energy cost function. The results show that our algorithm will be much closed to final value (up to computer accuracy) with less effort compare to optimal solution. The results as below:

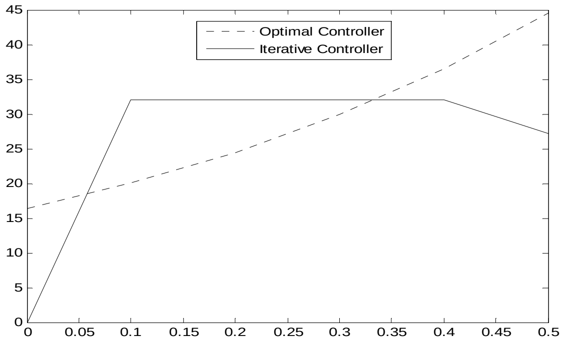

Fig. 3- Optimal / Iterative Controller (which is bounded on 32)

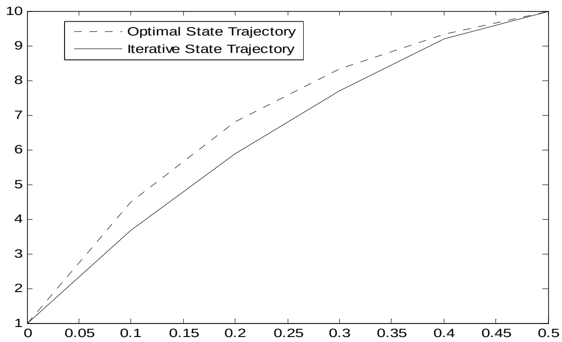

Fig. 4- Optimal / Iterative State Trajectory

It is noticeable that in relation (1), the error, Δe^i, in each iteration is being defined in accordance with the control problem. In a tracking problem, the error is defined as difference between the states and their desired values as stated in the following relation:

$$\Delta e^i(t) = x^i(t) - x_d^i(t) \tag{3}$$

If there would be stability problem, the error is defined in a way that satisfies an inequality related to the Lyapunov theorem condition. If there exist a fixed final state optimal control problem for a linear system, the error might be defined as below:

$$\Delta e^i(t) = x^i(t) - x(T) \tag{4}$$

Where, T in the above relation represents the total time interval.
The new error definitions mentioned in the above problems are considered in the rest of this paper.

3-1-Error in Optimal Control Problem

Consider again the fixed final state optimal control problem shown in relation (2). The error needed for updating the controller is defined as relation (4). We evaluate the value of cost function for both our and optimal control methods as below:

Cost function with learning controller = 48.3226;
Cost function with optimal controller = 54.7614;

It seems that our learning controller shows better performance. It is noticeable that the state trajectory with our control methodology never reaches desired final value exactly and it is the main reason that the minimum energy cost function is more minimized with our control scheme rather than optimal control.
We also can reach very fast to desired value if the bound of controller increases, as results have been shown in below:

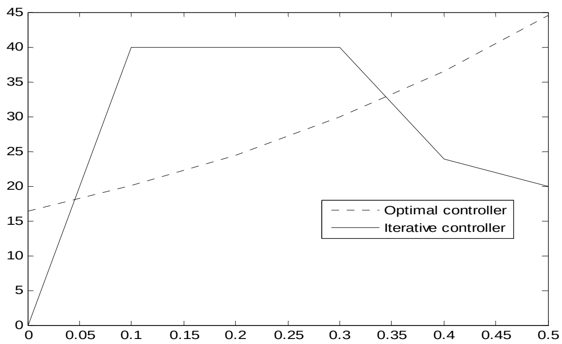

Fig. 5- Optimal / Iterative Controller (which is bounded on 40)

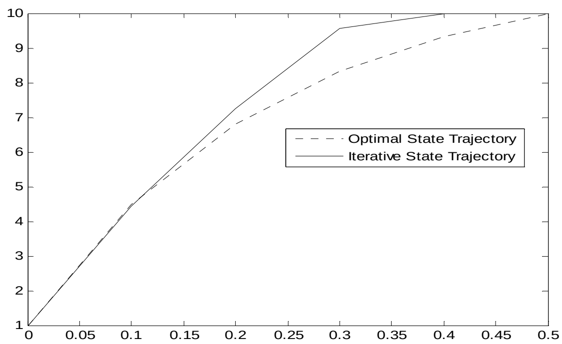

Fig. 6- Optimal / Iterative State Trajectory

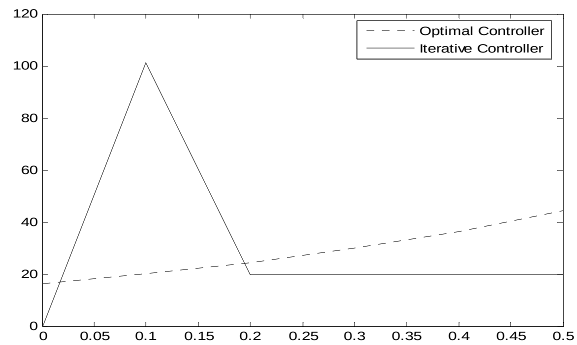

Fig. 7- Optimal/ Iterative Controller (with no bound on controller)

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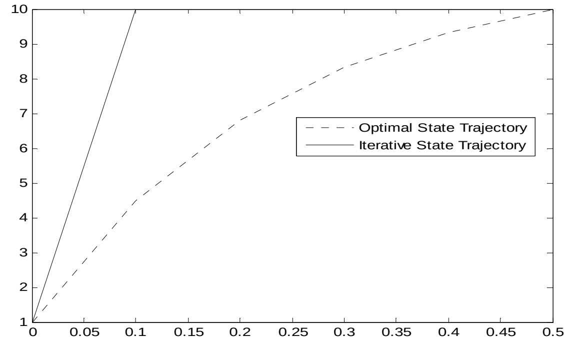

Fig. 8- Optimal / Iterative State Trajectory

Let us once more applied our algorithm in simpler equation as follow:

$$\dot{x} = b(t) \times u;$$
$$T = total\ \ time\ = 10\,s;$$
$$c = 0.01; tc = 10; x(T) = 0.1, x(0) = 200$$
$$b(t) = [-1/c\ * t - 1/c\ * \exp(-c\ * (tc\ - t))]$$
$$- [1/c^{\wedge}2\ * \exp(-c\ * tc)];$$

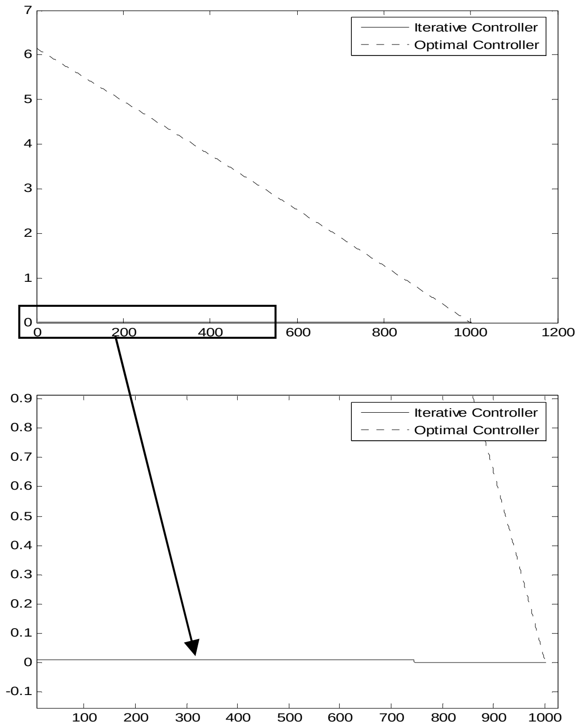

Fig. 9- Optimal / Iterative Controller (with bound on controller u < 0.01)

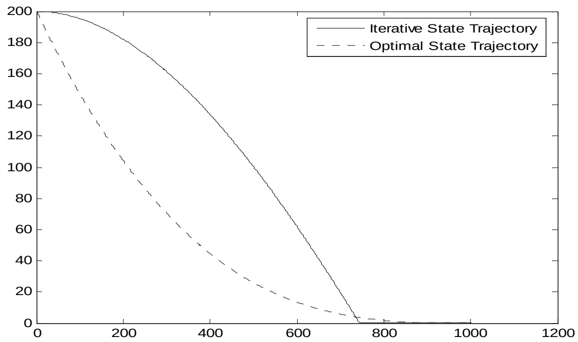

Fig. 10- Optimal / Iterative State Trajectory

Fig. 9 clearly shows the capability of our controller to improve of minimum energy cost function in comparison with optimal controller.
Our control scheme is also applicable to apply on nonlinear systems. We implement our algorithm on the famous chaotic systems as follow:

$$\dot{x}_1 = (25\alpha + 10)(x_2 - x_1)$$
$$\dot{x}_2 = (28 - 35\alpha)x_1 - x_1x_3 + (29\alpha - 1)x_2$$
$$\dot{x}_3 = x_1x_2 - \frac{\alpha + 8}{3}x_3$$
$$T = 0.5s,\ x(T) = [0;0;0]\ ;\ x(0) = [-10;2;1]\ ;$$

where α is a variable parameter and $\alpha \in [0,1]$. Variation of α, makes three chaotic system with different properties. When $\alpha \in [0,0.8)$ the chaotic system is called LORENZ, for $\alpha = 0.8$ it is defined as LU and $\alpha \in (0.8,1]$ is expressed as CHEN [2].

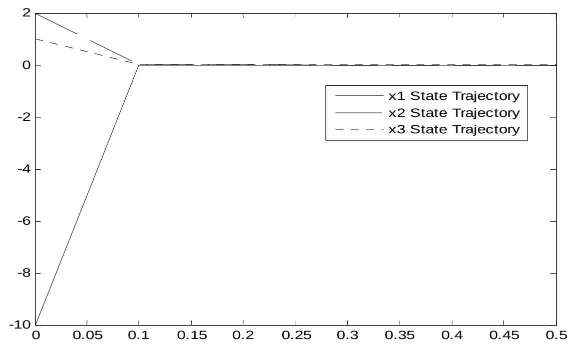

Fig. 11- State Trajectory for LORENZ system with no bounds on controller

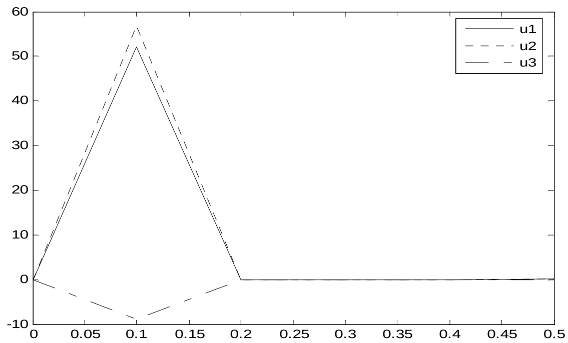

Fig. 12- Iterative Controller (with no bound on controller)

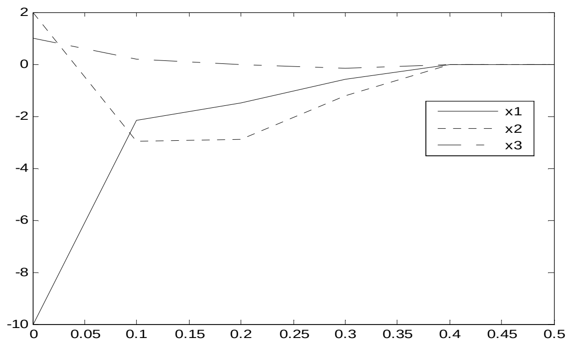

Fig. 13- State Trajectory for LORENZ system with bounds on controller

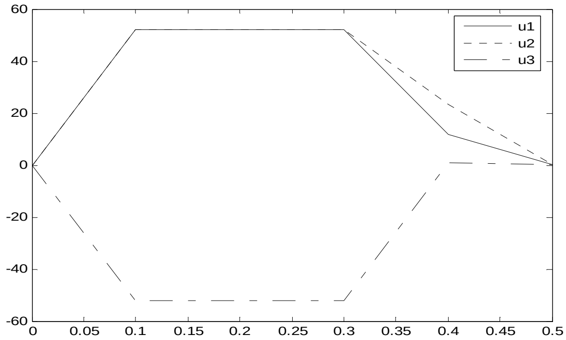

Fig. 14- Iterative Controller with bound on controller u < 52

III. CONCLUSION

The new iterative control method is introduced in this paper. Movement through index axis and updating the controller while the system is assumed to have static behavior between two time steps are principles of our control scheme in this paper.

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